Temos o problema de contorno definido por, para $(x, y) \in (0,1)^2$,

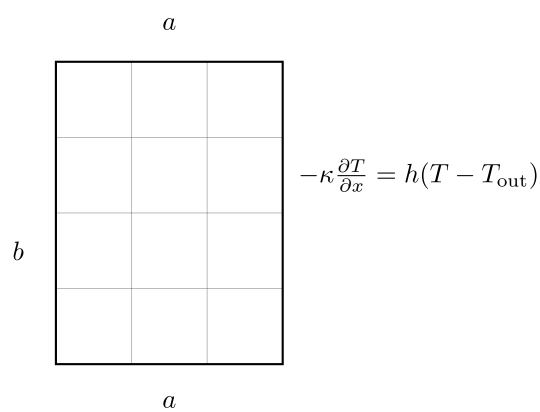
$$-\nabla(\kappa \nabla T) = 0\,; \tag{1}$$

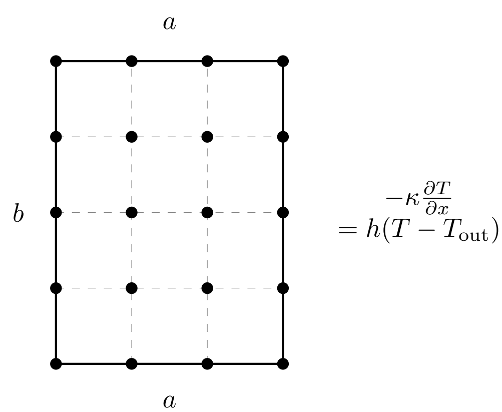
$$T(x,0) = T(x,1) = a\,; \tag{2}$$

$$T(0,y) = b\,;\; -\kappa \frac{\partial T}{\partial x}(1,y) = h(T - T_{\text{out}})\,. \tag{3}$$

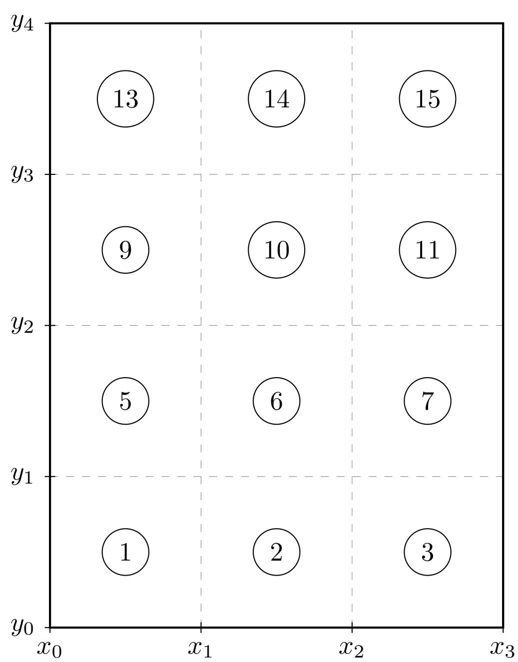
Consider the boundary conditions:

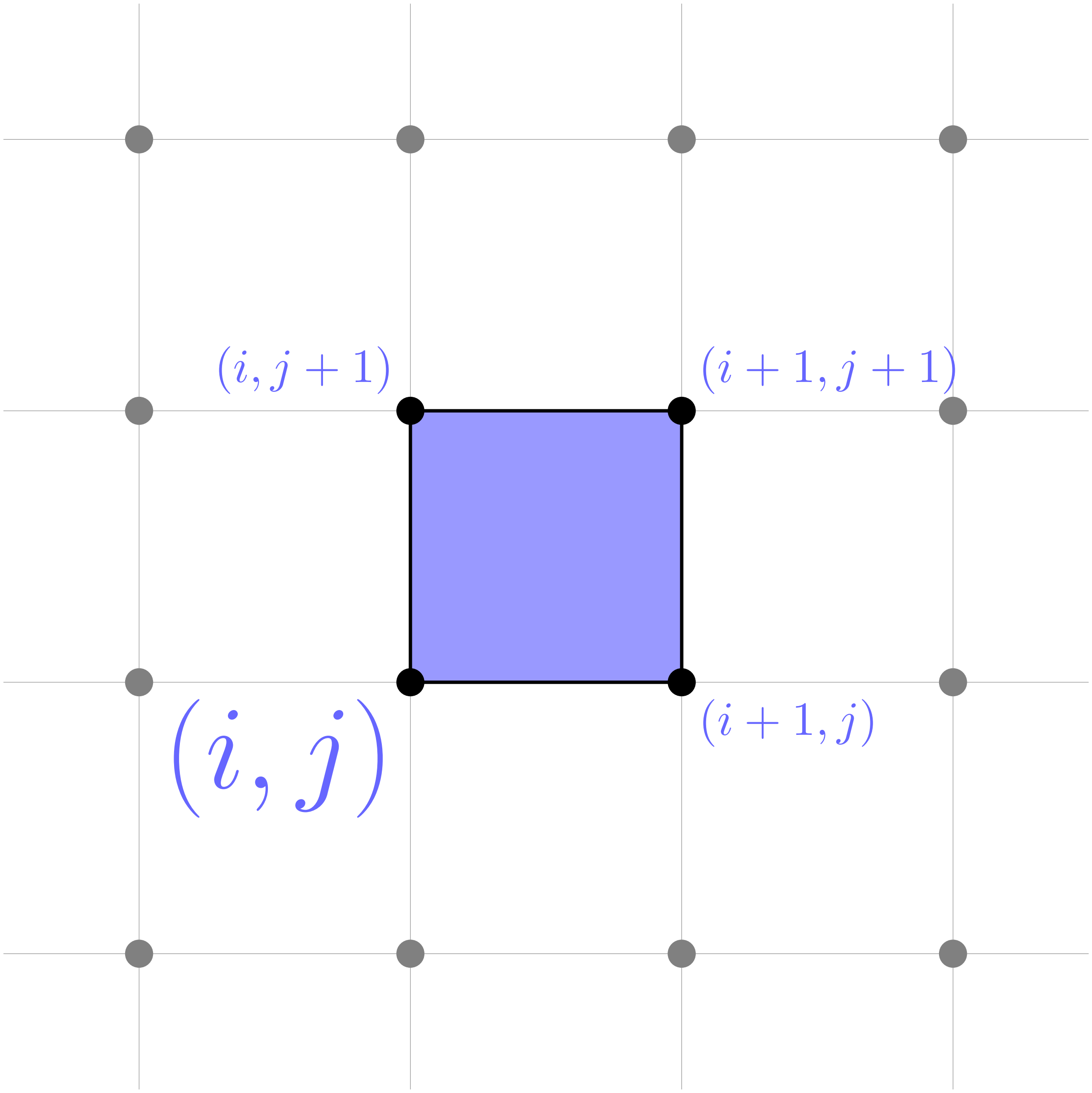
$$T(x,1) = a$$
$$T(0,y) = b$$
$$-\kappa \frac{\partial T}{\partial x}(1,y) = h(T - T_{\text{out}})$$





$$
\begin{aligned}
-\nabla^2 u(x,y) &= f(x,y), && \text{(PDE)}\\
u(x,0) &= g_1(x), && \text{(Dirichlet condition at } y=0\text{)}\\
u(x,h) &= g_2(x), && \text{(Neumann condition at } y=h\text{)}\\
-\frac{\partial u}{\partial y}(x,0) &= h_1(x), && \text{(Neumann condition at } y=0\text{)}\\
\frac{\partial u}{\partial y}(x,h) &= h_2(x), && \text{(Dirichlet condition at } y=h\text{)}
\end{aligned}
$$





Let α_x, α_y, and β be given constants, and a, b, and T_{out} be given boundary conditions. Then the linear system $AT = b$ for the given equations can be written as:

$$
A = \begin{bmatrix}
-2(\alpha_x+\alpha_y) & \alpha_x & 0 & \alpha_y & 0 & 0 & 0 & 0 & 0 \\
\alpha_x & -2(\alpha_x+\alpha_y) & \alpha_x & 0 & \alpha_y & 0 & 0 & 0 & 0 \\
0 & \alpha_x & -(2\alpha_x(1+\beta/2)+2\alpha_y) & 0 & 0 & \alpha_y & 0 & 0 & 0 \\
\alpha_y & 0 & 0 & -2(\alpha_x+\alpha_y) & \alpha_x & 0 & \alpha_y & 0 & 0 \\
0 & \alpha_y & 0 & \alpha_x & -2(\alpha_x+\alpha_y) & \alpha_x & 0 & \alpha_y & 0 \\
0 & 0 & \alpha_y & 0 & \alpha_x & -(2\alpha_x(1+\beta/2)+2\alpha_y) & 0 & 0 & \alpha_y \\
0 & 0 & 0 & \alpha_y & 0 & 0 & -2(\alpha_x+\alpha_y) & \alpha_x & 0 \\
0 & 0 & 0 & 0 & \alpha_y & 0 & \alpha_x & -2(\alpha_x+\alpha_y) & \alpha_x \\
0 & 0 & 0 & 0 & 0 & \alpha_y & 0 & \alpha_x & -(2\alpha_x(1+\beta/2)+2\alpha_y)
\end{bmatrix}, \tag{4}
$$

$$
T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix},
$$

$$
b = \begin{bmatrix}
-(\alpha_x a + \alpha_y b) \\
-\alpha_y a \\
-\alpha_x \beta T_{\text{out}} - \alpha_y a \\
-\alpha_x b \\
0 \\
-\alpha_x \beta T_{\text{out}} \\
-(\alpha_x b + \alpha_y a) \\
-\alpha_y a \\
-(\alpha_x \beta T_{\text{out}} + \alpha_y a)
\end{bmatrix}.
$$