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Optimal preventive maintenance of leased equipment with corrective minimal repairs

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Abstract

For leased equipment, the lessor carries out the maintenance of the equipment. Usually, the contract of lease specifies the penalty for equipment failures and for repairs not being carried out within specified time limits. This implies that optimal preventive maintenance policies must take these penalty costs into account and properly traded against the cost of preventive maintenance actions. The costs associated with failures are high as unplanned corrective maintenance actions are costly and the resulting penalties due to lease contract terms being violated. The paper develops a model to determine the optimal parameters of a preventive maintenance policy that takes into account all these costs to minimize the total expected cost to the lessor for new item lease. The parameters of the policy are (i) the number of preventive maintenance actions to be carried out over the lease period, (ii) the time instants for such actions, and (iii) the level of action.

Keywords: Unreliable equipment; Lease; Preventive maintenance

1. Introduction

Businesses need different types of equipment to produce products and services. Every equipment is unreliable in the sense that it degrades with age and/or usage, and ultimately fails (Blischke and Murthy, 2000). Equipment failures have a significant impact on the business performance. Preventive maintenance (PM) actions are used to control equipment degradation and failures. Corrective maintenance (CM) actions are

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used to restore failed equipment back to operational status. PM actions result in additional costs and are worthwhile only if this is less than the reduction in CM costs. A variety of models were developed to determine optimal PM effort. References to these can be found in review/survey papers (McCall, 1965; Pierskalla and Voelker, 1976; Sherif and Smith, 1976; Jardine and Buzacott, 1985; Gits, 1986; Thomas, 1986; Valdez-Flores and Feldman, 1989; Cho and Parlar, 1991; Pintelton and Gelders, 1992; Dekker et al., 1997; Scarf, 1997).

Prior to 1970, equipment was mostly owned and maintained in-house. Since 1970, businesses began to view maintenance as being not a core element of the business and started outsourcing it (Murthy and Yeung, 1995; Martin, 1997; Murthy and Asgharizadeh, 1999). This trend accelerated as the complexity of equipment increased which made it uneconomical to carry out in-house maintenance due to the need for expensive maintenance equipment and highly trained maintenance staff.

Since 1990, there has been a trend towards leasing equipment rather than owning the equipment. There are several reasons for leasing as opposed to owning such as the rapid technological obsolescence and the high cost of ownership (Nisbet and Ward, 2001). As a result, maintenance was no longer an issue for the lessee (leasing the equipment), but for the lessor (owner of the equipment) who carried out the maintenance. In other words, the equipment (a physical item) was bundled with maintenance (a service) and offered by the lessor as a package to the lessee under a contract.

The leasing of equipment raises several new issues for both the lessor and the lessee (Desai and Purohit, 1998; Kleiman, 2001). Often, the contract specifies the penalties should the leased equipment not perform as required (for example, failing frequently) or the maintenance service not being satisfactory (for example, repairs are not performed within reasonable times limits) as these affect the business performance of the lessee. In this paper we focus our attention on maintenance of leased equipment and study the impact of the terms of the contract on the optimal maintenance strategies for the lessor.

The outline of the paper is as follows. We start with a brief discussion of equipment leasing in Section 2. In section 3 we give the details of a model formulation. The details of the model analysis are given in Section 4. Section 5 deals with a numerical example where we present some results of sensitivity analysis. We conclude with a brief discussion of some topics for future research in Section 6.

2. Equipment leasing

Equipment leasing involves the loan of equipment owned by the lessor to the lessee under a lease contract. As such, it involves four elements which interact with each other as shown in Fig. 1. In this section we briefly discuss each of the four elements.

2.1. Equipment

Industrial and commercial products such as trucks, pumps, and computers are typically leased to assist the lessee with its business operations. The equipment generates revenue when it is in working state and the lessee incurs a loss (for example, idle workforce, dissatisfied customers, etc.) when the equipment is in failed state.

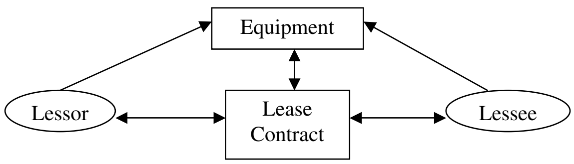

Fig. 1. Conceptual model of equipment leasing.

In general, equipment is complex in the sense that it can be viewed as a system comprising of several components. Equipment failures occur due to the failure of one or more of its components.

2.2. Lease contract

The contract deals with the terms and conditions of the lease. These include the period of the lease, the performance requirement that the leased equipment should meet, the actions that each party (lessor and lessee) is obligated to take and the cost to the lessee for leasing the equipment and the payment terms. The performance requirements can involve several measures such as (i) the upper limit on the number of failures over the lease period, (ii) the time interval between successive failures, (iii) the time to repair each failure and so on. When these are not met the lessor incurs penalties which are stated explicitly in the contract.

2.3. Lessee

Basically, the lessee has to decide on choosing the best lease arrangement from a set of alternate options available. The decision is based on an objective function that takes into account the lease price and the implications of the terms of the contract on the operations, customer satisfaction and overall business performance.

2.4. Lessor

The lessor has to address several issues some of which are strategic and others operational. At the strategic level, issues deal with the size and composition of the equipment fleet, the number and the location of lease centres, workshop facilities and, warehouse for spares. At the operational level, issues involve pricing, marketing, scheduling of jobs at the repair facility and so on.

2.5. Maintenance of leased equipment

The maintenance of lease items is an operational issue of great importance to the lessor as it not only affects the profits, but also the satisfaction of lessees and this in turn impacts on future business.

As mentioned earlier equipment degrades with age and/or usage and fail. The costs associated with failures of leased equipment are high for two reasons. First, since corrective maintenance actions are unplanned maintenance actions, each such action usually costs a lot more than a planned PM action. Second, failures can result in penalty costs due to the reliability performance specified in the lease contract being violated. PM actions can reduce this cost. This implies that the optimal PM actions need to be determined through a proper trade-off between the CM and PM costs.

One can define many different PM policies for leased equipment. In the next section we give the details of one such policy. The policy is characterized by a set of parameters $\theta = \{k, \underline{t}, \underline{\delta}\}$ where the last two are k-dimensional vectors. The parameters need to be selected optimally and this is the focus of the remainder of the paper.

3. Model formulation

We use the following notation:

$F(t)$ failure distribution function
$f(t)$ failure density function associated with $F(t)$

$r(t)$ failure rate [hazard] function associated with $F(t)$
$\lambda_0(t)$ failure intensity function with no PM $[=r(t)]$
$\lambda(t)$ failure intensity function with PM actions
$\Lambda(t)$ cumulative failure intensity function $[=\int_0^t \lambda(x)\mathrm{d}x]$
$N(t)$ number of failures over $[0, t)$
Y time to repair
$G(y)$ repair time distribution function
$g(y)$ repair time density function $[=\mathrm{d}G(y)/\mathrm{d}y]$
L lease period
k number of PM actions over the lease period
t_j time instant for jth PM action
δ_j reduction in intensity function due to jth PM action
$C_p(\delta)$ cost of PM action resulting in a reduction δ in intensity function
TC_p total cost of PM actions
C_f average cost of CM action to rectify failure
TC_f total cost of CM actions
τ repair time limit [parameter of lease contract]
C_t penalty cost per unit time if repair not completed within τ [Penalty-1]
C_n penalty cost per failure if $N(t) > 0$ [Penalty-2]
ϕ_1 total cost due to Penalty-1
ϕ_2 total cost due to Penalty-2
ϕ_3 total cost due to Penalty-1 and Penalty-2
J total expected cost to the lessor

In this section we give the details of the model formulation. We confine our attention to the case of new equipment lease. The lease of used equipment allows for additional options and is discussed in the last section.

3.1. Lease contract

The equipment is leased for a period L. The contact involves the following two penalties for the lessor. The lessor incurs the first type of penalty (Penalty-1) if he fails to restore the equipment from failed state to working state within a reasonable time. We model this as follows. Let Y (a random variable) denote the time to restore the equipment from failed state to working state. The penalty cost is given by $C_t\{\max[0, Y-\tau]\}$, where both $C_t > 0$ and $\tau > 0$. This implies that the lessor incurs no penalty if the failed equipment is restored back to working state within a period τ. $\tau \to \infty$ implies no penalty since $Y < \infty$. The lessor incurs the second type of penalty (Penalty-2) if failures occur during the lease period, $N(L)$. The penalty cost is given by $C_n\{\max[0, N(L)]\}$. Note that $N(L) = 0$ implies no penalty for failures over the lease period.

3.2. Equipment failures

The modeling of equipment failures can be done at two levels—system level and component level. In system level modeling, failures of equipment are viewed as random events over time and the specific component that failed and the failure mode is not modeled. In component level modeling, the failures of each component are modeled separately. In either case, the modeling can be done by viewing the system (or component) as a black-box or a white-box. In the former case, the time to failure is modeled by a distribution function, and in the latter case, the physical mechanism leading to failure is modeled explicitly. We confine our attention to modeling at the system level and using a black-box approach.

The time to first failure is a random variable and modeled by a distribution function $F(t)$. The failure density function $f(t)$ is given by $f(t) = \mathrm{d}F(t)/\mathrm{d}t$ and the hazard function $r(t)$ is given by $r(t) = f(t)/[1-F(t)]$. Subsequent failures of the equipment depend on the type of CM actions (to fix failures) and the PM actions (to avoid failures).

We first model the CM actions. We assume that all failures are rectified through minimal repair. Under minimal repair, the hazard function immediately after repair is the same as that just before failure (Barlow and Hunter, 1960). We further assume that the time needed to rectify failed equipment is small in relation to the mean time between failures and such that it can be ignored. In this case, equipment failures with no PM actions occur according to a non-homogeneous Poisson process (NHPP) with intensity function $\lambda_0(t) = r(t)$ where $r(t)$ is the hazard function associated with the distribution function $F(t)$ (Murthy, 1991).

3.3. PM policy

We consider the following PM Policy. The equipment is subjected to k PM actions over the lease period. The time instants at which these actions are carried out are given by $\{t_j, 1 \leqslant j \leqslant k\}$ with $t_i < t_j$ for $i < j$. At the jth PM action, the intensity function is reduced by δ_j. This is constrained so that the failure rate after PM action is never less than that for a new item. The policy is characterized by the set of parameters $\theta \equiv \{k, t_j, \delta_j, 1 \leqslant j \leqslant k\}$.

As a result, the failures over the lease period occur according to a NHPP with intensity function given by

$$\lambda(t) = \lambda_0(t) - \sum_{i=0}^{j} \delta_i \quad \text{for } t_j \leqslant t < t_{j+1} \tag{1}$$

with $t_0 = \delta_0 = 0$ and the δ_j constrained as follows

$$0 \leqslant \delta_j \leqslant \lambda_0(t_j) - \sum_{i=1}^{j-1} \delta_i, \quad 1 \leqslant j \leqslant k. \tag{2}$$

The cost of a PM action is a function of δ. We assume that it is given by $C_p(\delta) = a + b\delta$ where $a > 0$, is the fixed cost and $b \geqslant 0$, is the variable cost. Fig. 2 shows a plot of intensity function for failure with and without PM actions.

3.4. Cost to the lessor

The cost to the lessor is comprised of the following three costs:

(i) *Cost of CM actions*
Let $N(L)$ denote the number of failures over the lease period. The cost of CM actions is assumed as an average cost of a rectification, C_f, given by

$$TC_f = C_f N(L). \tag{3}$$

(ii) *Cost of PM actions*
The cost of PM actions is given by

$$TC_p = \sum_{j=1}^{k} (a + b\delta_j). \tag{4}$$

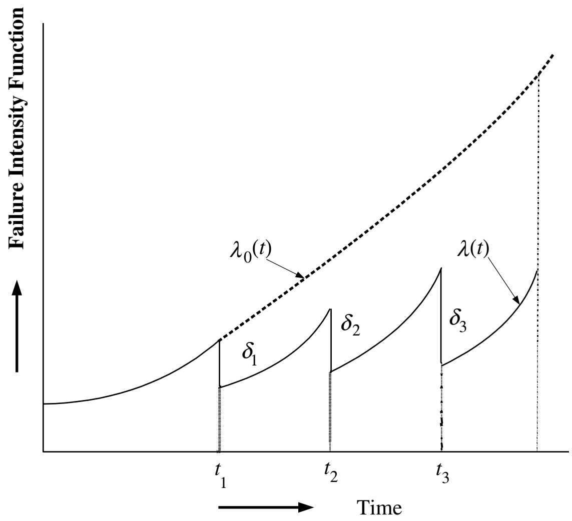

Fig. 2. Plot of failure intensity function [$k = 3$].

(iii) *Penalty costs*
Penalty-1 cost is a function of the repair time Y (a random variable with a distribution $G(y)$ called the repair time distribution) and τ.

Let Y_i denote the time to rectify the ith failure, $1 \leqslant i \leqslant N(L)$. Then, the total Penalty-1 cost incurred is given by

$$\phi_1(N(L), Y_i, \tau) = C_t \left\{ \sum_{i=1}^{N(L)} \max[0, Y_i - \tau] \right\}. \tag{5}$$

When the failures occur over the lease period, the lessor incurs the Penalty-2 costs and this is given by

$$\phi_2(N(L)) = C_n\{\max[0, N(L)]\}. \tag{6}$$

As a result, the total penalty cost is the sum of these two and is given by

$$\phi_3 = \phi_1 + \phi_2 = C_t \left\{ \sum_{i-1}^{N(L)} \max[0, Y_i - \tau] \right\} + C_n\{\max[0, N(L)]\}. \tag{7}$$

4. Model analysis

4.1. Expected cost to the lessor

With no PM actions, failures over the lease period occur according to a NHPP with intensity function, $\lambda_0(t)$. The expected number of failures over the lease period is given by

$$E[N(L)] = \Lambda_0(L) = \int_0^L \lambda_0(t)\mathrm{d}t. \tag{8}$$

With PM actions, the expected number of failures over the lease period is also given by a NHPP process with intensity function given by (1). As a result, we have

$$E[N(L)] = \Lambda(L) = \Lambda_0(L) - \sum_{j=1}^{k} \delta_j (L - t_j). \tag{9}$$

Note that the second term in the right-hand side of (9) represents the reduction in the expected number of failures due to PM Policy.

From (3), the expected cost resulting from CM actions is given by

$$E(TC_f) = C_f \Lambda(L) \tag{10}$$

with $\Lambda(L)$ given by (9).

From (5), the expected costs associated with Penalty-1 cost is given by

$$E[\phi_1(N(L), Y_i, \tau)] = C_t \Lambda(L) \left\{ \int_\tau^\infty (y - \tau) g(y) \mathrm{d}y \right\}. \tag{11}$$

On integrating by parts we have

$$\int_\tau^\infty (y - \tau) g(y) \mathrm{d}y = [(y - \tau) G(y)]_\tau^\infty - \int_\tau^\infty G(y) \mathrm{d}y = \int_\tau^\infty [1 - G(y)] \mathrm{d}y. \tag{12}$$

Using this in (11) results in

$$E[\phi_1(N(L), Y_i, \tau)] = C_t \Lambda(L) \left\{ \int_\tau^\infty (1 - G(y)) \mathrm{d}y \right\}. \tag{13}$$

From (6), the expected costs associated with Penalty-2 cost is given

$$E[\phi_2(N(L))] = C_n \sum_{n=0}^{\infty} n P[N(L) = n]. \tag{14}$$

Since failures occur according to a NHPP with intensity function given by (1), we have from Murthy (1991)

$$P[N(L) = n] = \frac{\exp^{-\Lambda(L)} [\Lambda(L)]^n}{n!} \tag{15}$$

for $n = 0, 1, 2, \ldots$ with $\Lambda(L)$ given by (9).

Using (15) in (14) results in

$$E[\phi_2(N(L))] = C_n \Lambda(L). \tag{16}$$

Combining all these costs yields the total expected cost given by

$$J(k, \underline{t}, \underline{\delta}) = C_f \Lambda(L) + \sum_{j=1}^{k} (a + b\delta_j) + C_t \Lambda(L) \left\{ \int_\tau^\infty (1 - G(y)) \mathrm{d}y \right\} + C_n \Lambda(L). \tag{17}$$

4.2. Optimal PM action

The optimal parameters for the policy are obtained using a three-stage process as indicated below:

Stage 1: Fix k and $\underline{t}$. As a result, $J(k,\underline{t},\underline{\delta})$ is only a function of $\underline{\delta}$. The optimal values $\underline{\delta}^*(k, \underline{t}) = \{\delta_1^*, \delta_2^*, \ldots, \delta_k^*\}$ are the ones that minimise $J(k, \underline{t}, \underline{\delta})$. Note that they are functions of k and $\underline{t}$.
Stage 2: Fix k. The optimal $\underline{t}^*(k) = \{t_1^*, t_2^*, \ldots, t_k^*\}$ are obtained by minimising $J(k\, \underline{t}, \underline{\delta}^*\ (k, \underline{t}))$.
Stage 3: The optimal k^* is now obtained by minimising $J(k, \underline{t}^*(k), \underline{\delta}^*\ (k, \underline{t}^*\ (k)))$.

The following three special cases are considered.

4.3. Special Case 1: [no penalty]

The total expected cost to the lessor is given by (17) with $C_n = C_t = 0$. As a result, we have

$$J(k, \underline{t}, \underline{\delta}) = C_f \left[\Lambda_0(L) - \sum_{j=1}^{k} \delta_j (L - t_j) \right] + ak + b \sum_{j=1}^{k} \delta_j. \tag{18}$$

Stage 1: Fix k and $\underline{t}$. As a result the optimisation problem reduces to finding $\underline{\delta}$ that minimises

$$J_1(\underline{\delta}) = J(k, \underline{t}, \delta(k, \underline{t}) | k, \underline{t}) = C_f \Lambda_0(L) + ak - C_f \sum_{j=1}^{k} \delta_j \left(L - t_j - \frac{b}{C_f} \right) \tag{19}$$

subject to the constraints

$$0 < t_1 < t_2 < \cdots < t_k < \left(L - \frac{b}{C_f} \right) \quad \text{and} \quad 0 \leqslant \delta_j \leqslant \lambda(t_j^-) = \lambda_0(t_j) - \lambda_0(t_{j-1}), \quad j = 1, 2, \ldots, k. \tag{20}$$

$J_1(\underline{\delta})$ is linear in $\underline{\delta}$ and constrained as indicated in (20). As a result, the optimal values are the extremal points of the constraint intervals. This yields

$$\delta_j^* = \begin{cases} \lambda_0(t_j) - \lambda_0(t_{j-1}) & \text{if } L - t_j - (b/C_f) > 0 \quad \text{for } 1 \leqslant j \leqslant k, \\ 0 & \text{if } L - t_j - (b/C_f) \leqslant 0 \quad \text{for } j > k. \end{cases} \tag{21}$$

Define $\widetilde{L} = L - (b/C_f)$. Then, the optimal PM action at t_j is to reduce the failure intensity by the maximum amount if $t_j < \widetilde{L}$ and not to carry out any PM action if $t_j \geqslant \widetilde{L}$. Note that $\widetilde{L}$ decreases as b increases and/or C_f decreases. When $\widetilde{L} < 0$ then no PM action is the optimal strategy. This situation arises when b is large (variable component of PM cost) or C_f (CM cost of each repair) is small so that it is cheaper to fix failures as opposed to carrying out PM actions to reduce the likelihood of failures.
Stage 2: Let $\tilde{k}$ be the largest j such that $t_j < \widetilde{L}$. Define $t_{\tilde{k}+1} = L$. Then, $J(k, \underline{t}, \underline{\delta})$ can be rewritten as

$$J_2(\underline{t}) = J(\tilde{k}, \underline{t}(\tilde{k}), \underline{\delta}^*(\tilde{k}, \underline{t}(\tilde{k})) | \tilde{k}) = C_f \left\{ \Lambda_0(L) - \left(L - \frac{b}{C_f} \right) \lambda_0(t_{\tilde{k}}) + \sum_{j=1}^{\tilde{k}} t_j [\lambda_0(t_j) - \lambda_0(t_{j-1})] \right\} + \tilde{k} a. \tag{22}$$

The optimal values (t_j^*) are obtained from the usual first order condition—differentiating $J_2(\underline{t})$ with respect to t_j, $1 \leqslant j \leqslant \tilde{k}$, and setting them to zero.
Stage 3: k^*, the optimal $\tilde{k}$ is obtained by minimising $J_3(\tilde{k})$ given by

$$J_3(\tilde{k}) = J(\tilde{k}, \underline{t}^*(\tilde{k}), \underline{\delta}^*(\tilde{k}, \underline{t}^*(\tilde{k}))) = C_f \left[\Lambda_0(L) - \sum_{j=1}^{\tilde{k}} \delta_j^* (L - t_j^*) \right] + \sum_{j=1}^{\tilde{k}} (a + b\delta_j^*). \tag{23}$$

This is a complex function and it is difficult to show that $J_3(\tilde{k})$ is convex in $\tilde{k}$. Note that the maximum reduction in the expected CM costs is given by $C_f \Lambda_0(L)$. The cost of PM actions is greater than $a\tilde{k}$. As a result, $\tilde{k} < k_m$ where k_m is the smallest integer greater than $C_f\, \Lambda_0(L)/a$. This implies that an enumerative search for k varying from 0 to k_m will yield the global optimal k^*.

4.3.1. Weibull intensity function

In this case $\lambda(t)$ is given by

$$\lambda_0(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \tag{24}$$

with the scale parameter $\alpha > 0$ and shape parameter $\beta > 1$ (indicating an increasing failure rate). Without loss of generality we assume $\alpha = 1$. If $\alpha \neq 1$ then a change in time scale given by $t' = t/\alpha$ results in failures occurring with an intensity function given by (24) with scale parameter equal to 1. In this case, the lease period L changes to $L' = L/\alpha$. In the remainder of the paper we assume $\alpha = 1$.

Stage 2: Define

$$V_j = \frac{t^*_{j-1}}{t^*_j}, \quad j = 1, 2, \ldots, \tilde{k} \tag{25}$$

with $t_0 = 0$. From the first order conditions, and after some analysis, we have the V_j's given by the following recursive relationship:

$$V_{j+1} = \frac{\beta - 1}{\beta - V_j^{\beta-1}}, \quad j = 1, 2, \ldots, \tilde{k} - 1 \tag{26}$$

with $V_1 = 0$ and

$$t^*_{\tilde{k}} = \left(\frac{\beta - 1}{\beta - V_{\tilde{k}}^{\beta-1}}\right)\tilde{L}. \tag{27}$$

Stage 3: The following enumerative approach can be used to obtain the optimal k.

Step 1: Generate V_j, $j = 1, 2, 3, \ldots$ using (25).
Step 2: Set $\tilde{k} = 0$ and compute $J(0, \underline{t}^*, \underline{\delta}^*) = C_f \Lambda_0(L)$.
Step 3: $\tilde{k} \leftarrow \tilde{k} + 1$.
Step 4: Compute $t^*_{\tilde{k}}$ and $t^*_j, j = \tilde{k} - 1, \tilde{k} - 2, \ldots, 1$ using (27) and (25).
Step 5: Evaluate $J_3(\tilde{k})$ using (23).
Step 6: If $\tilde{k} < k_m$, then go to step 3; else stop and search the computed values of $J_3(\tilde{k})$ to obtain k^*.

4.4. Special Case 2: [Penalty-1]

The total expected cost to the lessor is given by (17) with $C_n = 0$. As result, we have

$$J(k, \underline{t}, \underline{\delta}) = \left[C_f + C_t \int_\tau^\infty [1 - G(y)]\mathrm{d}y\right]\Lambda(L) + ak + b\sum_{j=1}^{k} \delta_j. \tag{28}$$

Note that this is identical to Special Case 1 except that instead of C_f we have $\widetilde{C}_f = C_f + C_t \int_\tau^\infty [1 - G(y)]\mathrm{d}y$. Hence, the optimal parameters for this case can be obtained as indicated earlier.

When $\tau \to \infty$ corresponds to no penalty associated with repair time. In this case $\widetilde{C}_f = C_f$ so that it reduces to the Special Case 1 as to be expected.

4.5. Special Case 3: [Penalty-2]

The total expected cost to the lessor is given by (17) with $C_t = 0$. As a result, we have

$$J(k, \underline{t}, \underline{\delta}) = (C_f + C_n)\Lambda(L) + ak + b\sum_{j=1}^{k} \delta_j. \tag{29}$$

Note that this is identical to Special Case 1 except that instead of C_f we have $\widetilde{C}_f = C_f + C_n$. Hence, the optimal parameters for this case can be obtained as indicated earlier.

When there is no penalty for failures ($C_n = 0$), then $\widetilde{C}_f = C_f$ so that it reduces to the Special Case 1 as to be expected.

5. Numerical example

The equipment failure distribution is given by a two-parameter Weibull with the failure rate given by (1) with $\beta = 2$ and $\alpha = 1$. The repair time distribution is also a two-parameter Weibull

$$G(y) = 1 - \exp\left[-\left(\frac{y}{\varphi}\right)^m\right], \quad 0 \leqslant y < \infty \tag{30}$$

with the shape parameter $m < 1$ (implying decreasing repair rate) and the scale parameter $\varphi > 0$. We assume that $m = 0.5$ and $\varphi = 0.5$ so that the mean time to repair is 1 day.

Let the nominal values for the remaining model parameters be as follows: $L = 5$ (years), $C_f = 100$ ($), $C_n = 200$ ($), $C_t = 300$ ($), $\tau = 2$ (days), $a = 100$ ($), and $b = 50$ ($).

Note that $\widetilde{L} = 5 - 50/421.80 = 4.88$ and $k_m = \lceil 421.80(5)^2/100 \rceil = 105$. Using the approach discussed in Section 4.2, $J_3(k)$ versus k for $1 \leqslant k \leqslant k_m$, is shown in Fig. 3. As can be seen, $k^* = 9$ and the total expected maintenance cost to the lessor, $J(k^*, \underline{t}^*, \underline{\delta}^*)$, is 2,399.16 ($). The optimal $\{\underline{t}^*\}$ and $\{\underline{\delta}^*\}$ given in Table 1.

Table 2 gives expected number of failures over $[t^*_{j-1}, t^*_j)$ for $1 \leqslant j \leqslant k^* + 1$ (with $t^*_0 = 0$ and $t^*_{k^*+1} = L$) with no PM and with optimal PM. Note that with no PM actions, the expected number of failures over time increases more rapidly than with PM action. As a result, the expected cost with no PM action is 10,544.96 ($). The use of optimal PM action reduces this to 2,399.16 ($) due to the decrease in the occurrence of failures. Note that with the optimal PM strategy, the expected number of failures between PM actions is the same for all intervals except the last because $\beta = 2$. However, this is not the case when $\beta \neq 2$.

5.1. Sensitivity studies

The effect of various model parameters on the optimal solution was studied by considering several different combinations of parameter values as indicated below.

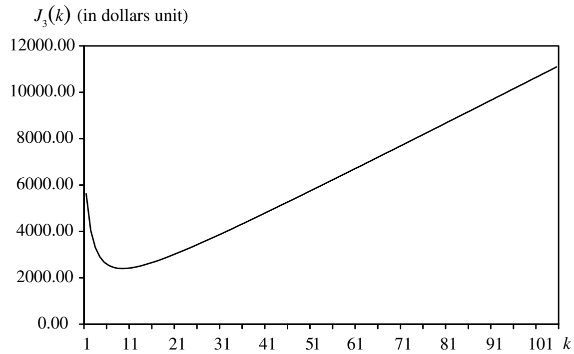

Fig. 3. $J_3(k)$ versus k for $1 \leqslant k \leqslant k_m (= 105)$.

Table 1
Optimal t_j^* and δ_j^*

j	t_j^*	δ_j^*
1	0.4881	0.9763
2	0.9763	0.9763
3	1.4644	0.9763
4	1.9526	0.9763
5	2.4407	0.9763
6	2.9289	0.9763
7	3.4170	0.9763
8	3.9052	0.9763
9	4.3933	0.9763

Table 2
Expected number of failures in different intervals with no PM and optimal PM

j	t_j^*	No PM		Optimal PM	
		$E[N(t_j^*)]$	$E[N(t_j^*)] - E[N(t_{j-1}^*)]$	$E[N(t_j^*)]$	$E[N(t_j^*)] - E[N(t_{j-1}^*)]$
1	0.4881	0.2382	0.2382	0.2382	0.2382
2	0.9763	0.9592	0.7149	0.4765	0.2382
3	1.4644	2.1445	1.1913	0.7148	0.2382
4	1.9526	3.8126	1.6682	0.9531	0.2382
5	2.4407	5.9570	2.1444	1.1913	0.2382
6	2.9289	8.5785	2.6214	1.4296	0.2382
7	3.4170	11.6759	3.0974	1.6678	0.2382
8	3.9052	15.2506	3.5747	1.9061	0.2382
9	4.3933	19.3011	4.0505	2.1444	0.2382
10	5.0000	25.0000	5.6989	2.5124	0.3680

$$L = 3, 5, 7 \text{ (years)},$$
$$\beta = 1.5, 2.0, 3.0,$$
$$C_n = 0, 100, 200, 300 \text{ (\$)},$$
$$\tau = 1, 2, 3, \infty \text{ (days)}.$$

Table 3 gives the k^* and $J(k^*, \underline{t}^*, \underline{\delta}^*)$. The following observations can be made:

1. As the lease period (L) increases, both the optimal number of PM actions, k^* and the total expected cost $J(k^*, \underline{t}^*, \underline{\delta}^*)$ increase.
2. As τ increases, $J(k^*, \underline{t}^*, \underline{\delta}^*)$ decreases and k^* is non-increasing. This is to be expected as the cost associated with Penalty-1 decreases.
3. As C_n increases, $J(k^*, \underline{t}^*, \underline{\delta}^*)$ increases. This is due to higher cost resulting from Penalty-2. k^* increases as to be expected.
4. As β increases, both k^* and $J(k^*, \underline{t}^*, \underline{\delta}^*)$ increase.

5.2. Effect of penalty

The case of no penalty corresponds to $C_n = 0$ and $\tau \to \infty$. In this case, for $\beta = 2$ and $L = 5$, we have from Table 3 $k^* = 4$ and $J(k^*, \underline{t}^*, \underline{\delta}^*) = 1{,}280.00$ (\$). With only Penalty-1 ($\tau = 2$), k^* increases to 6 and

Table 3
Effect of parameter variations on k^* and $J(k^*, \underline{t}^*, \underline{\delta}^*)$

β	L	C_n	$\tau = 1$		$\tau = 2$		$\tau = 3$		$\tau \to \infty$	
			k^*	$J(k^*, \underline{t}^*, \underline{\delta}^*)$	k^*	$J(k^*, \underline{t}^*, \underline{\delta}^*)$	k^*	$J(k^*, \underline{t}^*, \underline{\delta}^*)$	k^*	$J(k^*, \underline{t}^*, \underline{\delta}^*)$
1.5	3	0	2	$672.67	2	$597.90	2	$553.08	1	$391.40
	5	0	4	$1,002.27	3	$907.63	3	$838.08	2	$615.31
	7	0	5	$1,299.26	5	$1,175.85	4	$1,090.82	3	$811.27
	3	100	3	$777.05	3	$723.00	3	$690.64	2	$567.81
	5	100	5	$1,161.19	4	$1,078.78	4	$1,024.47	3	$860.93
	7	100	6	$1,499.74	6	$1,395.28	5	$1,329.41	3	$1,166.35
	3	200	3	$876.51	3	$822.54	3	$790.25	3	$701.27
	5	200	5	$1,298.34	5	$1,223.91	5	$1,179.39	4	$1,042.31
	7	200	7	$1,673.27	7	$1,582.73	6	$1,525.26	6	$1,353.32
	3	300	4	$956.68	4	$914.48	4	$889.24	3	$800.86
	5	300	6	$1,417.99	6	$1,354.95	5	$1,316.52	5	$1,194.01
	7	300	8	$1,829.35	8	$1,749.46	7	$1,695.40	6	$1,545.76
2	3	0	4	$1,129.70	3	$1,015.59	3	$941.12	2	$683.33
	5	0	7	$1,992.32	6	$1,811.05	6	$1,693.48	4	$1,280.00
	7	0	10	$2,857.93	9	$2,606.67	8	$2,441.35	6	$1,878.57
	3	100	5	$1,308.58	4	$1,213.02	4	$1,153.91	3	$965.63
	5	100	8	$2,283.20	8	$2,131.42	7	$2,034.14	6	$1,732.14
	7	100	12	$3,257.54	11	$3,048.56	11	$2,915.23	9	$2,498.75
	3	200	5	$1,459.74	5	$1,377.76	5	$1,328.66	4	$1,173.33
	5	200	10	$2,531.77	9	$2,399.16	9	$2,317.58	7	$2,067.71
	7	200	14	$3,603.62	13	$3,420.79	13	$3,306.74	11	$2,959.03
	3	300	6	$1,594.09	6	$1,532.92	5	$1,479.76	5	$1,344.79
	5	300	11	$2,754.52	10	$2,636.10	10	$2,562.04	9	$2,344.38
	7	300	16	$3,915.20	15	$3,749.77	14	$3,647.09	13	$3,344.20
3	3	0	8	$2,933.54	6	$2,690.12	4	$2,521.39	3	$1,903.56
	5	0	19	$7,511.43	16	$7,009.92	16	$6,677.07	10	$5,437.03
	7	0	33	$13,825.00	29	$13,012.33	26	$12,464.34	18	$10,502.32
	3	100	3	$3,309.45	8	$3,114.90	8	$2,986.12	6	$2,577.27
	5	100	22	$8,294.98	20	$7,888.48	19	$7,624.23	15	$6,785.51
	7	100	39	$15,099.27	35	$14,436.45	33	$14,008.09	27	$12,650.42
	3	200	11	$3,626.96	10	$3,459.61	10	$3,354.86	8	$3,028.56
	5	200	26	$8,962.08	24	$8,610.66	23	$8,388.23	20	$7,712.87
	7	200	44	$16,190.33	41	$15,616.00	39	$15,252.81	34	$14,151.29
	3	300	12	$3,908.23	12	$3,760.90	11	$3,666.03	10	$3,389.32
	5	300	29	$9,554.11	27	$9,240.10	26	$9,043.97	23	$8,462.65
	7	300	49	$17,160.71	46	$16,646.58	45	$16,325.43	40	$15,373.65

the total expected cost increases to 1,811.05 ($). With only Penalty-2 ($C_n = 200$), k^* increases to 7 and the total expected cost increase to 2,067.71 ($). With both penalties ($\tau = 2$ and $C_n = 200$), k^* increases to 9 and the total expected cost increase to 2,399.16 ($). As can be seen, the effect of Penalty-2 is more significant that Penalty-1. This can change as the parameter values change.

6. Conclusions

For leased equipment, the lessor needs to take into account the penalty costs stated in the lease contract in determining optimal PM actions. In this paper we have developed a model where PM actions result in a reduction in the failure intensity function. The optimal parameters are determined by minimising a cost function that takes into account the CM and PM costs as well as the penalty costs. We present a numerical example that highlights the effect of penalty terms on the optimal PM strategy.

In this section we briefly discuss some future research.

6.1. Extensions to the current model

(1) In our model we have assumed that under Penalty-2 the lessor incurs a penalty for every failure. We can modify the model so that the penalty is incurred only when the number of failures over the lease period exceeds some specified number ($\eta \geqslant 0$). In this case, the Penalty-2 cost given by $C_n\{\max[0, N(L)-\eta]\}$. Hence, the expected cost associated with Penalty-2 is given by

$$E[\phi_2(N(L), \eta)] = C_n \sum_{n=\eta}^{\infty} (n-\eta) P[N(L)=n]. \tag{31}$$

Since $\Lambda(L)$ is a function of $\underline{\delta}$, we see that $J(k, \underline{t}, \underline{\delta})$ is no longer linear in $\underline{\delta}$. As a result, the analysis of this case is more complicated and currently under investigation.

(2) The occurrence of failures depends on factors such as usage intensity, operating environment and operator skills. These can vary across the lessee population. One way of modeling this is through the Cox regression model where the intensity function involves an extra term to reflect the effect of these variables. This makes the analysis more complex.

Other topics for future research are the following:

6.2. Optimal PM for used equipment lease

In this paper we have confined our attention to the leasing of new equipment. The model can be used for the lease of used equipment. However, with used equipment, the lessor has the option to carry out an upgrade (or overhaul) which improves the equipment reliability and hence reduce the likelihood of failures. This adds an extra dimension to the optimisation problem. One can look at three options: (i) upgrade and no PM action, (ii) no upgrade and PM actions, and (iii) upgrade and PM actions. The authors are currently investigating this topic.

6.3. New PM policies

The PM policy that we used in this paper is one of many different PM policies that the lessor can use. Three other policies that are currently under investigation are as follows:

Policy 1: The equipment is subjected to preventive maintenance action whenever the intensity function reaches a specified level, ρ. Each PM action reduces the intensity function by a fixed amount δ. Any failure over the lease period is rectified through minimal repair.

Policy 2: The equipment is subjected to preventive maintenance action periodically so that the jth PM action is carried at time $t_j = ju$. After each PM action, the intensity function is reduced to a level v. Any failure over the lease period is rectified through minimal repair.

Policy 3: Let $0 < \varsigma_1 < \varsigma_2 < L$. The equipment is subjected no PM action in over the interval $[0, \varsigma_1)$, periodic PM actions with period 2Δ in the interval $[\varsigma_1, \varsigma_2)$ and period Δ in the interval $[\varsigma_2, L)$. Each PM action reduces the intensity function to a specified level v. Any failure over the lease period is rectified through minimal repair.

The authors are currently studying these policies.

6.4. Some operational and strategic issues

We briefly mentioned some strategic and operational associated with leased equipment earlier in the paper. Some topics for future study are as follows:

- The lease contract involving more complex penalty terms. For examples, different upper limits on the number of failures for different intervals over the lease period, the time interval between subsequent failures, etc.
- The time to carry out CM actions depends on the availability of repair crew and spare parts. This raises several issues such as the optimal inventory levels for spares, number of repair crew etc. Large inventory and greater number of crews reduces the penalty cost but increases the inventory holding and operating costs. As a result, these must be selected optimally to achieve a proper trade-off.
- When the lessor offers a wide range of options to the lessee, he has to decide on the optimal pricing strategy for each option. This needs to take into account factors such as competition, demand, etc.
- From the lessor's point of view the size and variety of equipment to stock for leasing are important issues. The optimal choice with regards these and the replacement decisions need to take into account the different needs of different lessees and the investment needed for purchase of new stock.

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