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W3 Lesson 1



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Vectors and Linear Transformations

Machine Learning motivation

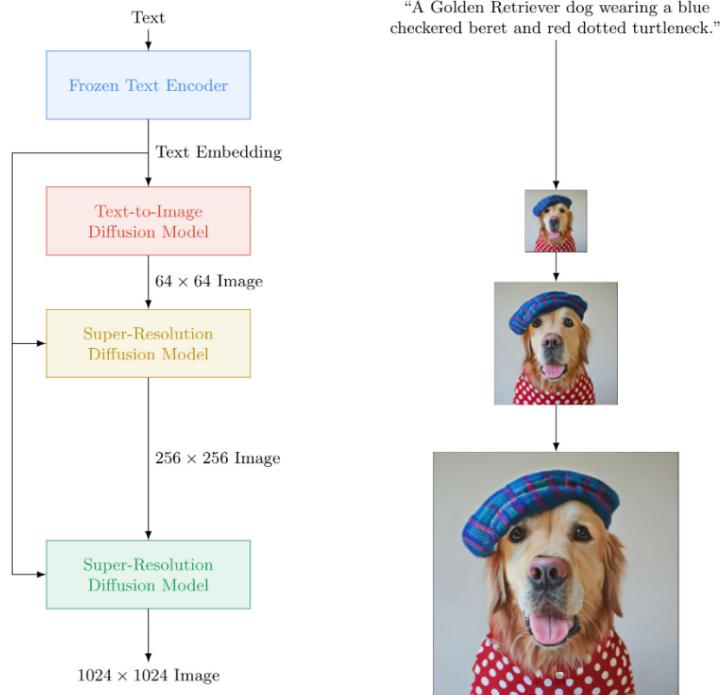
Neural Networks - AI generated images



AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."



The screenshot shows an AI interface for generating text from images. On the left, there is a 'Model' section containing a large clock icon. On the right, there is an 'Output' section with a text box containing the text "wall clock - wall clock." Above the text box, the number "3.0s" is displayed, indicating the time taken for the generation. There are also edit icons (pencil and X) at the top right of the output area.

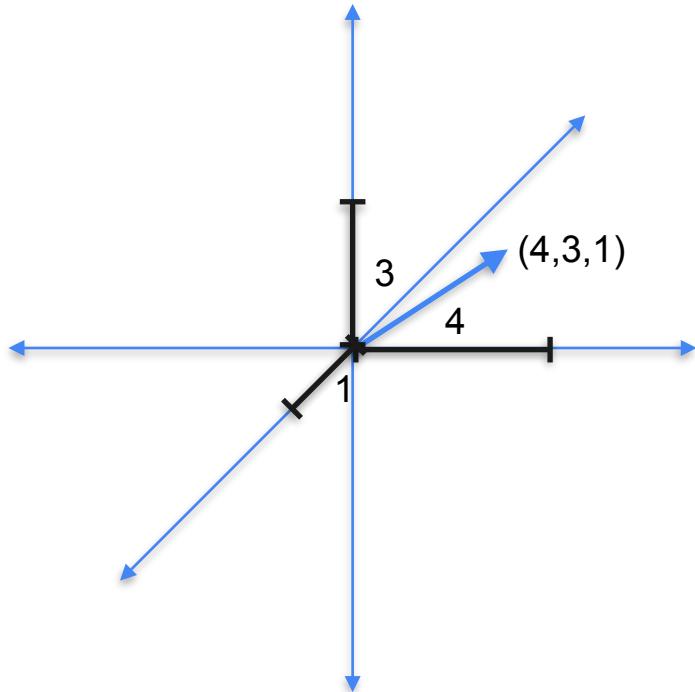
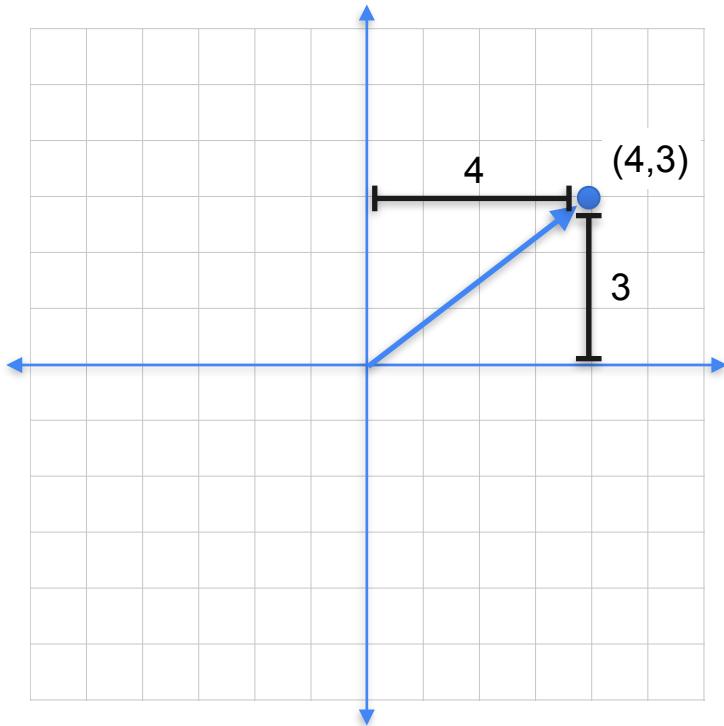


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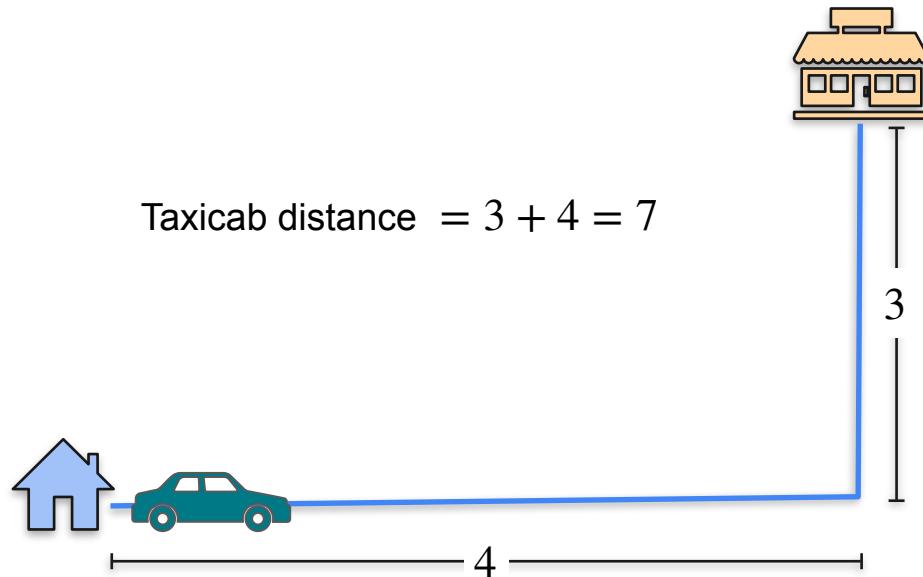
Vectors and Linear Transformations

Vectors and their properties

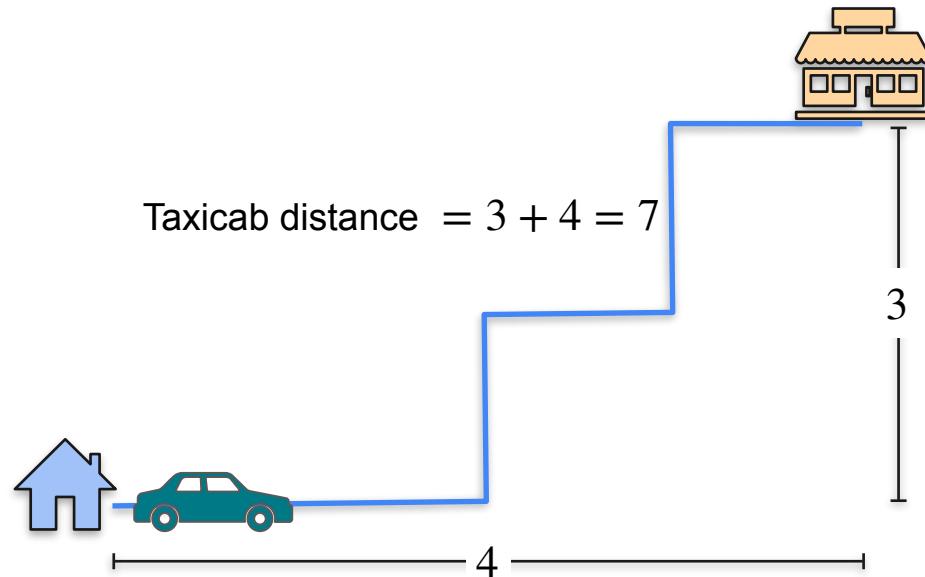
Vectors



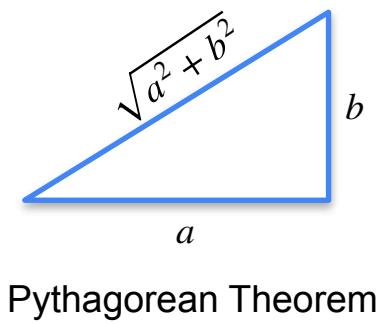
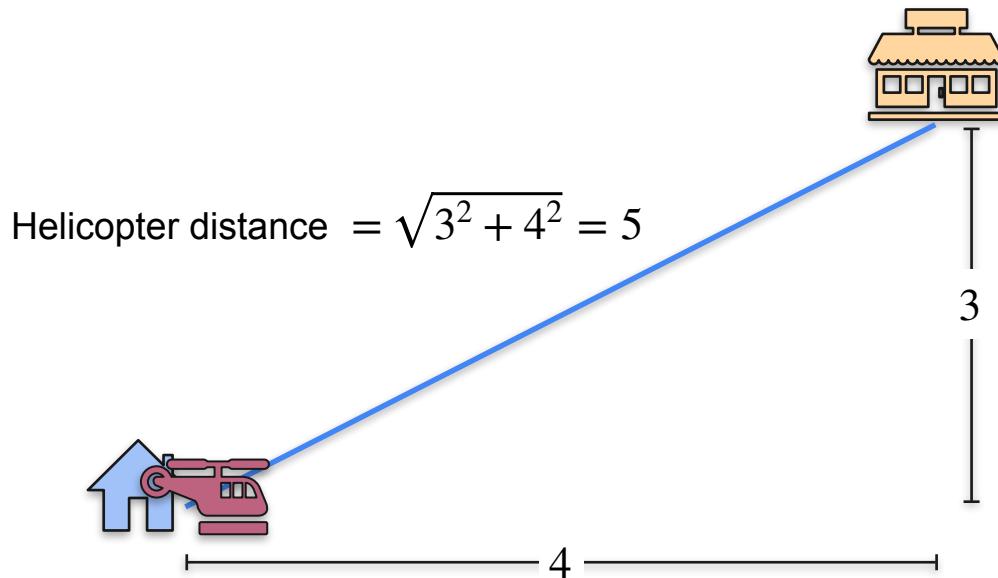
How to get from point A to point B?



How to get from point A to point B?

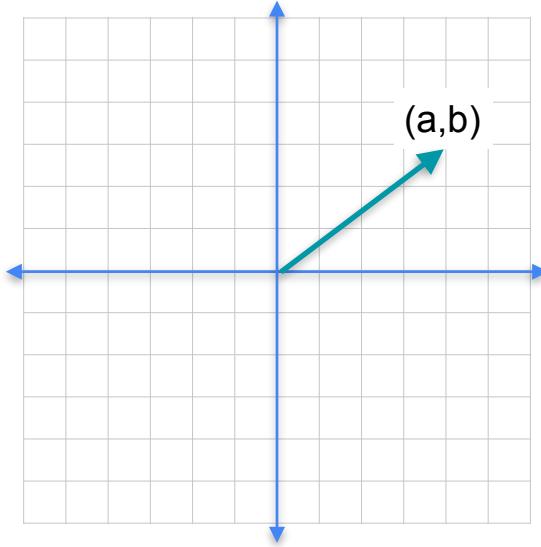


How to get from point A to point B?

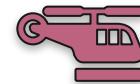


Pythagorean Theorem

Norms

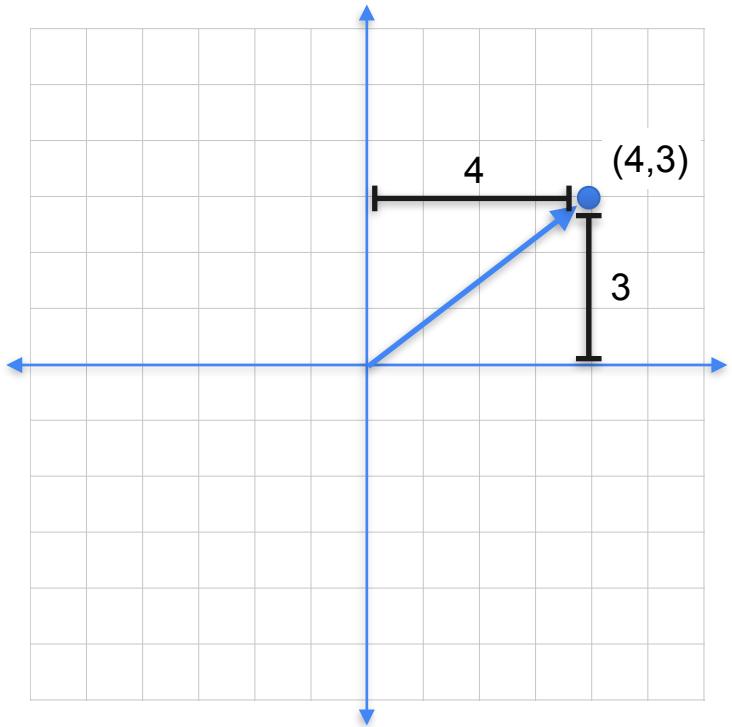


$$\text{L1-norm} = \|(a, b)\|_1 = |a| + |b|$$



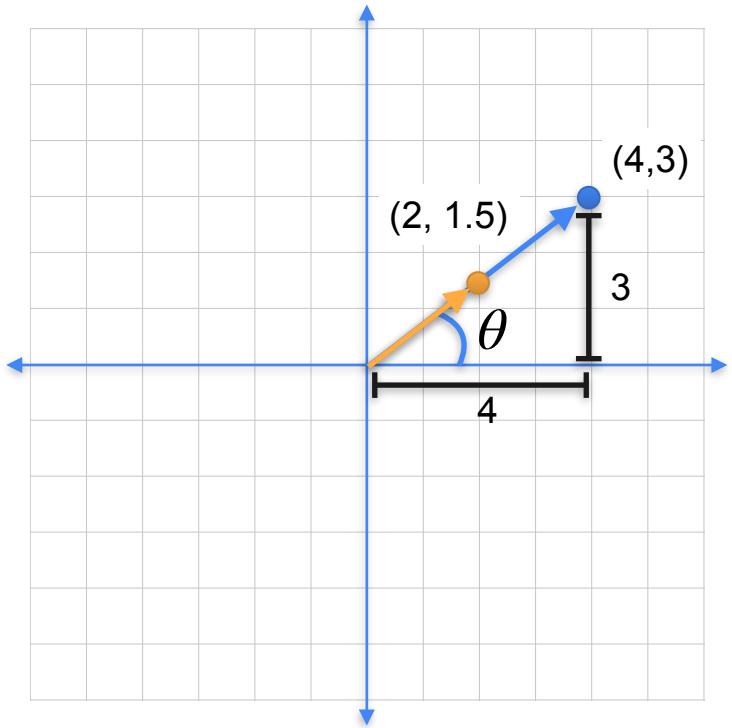
$$\text{L2-norm} = \|(a, b)\|_2 = \sqrt{a^2 + b^2}$$

Norm of a vector



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

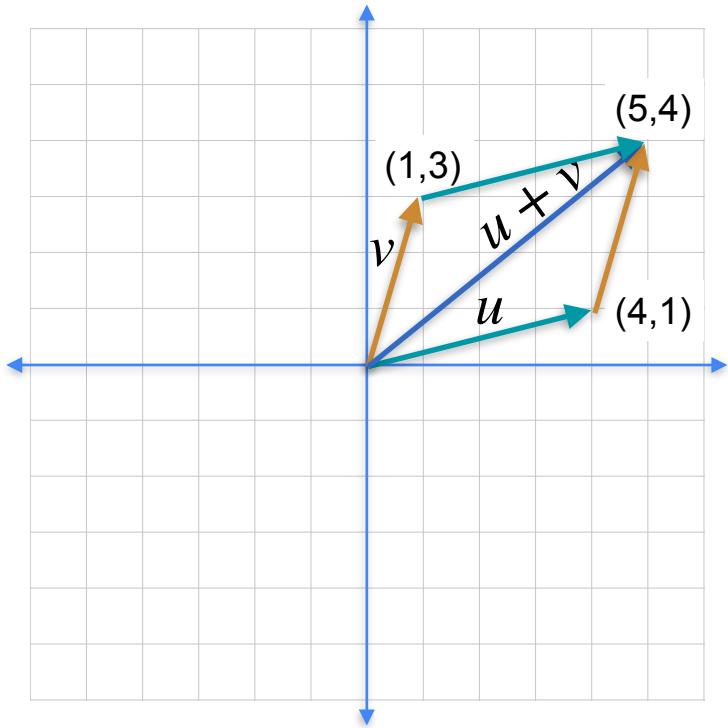


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Vectors and Linear Transformations

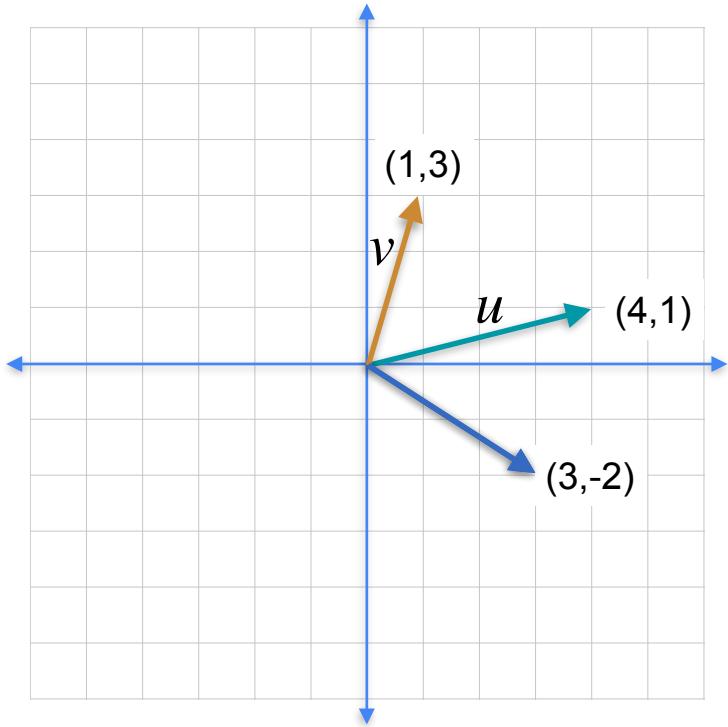
**Sum and difference of
vectors**

Sum of vectors



$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

General definition: sum and difference

Same number
of components

$$x = (x_1 \quad x_2 \quad \dots \quad x_n)$$
$$y = (y_1 \quad y_2 \quad \dots \quad y_n)$$

Sum

$$x + y = (x_1 + y_1 \quad x_2 + y_2 \quad \dots \quad x_n + y_n)$$

Sum component by component

Difference

$$x - y = (x_1 - y_1 \quad x_2 - y_2 \quad \dots \quad x_n - y_n)$$

Subtract component by component

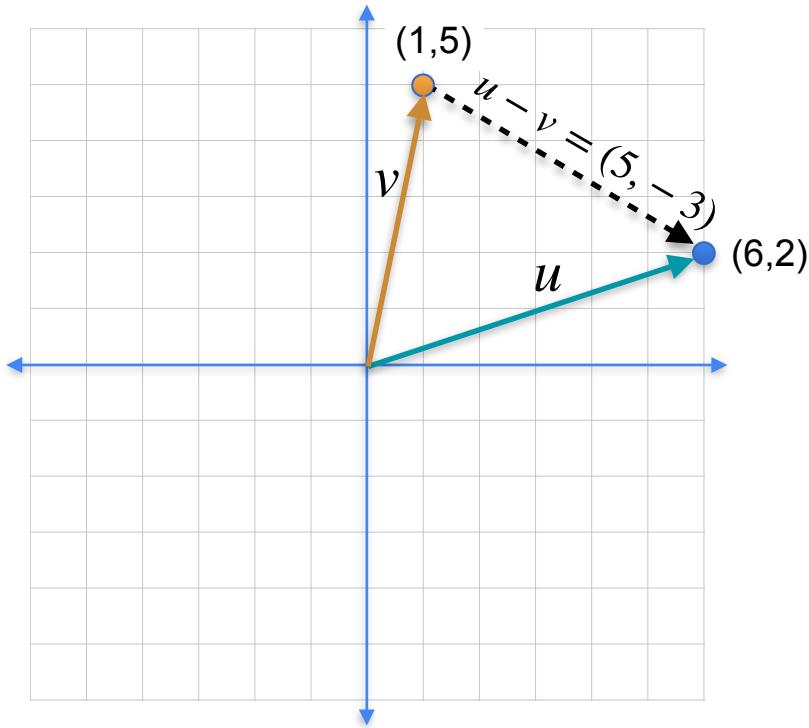


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Vectors and Linear Transformations

Distance between vectors

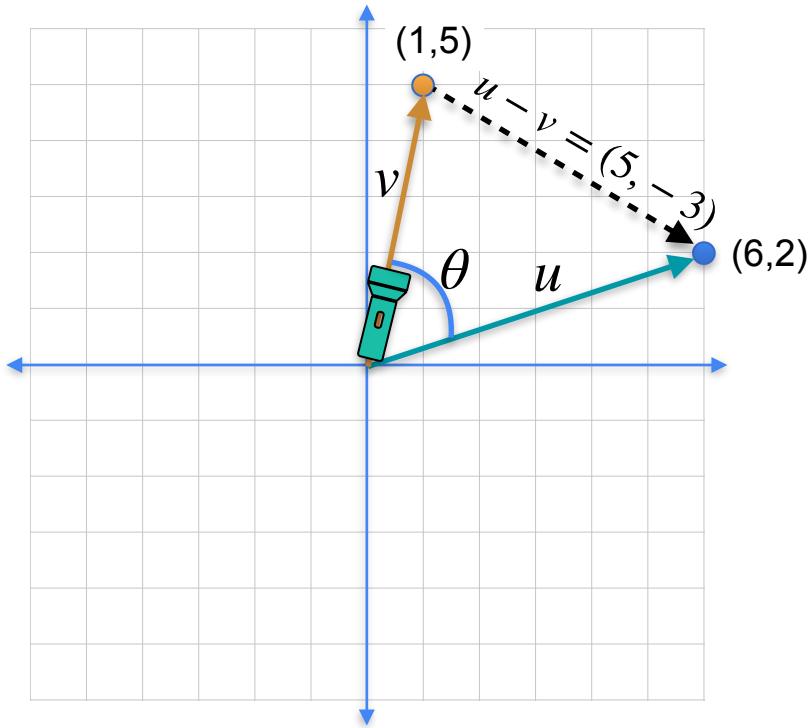
Distances



 L1-distance $\|u - v\|_1 = |5| + |-3| = 8$

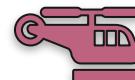
 L2-distance $\|u - v\|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances



L1-distance

$$\|u - v\|_1 = |5| + |-3| = 8$$



L2-distance

$$\|u - v\|_2 = \sqrt{5^2 + 3^2} = 5.83$$



Cosine distance

$$\cos(\theta)$$

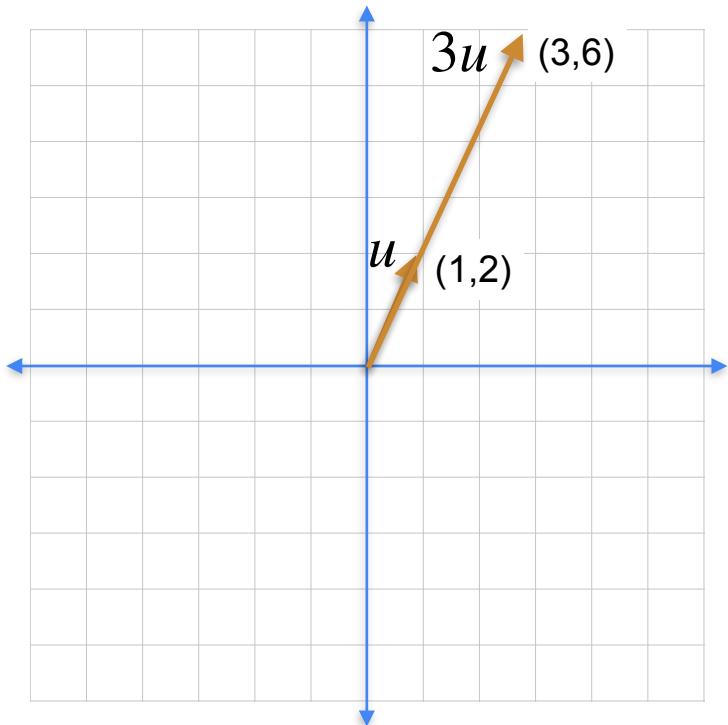


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Vectors and Linear Transformations

Multiplying a vector by a scalar

Multiplying a vector by a scalar

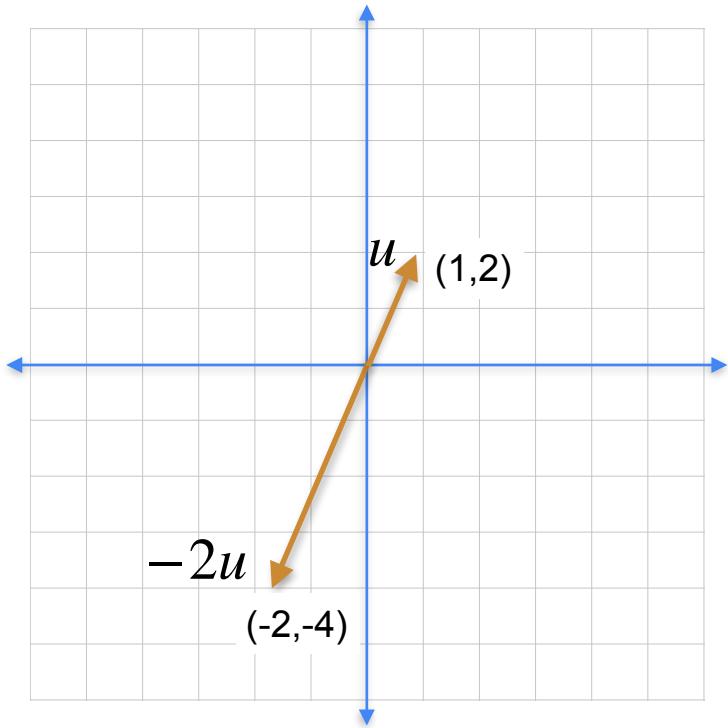


$$u = (1, 2)$$

$$\lambda = 3$$

$$\lambda u = (3, 6)$$

If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

General definition

Multiplication by a Scalar

$$x = (x_1 \quad x_2 \quad \dots \quad x_n)$$

$$\lambda x = (\lambda x_1 \quad \lambda x_2 \quad \dots \quad \lambda x_n)$$

Video 6: The dot product



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Vectors and Linear Transformations

The dot product

A shortcut for linear operations

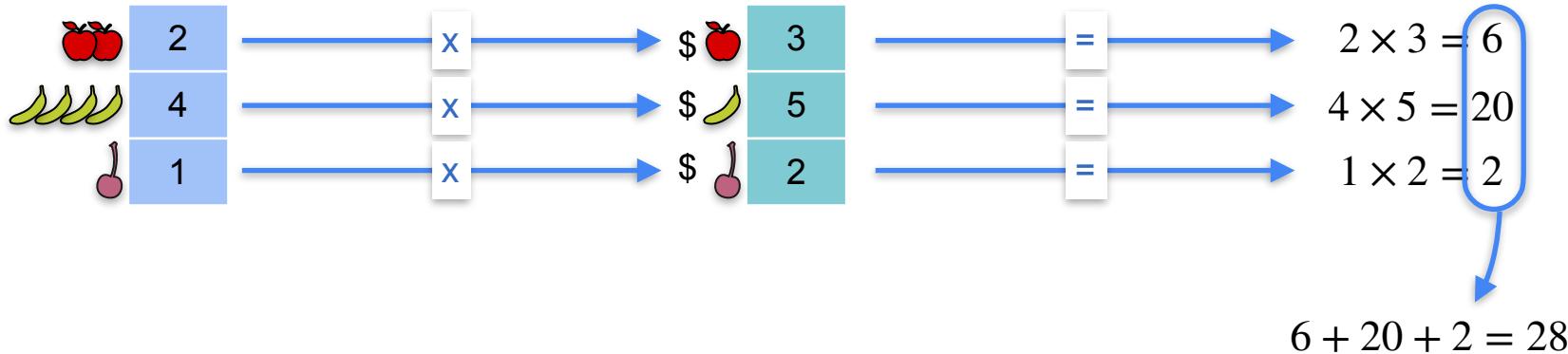
Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price
\$28



The dot product

$$\begin{array}{c} \text{apple} \\ \text{banana} \\ \text{cherry} \end{array} \cdot \begin{array}{c} \text{apple} \\ \text{banana} \\ \text{cherry} \end{array} = \$28$$

| | |
|---|---|
| 2 | 3 |
| 4 | 5 |
| 1 | 2 |

The dot product

The diagram shows two vectors being multiplied. The first vector, on the left, is represented by a blue grid with three rows. The top row contains two red apples, the middle row contains four yellow bananas, and the bottom row contains one red cherry. To the right of this grid are its corresponding numerical values: 2, 4, and 1, stacked vertically. A multiplication sign follows this vector. To the right of the multiplication sign is a second vector, represented by a teal grid with three rows. The top row contains one red apple with a dollar sign, the middle row contains one yellow banana with a dollar sign, and the bottom row contains one red cherry with a dollar sign. To the right of this grid are its corresponding numerical values: 3, 5, and 2, stacked vertically. An equals sign follows this vector, and finally, the result of the multiplication is shown as '\$28'.

$$\begin{matrix} \text{apple} \\ \text{banana} \\ \text{cherry} \end{matrix} \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} \text{apple} \\ \text{banana} \\ \text{cherry} \end{matrix} \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = \$28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

The dot product

The diagram shows two vectors of fruits being multiplied. The first vector (left) has three elements: an apple (value 2), four bananas (value 4), and one cherry (value 1). The second vector (right) has three elements: an apple (\$3), a banana (\$5), and a cherry (\$2). The multiplication is shown as a dot product calculation: $(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$.

| | | |
|---|---|---|
| 2 | 4 | 1 |
|---|---|---|

.

| |
|------|
| \$ 3 |
| \$ 5 |
| \$ 2 |

= \$28

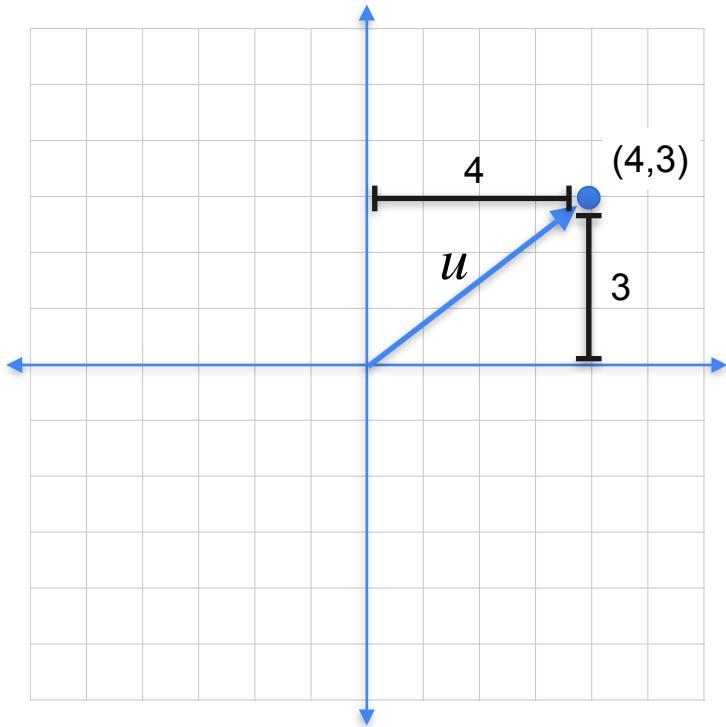
$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

The dot product

$$\begin{matrix} 2 & | & 4 & | & 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = 25$$

$$L2 - norm = \sqrt{dot\ product(u, u)}$$

$$\|u\|_2 = \sqrt{\langle u, u \rangle}$$

Vector Transpose

$$\begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

Vector Transpose

Transpose: convert columns to rows

$$\begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$\begin{matrix} 2 & 4 & 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

Vector Transpose

$$\begin{matrix} T \\ \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \end{matrix} =$$

Vector Transpose

$$\begin{matrix} & T \\ \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} & = & \begin{matrix} 2 & 4 & 1 \end{matrix} \end{matrix}$$

Vector Transpose

$$\begin{matrix} & T \\ \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} & = & \begin{matrix} 2 & 4 & 1 \end{matrix} & \begin{matrix} 2 & 4 & 1 \end{matrix} & = & T \end{matrix}$$

Vector Transpose

$$\begin{matrix} 2 \\ 4 \\ 1 \end{matrix}^T = \begin{matrix} 2 & 4 & 1 \end{matrix}$$

$$\begin{matrix} 2 & 4 & 1 \end{matrix}^T = \begin{matrix} 2 \\ 4 \\ 1 \end{matrix}$$

Matrix Transpose

$$\begin{matrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{matrix}^T = \text{Transpose the columns!}$$

Matrix Transpose

$$\begin{matrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{matrix}^T = \begin{matrix} 2 & 4 & 1 \end{matrix}$$

Matrix Transpose

$$\begin{matrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{matrix}^T = \begin{matrix} 2 & 4 & 1 \\ 5 & 7 & 3 \end{matrix}$$

Columns → Rows

3×2

3×2

General definition: dot product

Same number
of components

$$x = (x_1 \quad x_2 \quad \dots \quad x_n) \qquad \qquad y = (y_1 \quad y_2 \quad \dots \quad y_n)$$

$$x \cdot y = (x_1 \times y_1) + (x_2 \times y_2) + \dots + (x_n \times y_n)$$

$\langle x, y \rangle$ is another notation for the dot product

$$x \cdot y^T = (x_1 \quad x_2 \quad \dots \quad x_n) \cdot \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

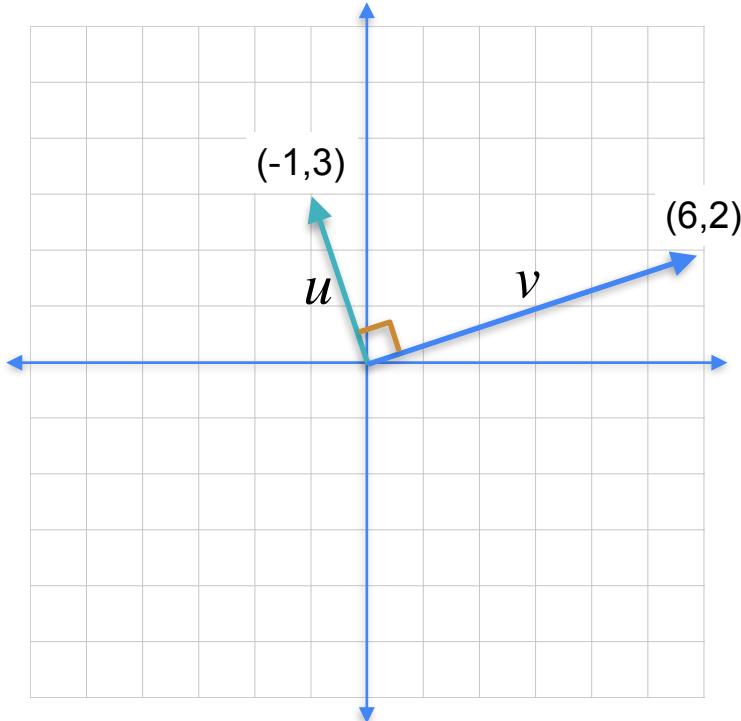


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Vectors and Linear Transformations

Geometric dot product

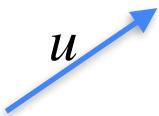
Orthogonal vectors have dot product 0



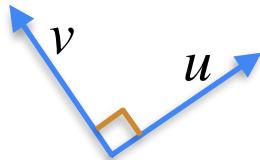
$$\begin{matrix} 6 & 2 \end{matrix} \times \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

$$\langle u, v \rangle = 0$$

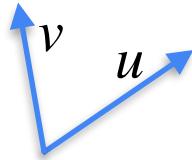
The dot product



$$\langle u, u \rangle = \|u\|^2$$

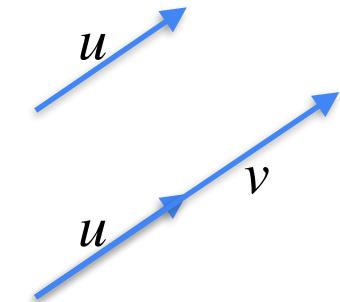


$$\langle u, v \rangle = 0$$

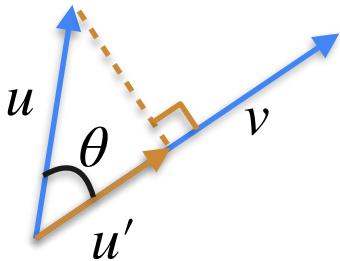


$$\langle u, v \rangle = ?$$

The dot product



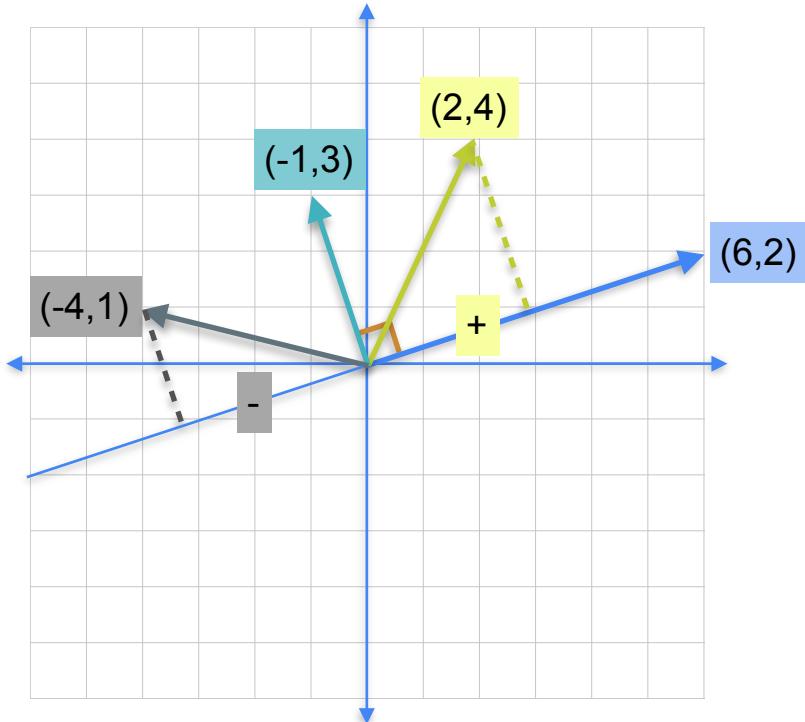
$$\langle u, u \rangle = \|u\|^2 = \|u\| \cdot \|u\|$$



$$\langle u, v \rangle = \|u\| \cdot \|v\|$$

$$\begin{aligned}\langle u, v \rangle &= \|u'\| \cdot \|v\| \\ &= \|u\| \|v\| \cos(\theta)\end{aligned}$$

Geometric dot product

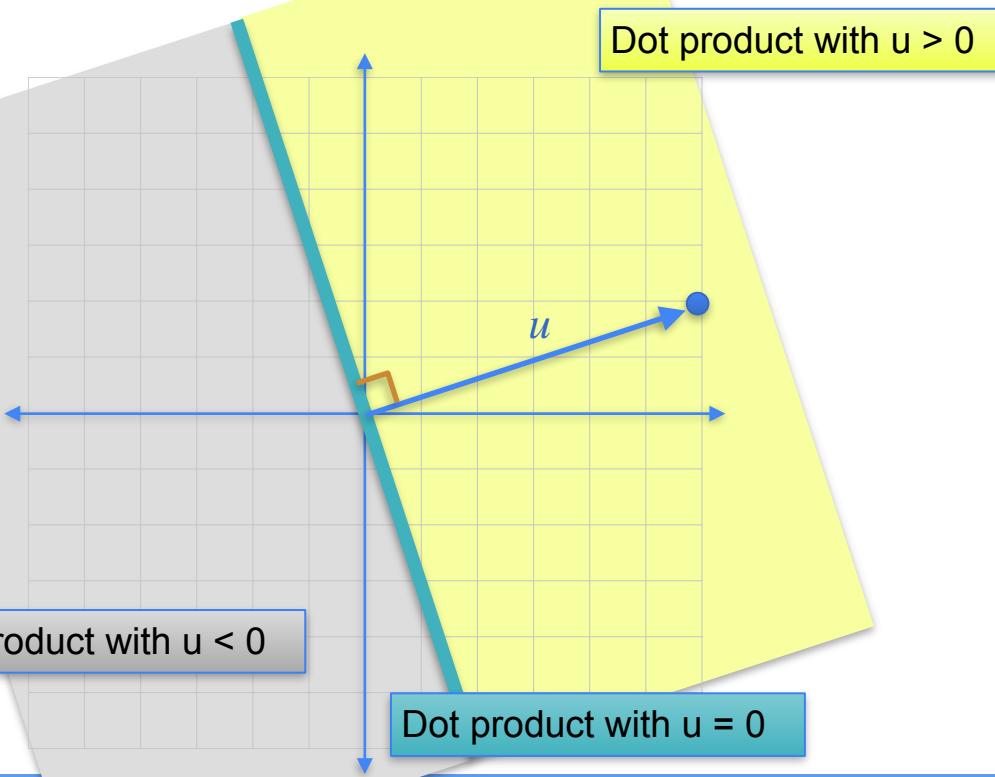


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \text{ Negative}$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$



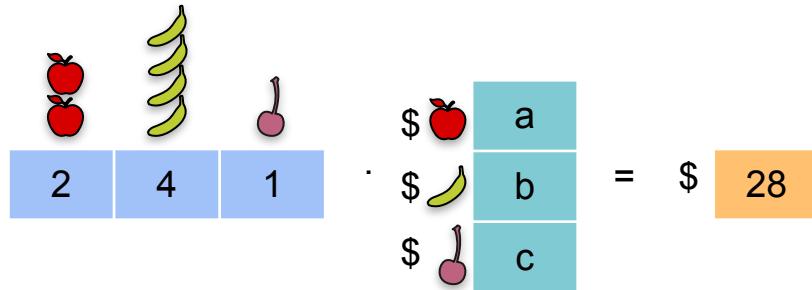
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Vectors and Linear Transformations

**Multiplying a matrix by a
vector**

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

$$a + b + c = 10$$

A diagram illustrating the equation $a + b + c = 10$. It shows two vectors being multiplied together. The first vector is composed of three blue boxes labeled 1, 1, and 1, each accompanied by a fruit icon: an apple, a banana, and a cherry respectively. The second vector is a column matrix with three rows, labeled a , b , and c from top to bottom. The entries are \$1 for a , \$1 for b , and \$1 for c , each preceded by a dollar sign. The result of the multiplication is \$10, shown in an orange box.

$$a + 2b + c = 15$$

A diagram illustrating the equation $a + 2b + c = 15$. It shows two vectors being multiplied together. The first vector is composed of three blue boxes labeled 1, 2, and 1, each accompanied by a fruit icon: an apple, two bananas, and a cherry respectively. The second vector is a column matrix with three rows, labeled a , b , and c from top to bottom. The entries are \$1 for a , \$2 for b , and \$1 for c , each preceded by a dollar sign. The result of the multiplication is \$15, shown in an orange box.

$$a + b + 2c = 12$$

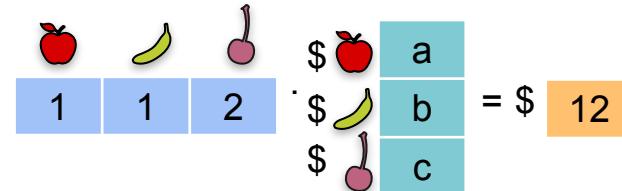
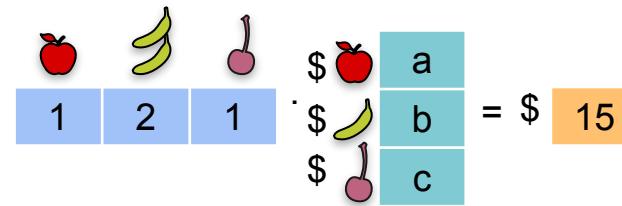
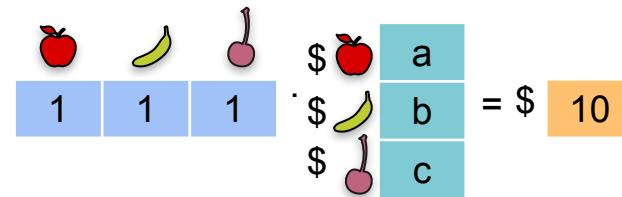
A diagram illustrating the equation $a + b + 2c = 12$. It shows two vectors being multiplied together. The first vector is composed of three blue boxes labeled 1, 1, and 2, each accompanied by a fruit icon: an apple, a banana, and two cherries respectively. The second vector is a column matrix with three rows, labeled a , b , and c from top to bottom. The entries are \$1 for a , \$1 for b , and \$2 for c , each preceded by a dollar sign. The result of the multiplication is \$12, shown in an orange box.

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

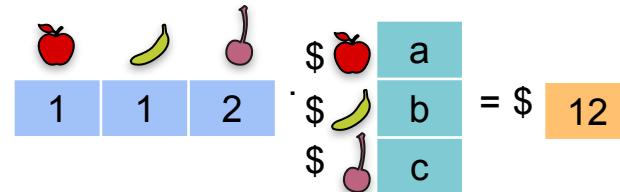
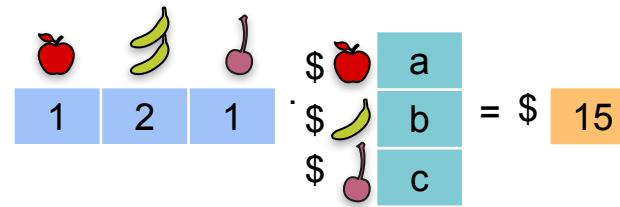
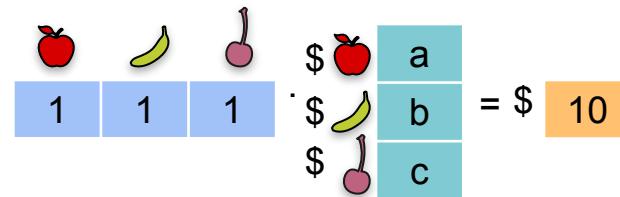


Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

| | | |
|---|---|---|
| | | |
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

·

| | | |
|----|---|----|
| \$ | a | 10 |
| \$ | b | 15 |
| \$ | c | 12 |

= \$

| | |
|----|----|
| \$ | 10 |
| \$ | 15 |
| \$ | 12 |

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$

Equations as dot product

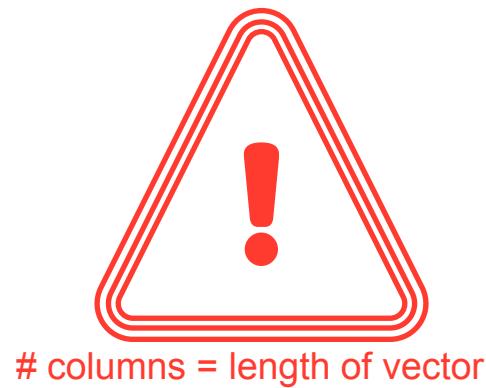
Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$

3×3 Length 3

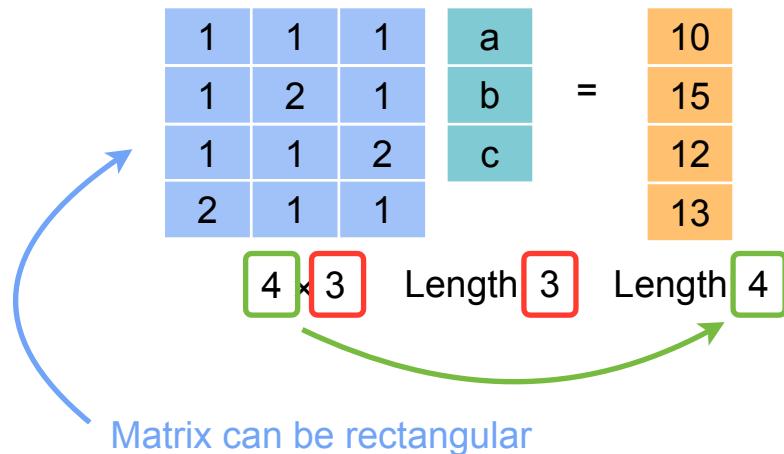


Matrix can be rectangular



Equations as dot product

Matrix product



columns = length of vector

W3 Lesson 2

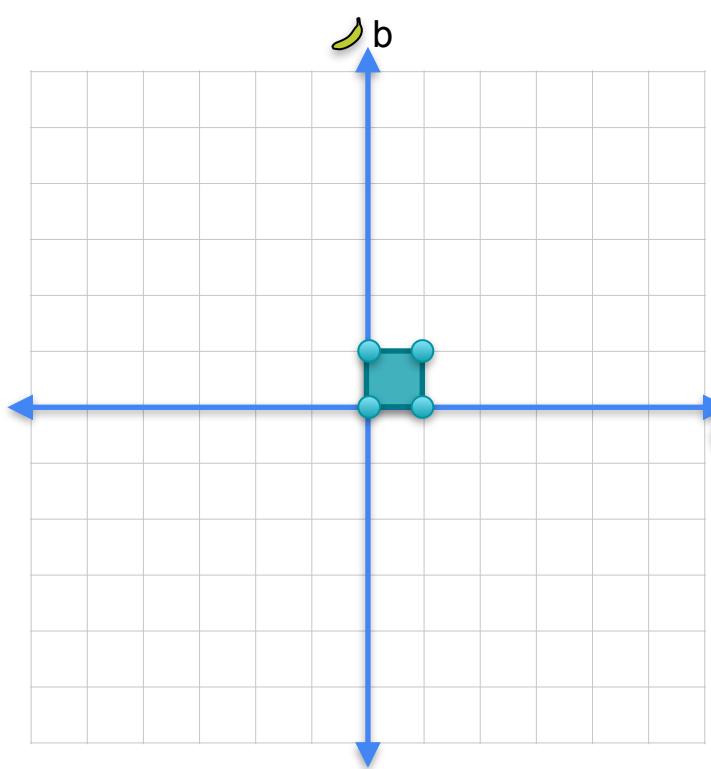


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Vectors and Linear Transformations

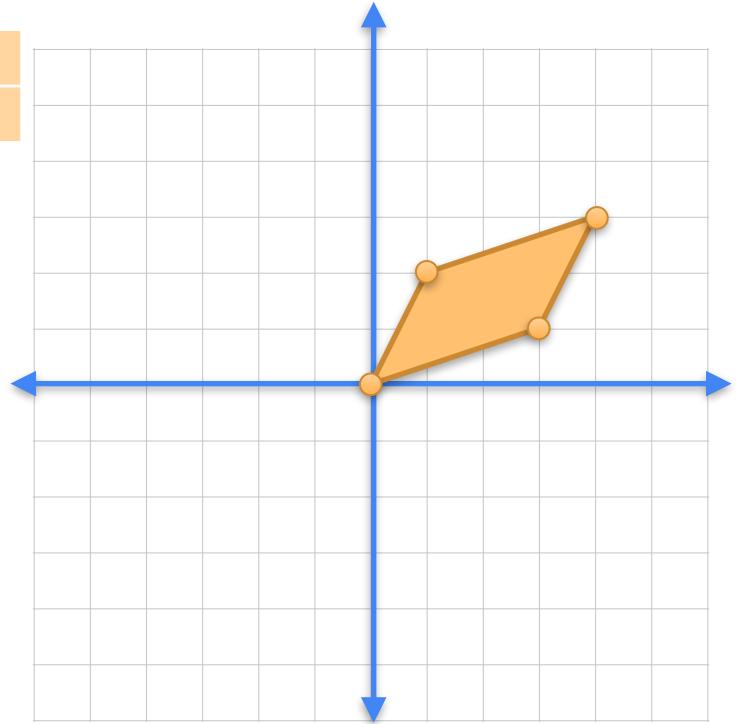
**Matrices as linear
transformations**

Matrices as linear transformations

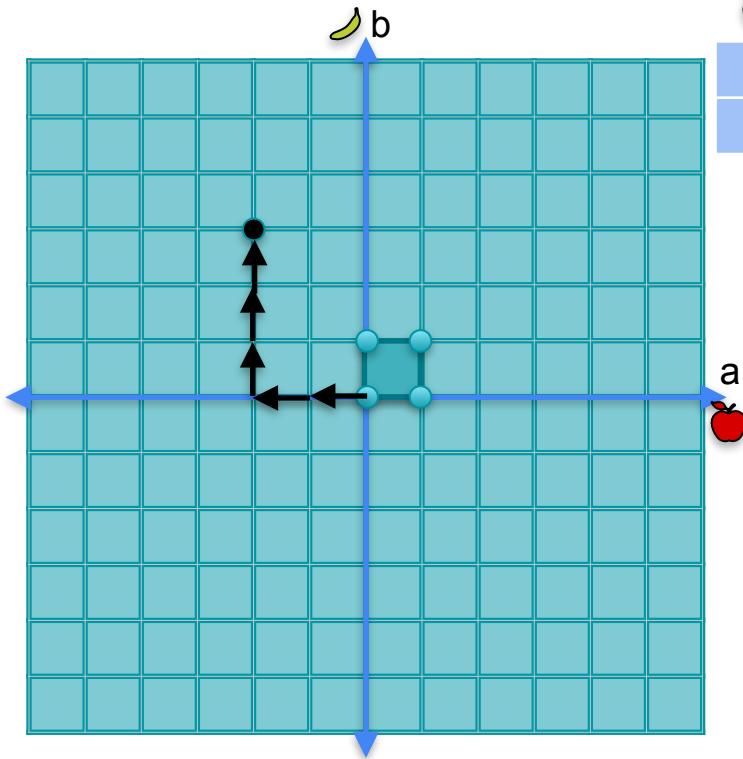


$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|c|} \hline 3 & 1 & 1 \\ \hline 1 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 & \\ \hline 3 & \\ \hline \end{array} \end{array}$$

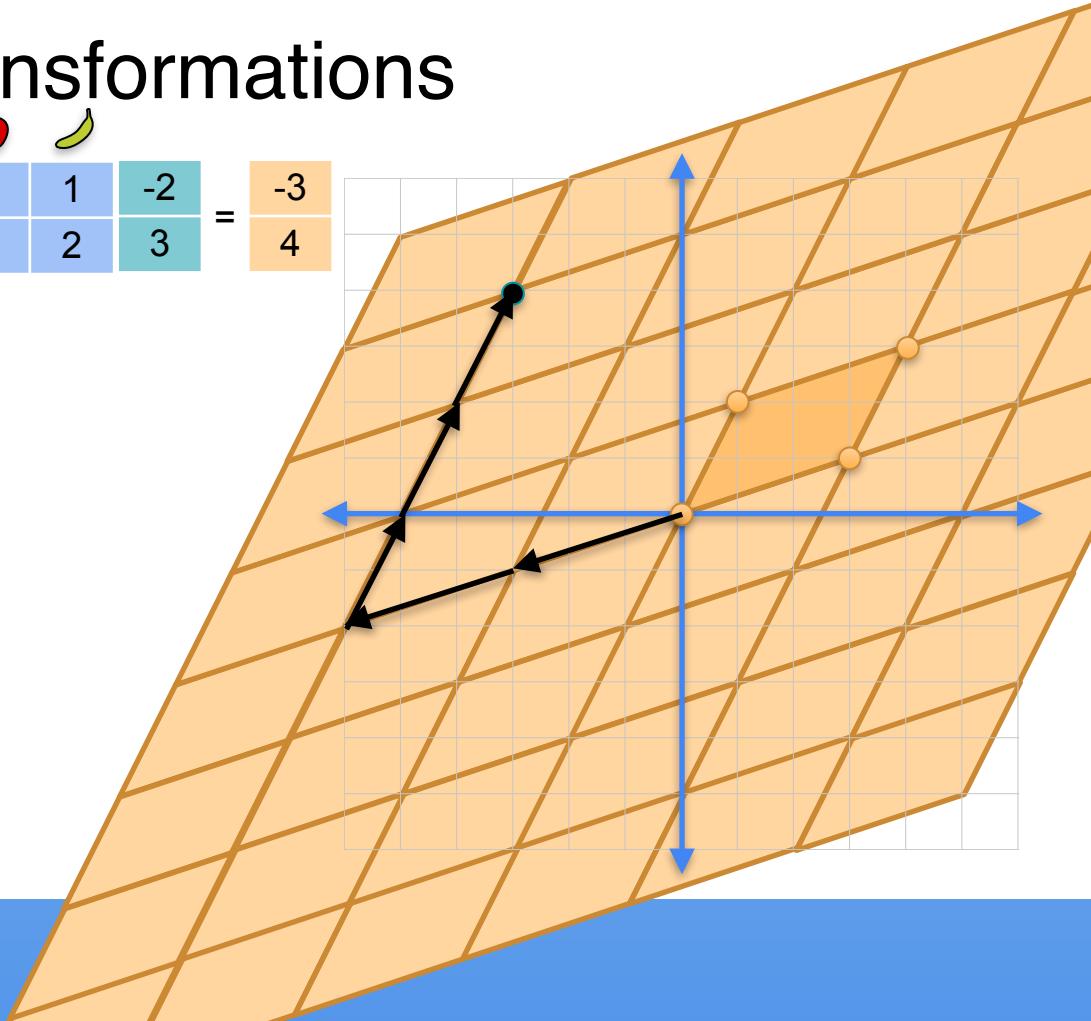
$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$



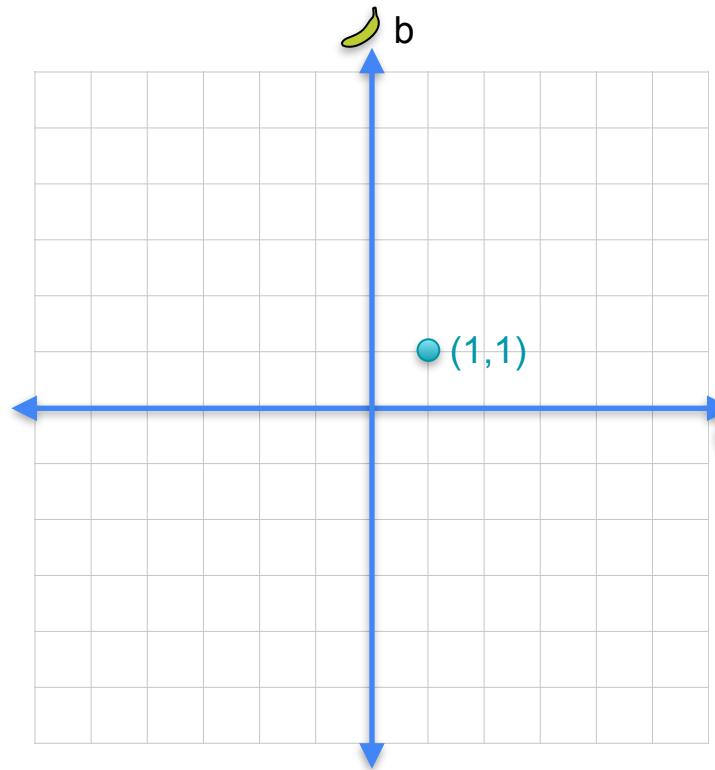
Matrices as linear transformations



$$\begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix}$$



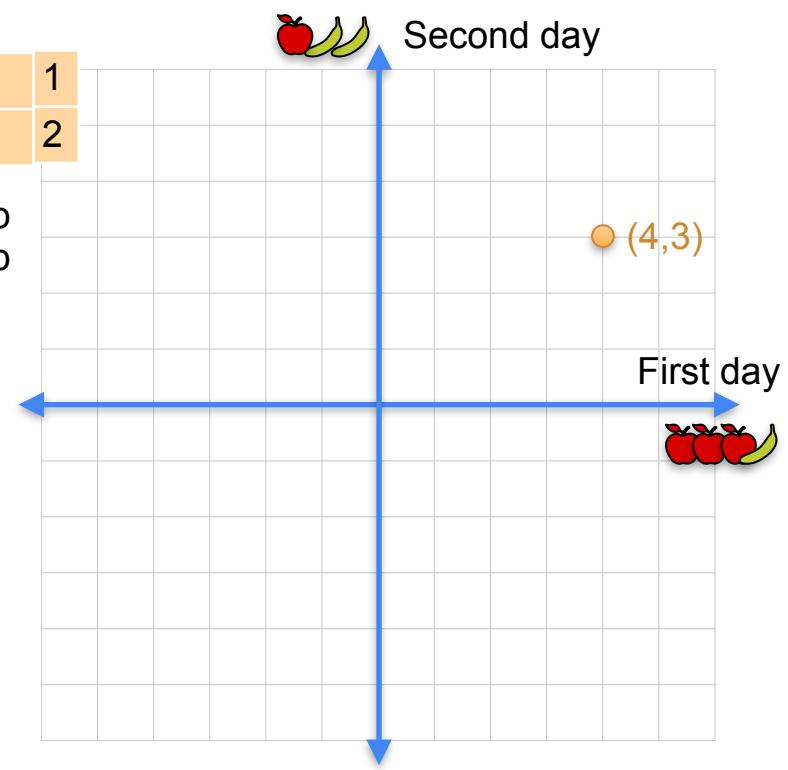
Systems of equations as linear transformations



| | | | | |
|---|---|---|---|---|
| | | | | |
| | | | | |
| 3 | 1 | 1 | 4 | 1 |
| 1 | 2 | 1 | 3 | 2 |
| | | | | |

=

First day: $3a + b$
Second day: $a + 2b$



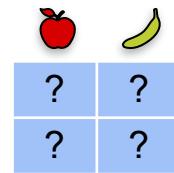
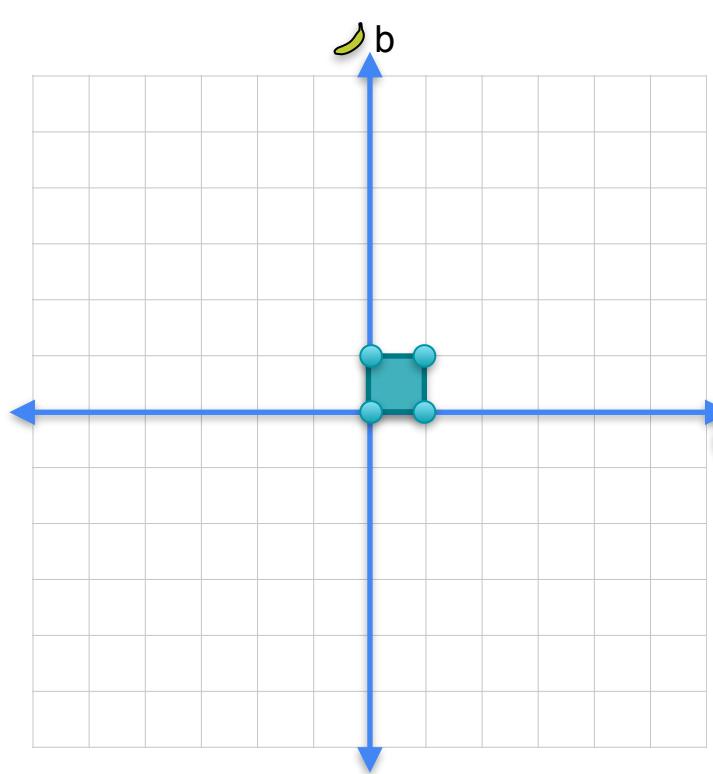


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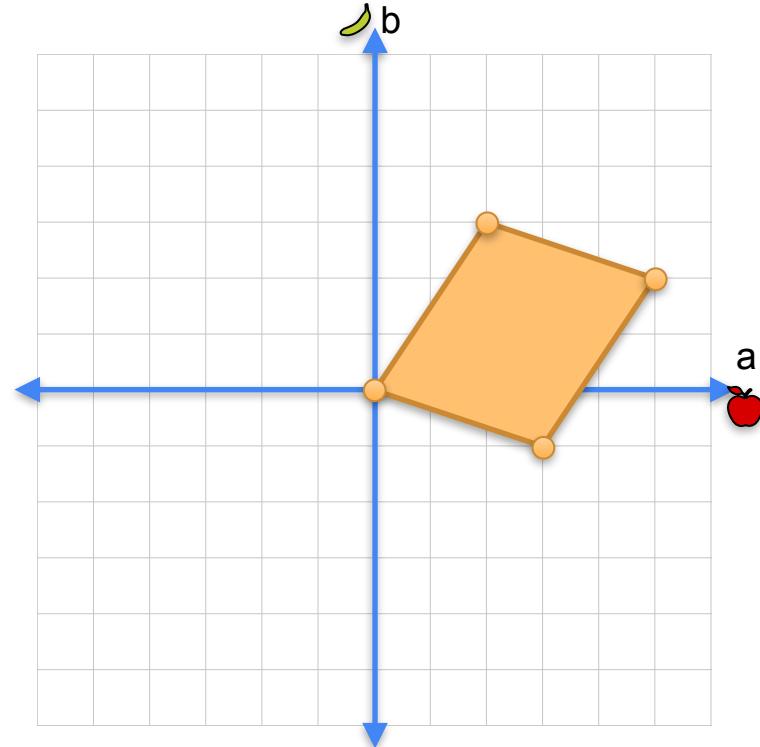
Vectors and Linear Transformations

**Linear transformations as
matrices**

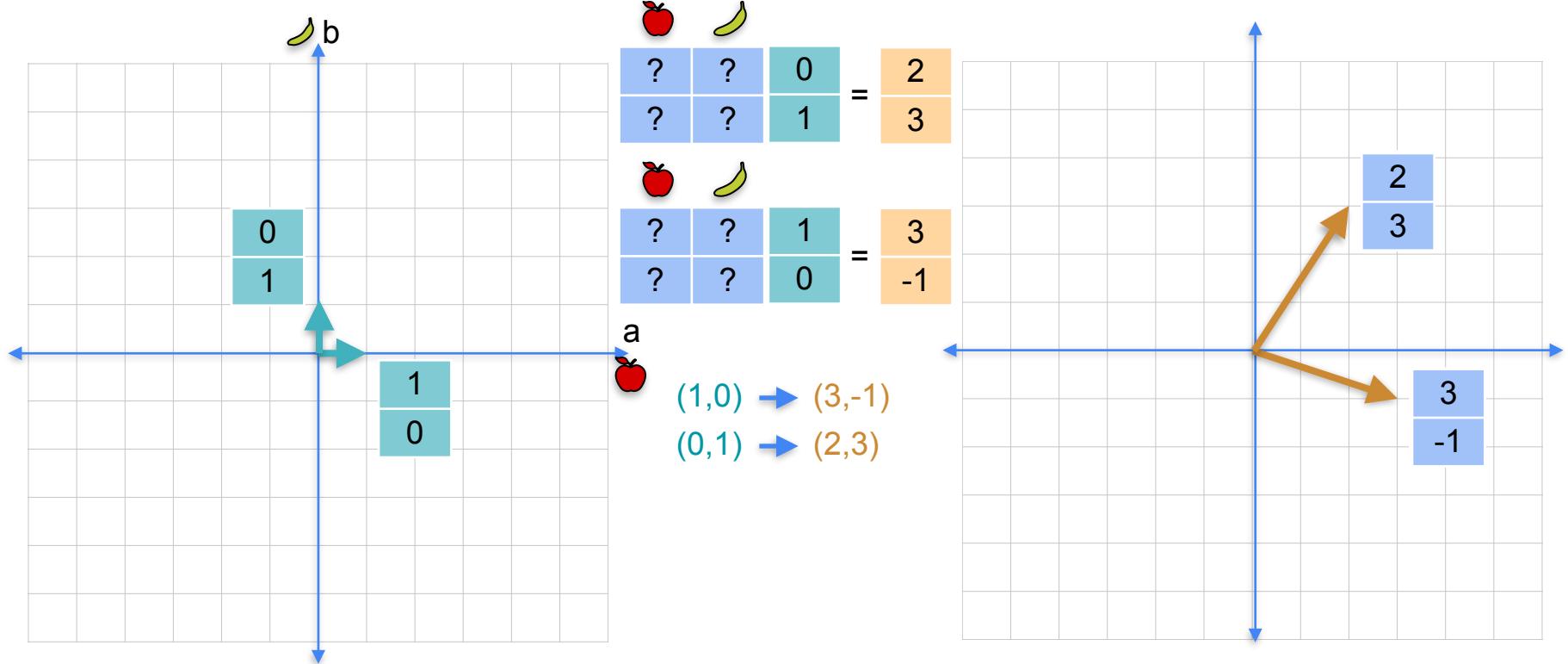
Linear transformations as matrices



- $(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,-1)$
 $(0,1) \rightarrow (2,3)$
 $(1,1) \rightarrow (5,2)$



Linear transformations as matrices



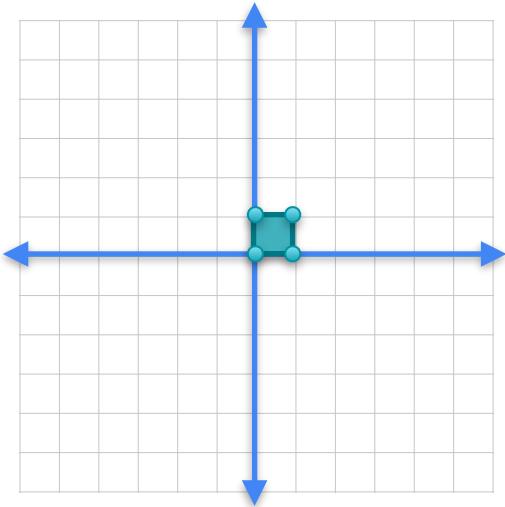


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Vectors and Linear Transformations

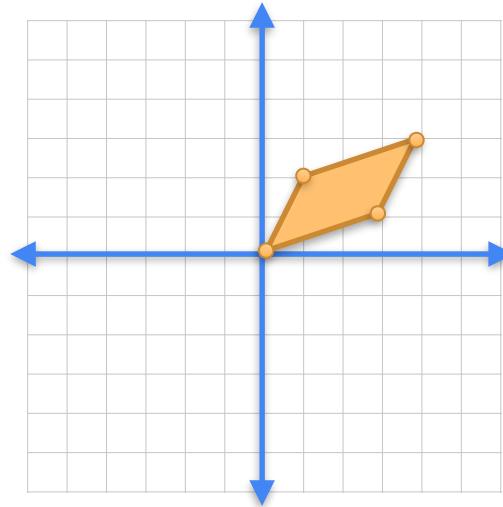
Matrix multiplication

Combining linear transformations

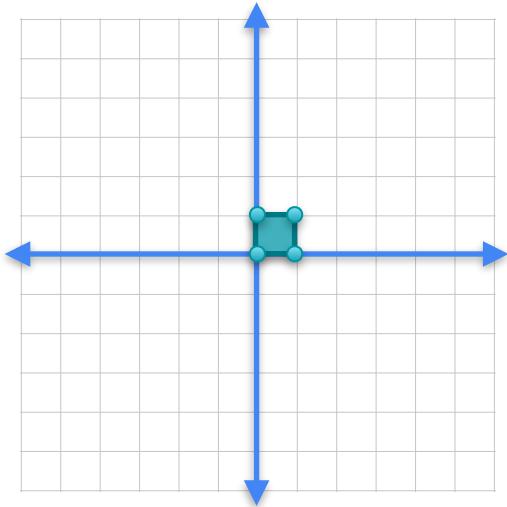


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

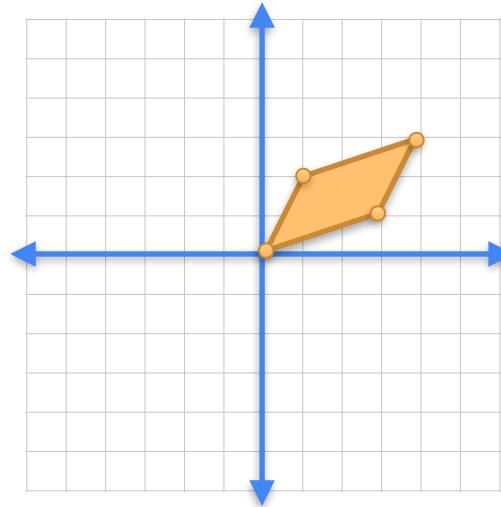


Combining linear transformations

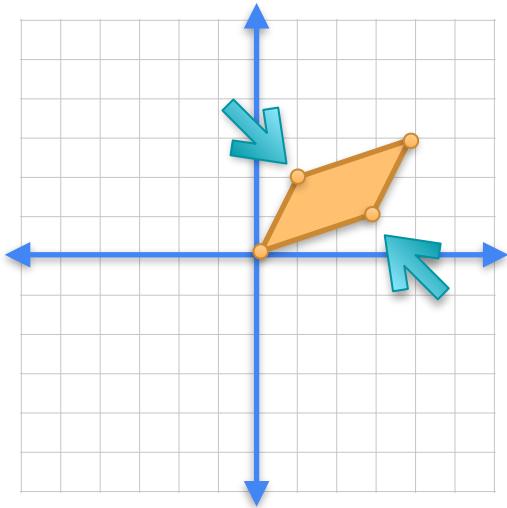


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

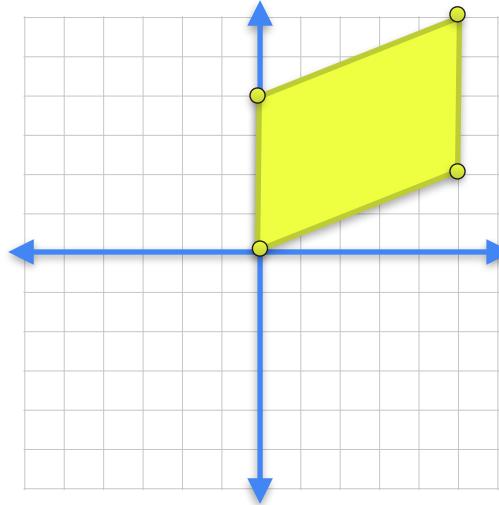


Combining linear transformations

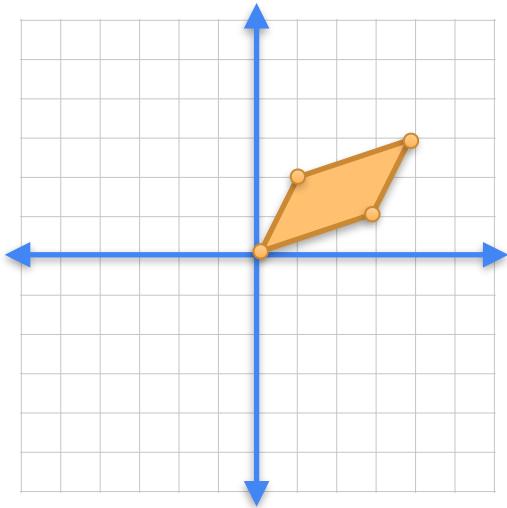


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

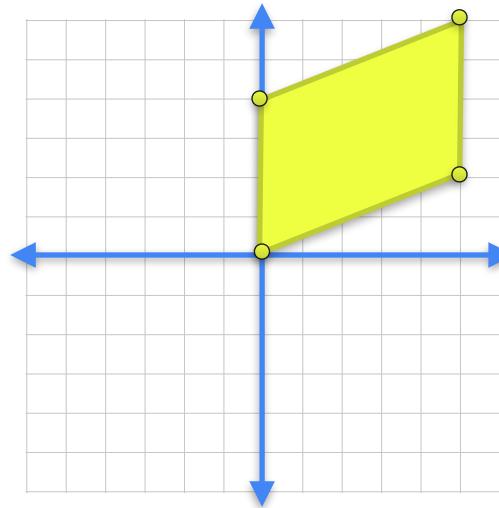


Combining linear transformations

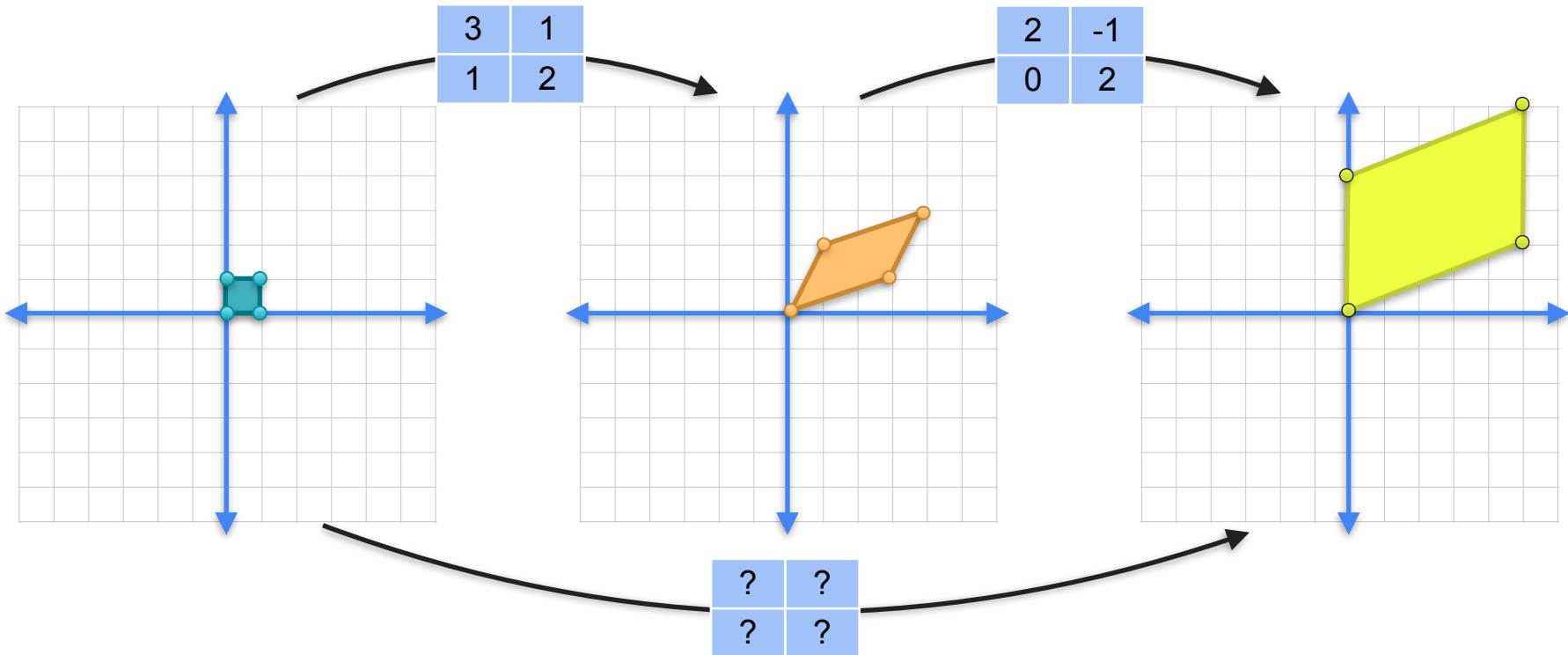


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

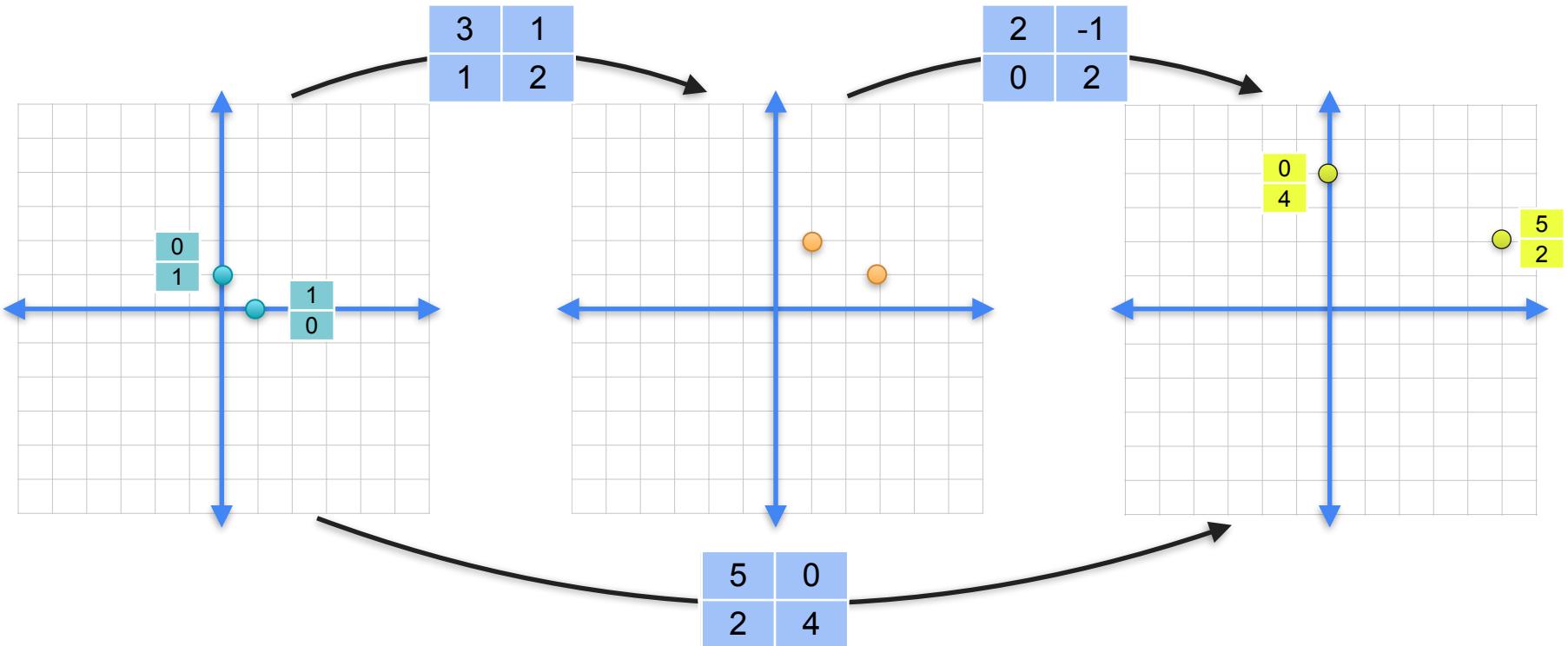
$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

Dimension of the matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 2 & -1 & 5 & 3 \\ 0 & 2 & 0 & 1 \\ 2 & -1 & 1 & 2 \\ 0 & 2 & 1 & 2 \end{matrix}$$

Dimension of the matrices

$$\begin{array}{c} \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} & = & \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array} \\ 2 \times 2 & 2 \times 2 & 2 \times 2 \end{array}$$

Dimension of the matrices

| | | |
|---|----|---|
| 3 | 1 | 4 |
| 2 | -1 | 2 |

2×3

| | | | |
|----|---|---|----|
| 3 | 0 | 1 | -2 |
| 1 | 5 | 2 | 0 |
| -2 | 1 | 4 | 0 |

3×4

=

| | | | |
|---|--|--|--|
| | | | |
| ? | | | |

2×4

Dimension of the matrices

$$\begin{array}{c|c} \begin{matrix} 3 & 1 & 4 \\ 2 & -1 & 2 \end{matrix} & \cdot \begin{matrix} 3 & 0 & 1 & -2 \\ 1 & 5 & 2 & 0 \\ -2 & 1 & 4 & 0 \end{matrix} = \begin{matrix} 2 & 9 & 21 & -6 \\ ? & -3 & 8 & -4 \end{matrix} \\ \begin{matrix} 2 \times 3 \\ 3 \times 4 \end{matrix} & \quad \quad \quad \begin{matrix} 2 \times 4 \end{matrix} \end{array}$$

- Columns of first matrix must match rows of second (numbers in red match)
- Result takes number of rows from first matrix (numbers in blue match)
- Result takes number of columns from second matrix (numbers in purple match)

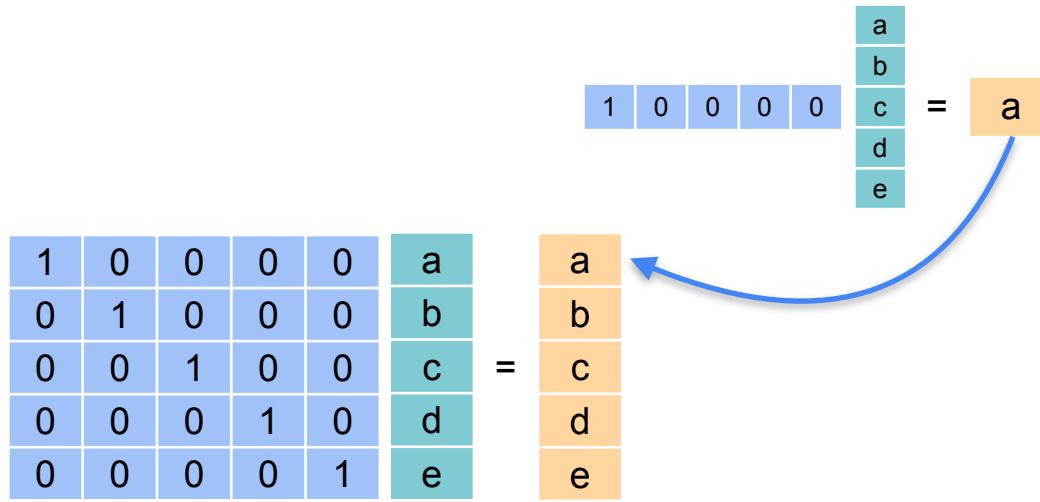


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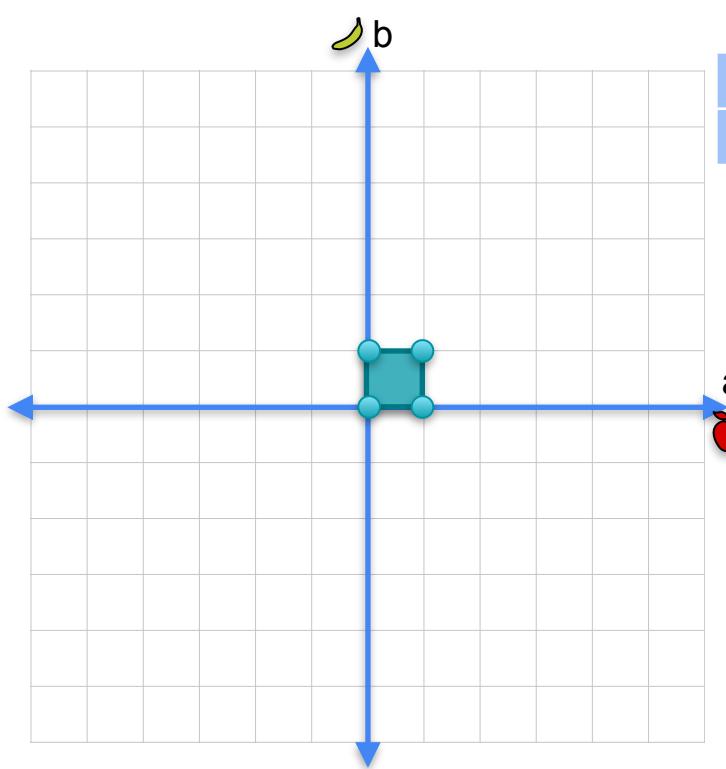
Vectors and Linear Transformations

The identity matrix

The identity matrix

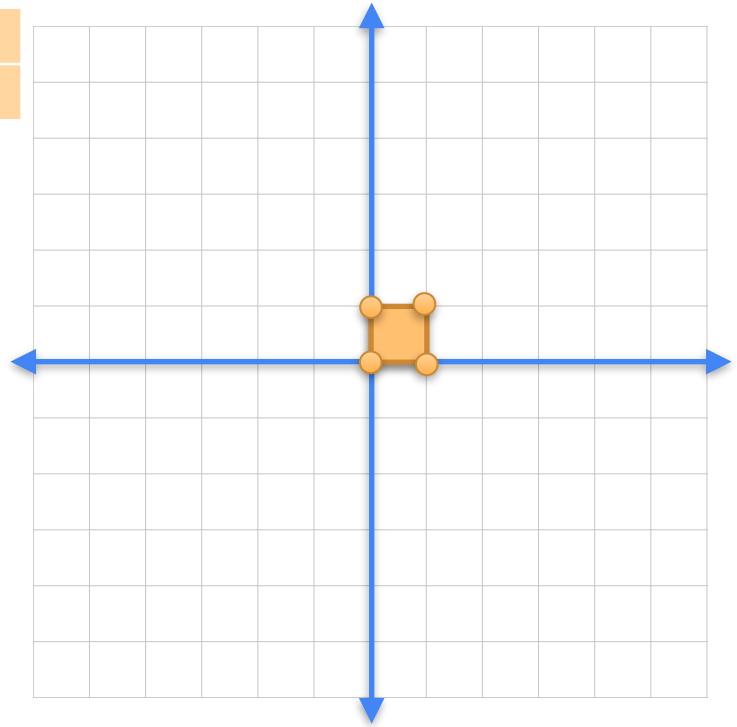


The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} x \\ y \end{matrix} \end{matrix} = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix} & \begin{matrix} x \\ y \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,0)$
 $(0,1) \rightarrow (0,1)$
 $(1,1) \rightarrow (1,1)$



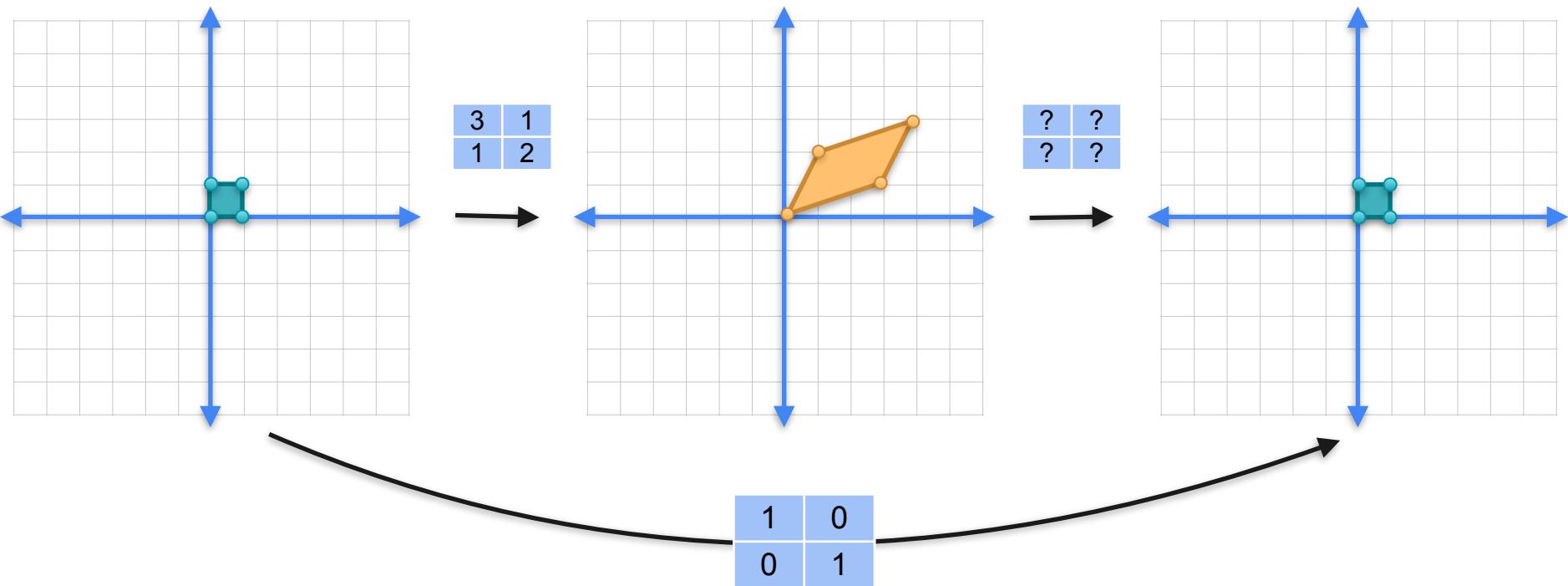


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Vectors and Linear Transformations

Matrix inverse

Matrix inverses



Multiplying matrices

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$
$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{matrix}$$


How to find an inverse?

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1c = 1 \quad a = \frac{2}{5}$$
$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3b + 1d = 0 \quad b = -\frac{1}{5}$$
$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2c = 0 \quad c = -\frac{1}{5}$$
$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1b + 2d = 1 \quad d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

| | |
|---|---|
| 5 | 2 |
| 1 | 2 |

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.



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Vectors and Linear Transformations

Which matrices have an inverse?

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5 ← Det = 8 →

Non-zero determinants

Det = 0 ← Zero determinant →



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Vectors and Linear Transformations

**Neural networks and
matrices**

Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____,
then the email is spam.

Goal: Find the best points and threshold

Lottery: ____ point

Win: ____ point

Threshold: ____ points

Quiz: Natural language processing

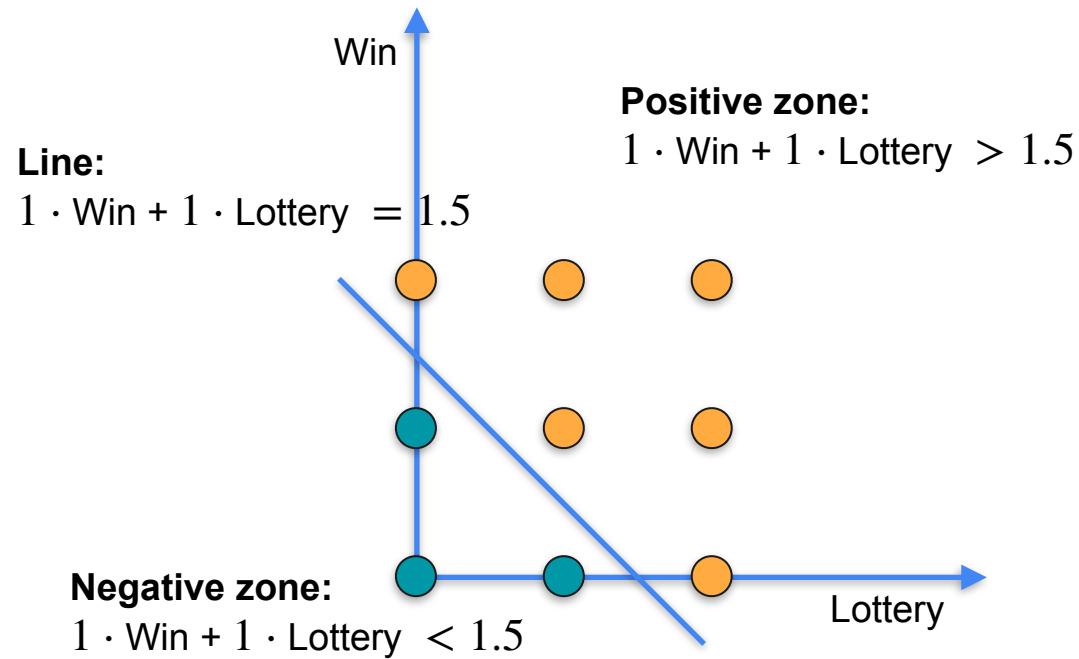
| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

| Score | > 1.5? |
|-------|--------|
| 2 | Yes |
| 3 | Yes |
| 0 | No |
| 2 | Yes |
| 1 | No |
| 1 | No |
| 4 | Yes |
| 2 | Yes |
| 3 | Yes |

Solution:
Lottery: 1 point
Win: 1 point
Threshold: 1.5 points

Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

| Model |
|-------|
| 1 |
| 1 |

= 3

Check: > 1.5?



Spam

Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

$$\begin{matrix} \text{Model} \\ 1 \\ 1 \end{matrix} = 1$$

Check: $> 1.5?$



Not spam

Matrix multiplication

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

$$\begin{matrix} \text{Model} \\ 1 \\ 1 \end{matrix} = \begin{matrix} \text{Prod} \\ 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix}$$

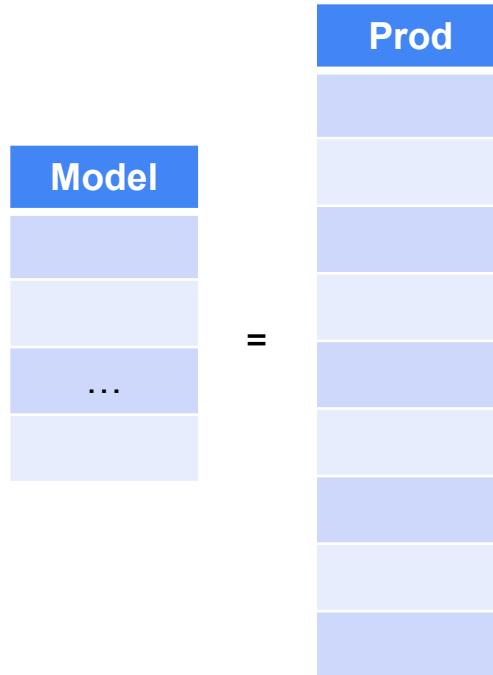
Check: >1.5?



| Check |
|-------|
| Yes |
| Yes |
| No |
| Yes |
| No |
| No |
| Yes |
| Yes |
| Yes |

Perceptrons

| Spam | Word1 | Word2 | ... | WordN |
|------|-------|-------|-----|-------|
| Yes | | | | |
| Yes | | | | |
| No | | | | |
| Yes | | | | |
| No | | | | |
| No | | | | |
| Yes | | | | |
| Yes | | | | |
| Yes | | | | |



Check:



| Check |
|-------|
| Yes |
| Yes |
| No |
| Yes |
| No |
| No |
| Yes |
| Yes |
| Yes |

Threshold and bias

| Spam | Lottery | Win | Bias |
|------|---------|-----|------|
| Yes | 1 | 1 | 1 |
| Yes | 2 | 1 | 1 |
| No | 0 | 0 | 1 |
| Yes | 0 | 2 | 1 |
| No | 0 | 1 | 1 |
| No | 1 | 0 | 1 |
| Yes | 2 | 2 | 1 |
| Yes | 2 | 0 | 1 |
| Yes | 1 | 2 | 1 |

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: > 0?

| Model |
|-------|
| 1 |
| 1 |
| -1.5 |

Bias

The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

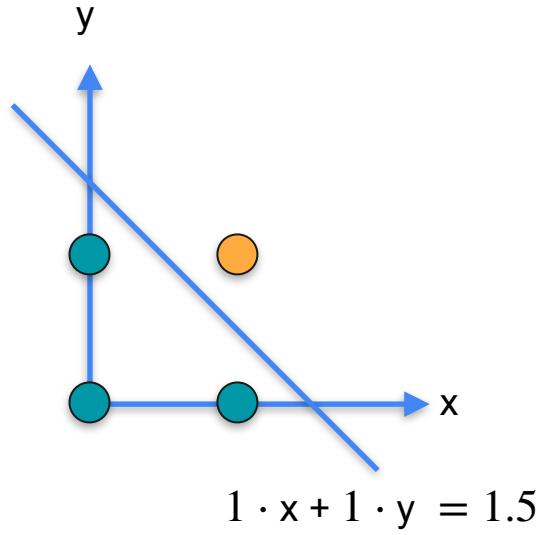
Check: $>1.5?$



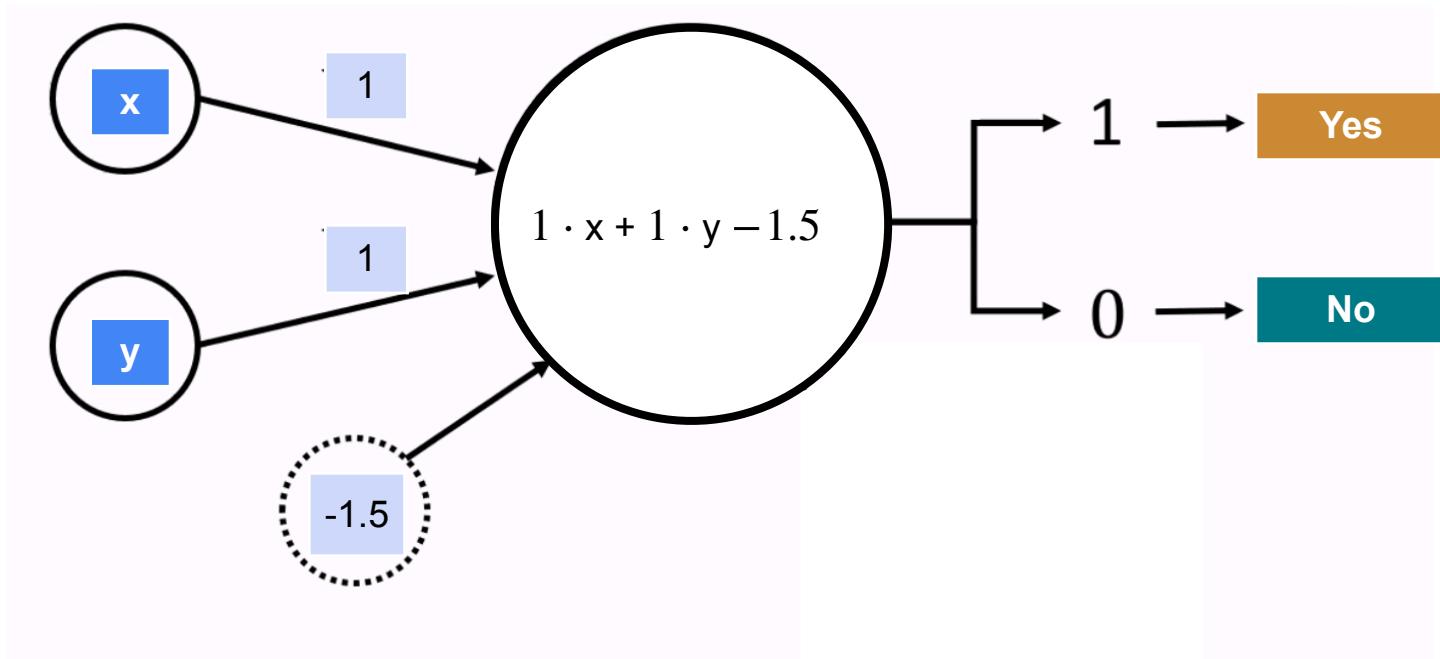
| Check |
|-------|
| No |
| No |
| No |
| Yes |

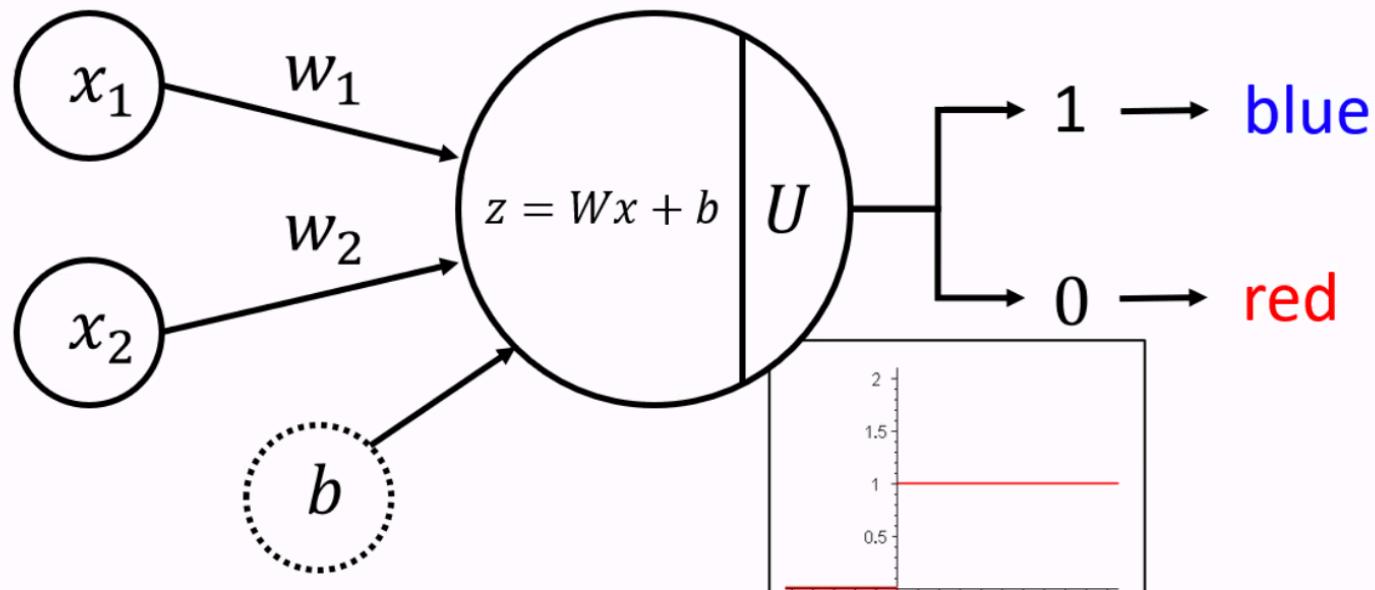
The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |



The perceptron







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Vectors and Linear Transformations

Conclusion