

# HARMONY: how to define a logic?

Logic studies laws governing valid inference.

Gentzen '35  
Dummett '76  
Martin-Löf '83

Helena is a woman  
All women are mortal  
Therefore Helena is mortal

woman(helena)  
 $\forall x, \text{woman}(x) \rightarrow \text{mortal}(x)$   
 $\text{mortal}(\text{helena})$

$P(x)$   
 $\forall x, P(x) \rightarrow Q(x)$   
 $Q(c)$

Judgement	Logic	Computation
A true	intuitionistic logic	$\lambda$ -calculus
A false	classical logic	?
A valid	modal logic	Staged computation
A true at time t	temporal logic	?
A true in world w	modal logic	distributed computation
A physical	linear logic	Concurrent computation

ML: The meaning of a proposition is determined by what counts as a verification of it.  
proof

Defining A

Consequences:

$\frac{A \text{ true} \quad B \text{ true}}{A \wedge B \text{ true}} \wedge I \quad \left| \quad \frac{A \wedge B \text{ true}}{A \text{ true}} \wedge E_1 \quad \frac{A \wedge B \text{ true}}{B \text{ true}} \wedge E_2$

Verificationist takes LHS as defn, RHS as derived

Pragmatist takes RHS as defn, LHS as derived

\* Want these to coincide  $\rightarrow$  ie be in "harmony"

If all introductions followed by elimination don't gain new information: Soundness.

Eg.  $\frac{\frac{A \quad B}{A \wedge B} \wedge I}{A} \wedge E_1 \Rightarrow A$   
(local) Soundness  
Local Reduction

Hypothetical Proof  
 $\frac{\frac{A \wedge (B \wedge C) \text{ true}}{B \wedge C \text{ true}} \wedge E_1}{B \text{ true}} \wedge E_2$

establishes derived inference rule:

$\frac{A \wedge (B \wedge C)}{B}$

or hypothetical judgement:

$A \wedge (B \wedge C) \vdash B$

$\frac{A \text{ true} \quad \dots \quad B \text{ true}}{A \supset B} \supset I^x$   
scope of x  
 $\frac{A \supset B \text{ true} \quad B \text{ true}}{A \text{ true}} \supset E$

If can get out pieces to construct by introduction then it is complete.

$\frac{A \wedge B}{A} \wedge E_1 \Rightarrow A$   
local Expansion  
Completeness

Soundness  
?: how do you figure out what terms to check?

$\frac{\frac{A \supset B}{A} \supset I^x \quad B}{A} \supset E$

$\frac{A \quad B}{A \wedge B} \wedge I \Rightarrow A \wedge B$

Completeness  
 $\frac{A \supset B \quad A}{B} \supset E$

$\frac{A \text{ true}}{A \vee B \text{ true}} \vee I_1 \quad \frac{B \text{ true}}{A \vee B \text{ true}} \vee I_2$

$\frac{A \vee B \text{ true} \quad \text{elim} \quad \text{time} \quad \text{time} \quad \text{time}}{C \text{ true}} \vee E$

Ex. i.  $A \vee (B \wedge C) \vdash (A \vee B) \wedge (A \vee C)$   
ii.  $A \supset (B \supset C) \vdash (A \supset B) \supset (A \supset C)$   
iii.  $A \supset (B \vee C) \vdash (A \supset B) \vee (A \supset C)$

Ex.  $A \oplus B \quad A \supset B, A \wedge B, A \vee B, 0A$   
 $0A \oplus B$  "iff"  $A \oplus B$   
Instead of A true