DS 3021 HW

Question 6

1) how to compute sample mean:

$$m(X) = \frac{1}{N}\sum_{i=1}^{N} x_i$$

definition of the mean:

$$m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

$$\frac{1}{N}\left(\sum a + \sum bx_i\right) =$$

$\sum a = N \times a$

$$\frac{1}{N}\left(Na + b\sum x_i\right) =$$

$$a + b\left(\frac{1}{N}\sum x_i\right) =$$

$$\boxed{a + b \times m(X)} = m(a+bX)$$

2) using sample covariance formula

$$\mathrm{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X)) =$$

$$\boxed{\frac{1}{N}\sum (x_i - m(X))^2} = s^2 = \text{sample variance of } X^2$$

3) using sample covariance formula

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$$

$a+b \times m(X) = m(a+bX)$ → $a + b \times m(Y) = m(a+bY)$

$$\mathrm{Cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - (a+bm(Y)))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(b(y_i - m(Y)))$$

$$= b\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$$

$$\boxed{\mathrm{cov}(X, a+bY) = b\,\mathrm{cov}(X,Y)}$$

4) $\text{cov}(a+bX, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i - m(a+bX))(a+by_i - m(a+bY))$

$= \frac{1}{N}\sum_{i=1}^{N}((\cancel{a}+bx_i) - (\cancel{a}+bm(x))((\cancel{a}+by_i) - (\cancel{a}+bm(Y))$

$= \frac{1}{N}\sum_{i=1}^{N}(b(x_i - m(X))(b(y_i - m(Y))$

$= b^2 \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$

$\boxed{\text{cov}(a+bX, a+bY) = b^2 \text{cov}(X,Y)}$

5)

→ Median: It's $\boxed{\text{true}}$ that the median $a+bX = a+b \times \text{med}(X)$. Since $b>0$, the order of the data points doesn't change. The middle point still stays in the middle.

→ IQR: $IQR(X) = Q_3 - Q_1$ $IQR(a+bX) = (\cancel{a}+bQ_3) - (\cancel{a}+bQ_1) = b(Q_3 - Q_1)$

$IQR(a+bX) = b\,IQR(X)$

$\boxed{IQR(a+bX) \neq a+b\,IQR(X)}$

6)

→ Example for X^2:

$X = \{0, 4\} \rightarrow m(X) = 2$ $X^2 = \{0, 16\} \rightarrow m(X^2) = 8$

$m(X)^2 = 4$

$\boxed{m(X^2) \neq m(X)^2}$

→ Example for $\sqrt{X}$:

$X = \{4, 16\} \rightarrow m(X) = 10$ $\sqrt{X} = \{2, 4\}$

$\sqrt{m(X)} = \sqrt{10} \approx 3.162$ $m(\sqrt{X}) = 3$

$\boxed{\sqrt{m(X)} \neq m(\sqrt{X})}$