Comparison of Direct Methods of NMPC for a Thrust-Vector-Controlled Drone

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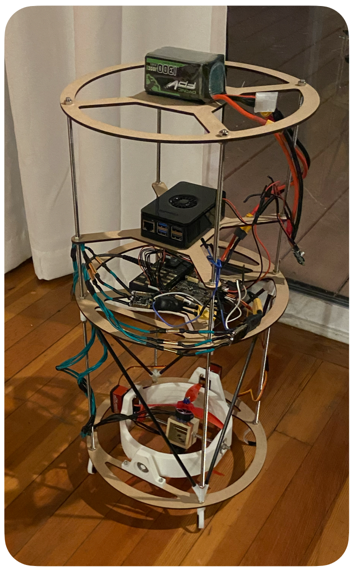
Summary

Orthogonal collocation on finite elements (Legendre-Radau) and Chebyshev pseudospectral have lower mean CPU time than RK multiple shooting for NMPC on a TVC drone and can be run at 50 Hz on a Raspberry Pi 5. Visual inspection of trajectories showed OC w/ LR produces more reliable trajectories than Chebyshev PS or MS w/ RK when identical time horizons and solver settings are used.

Motivation and Problem Description

A thrust-vector-controlled (TVC) drone is a safe, low-cost proxy for studying VTOL rocket control. Nonlinear model predictive control (NMPC) captures the vehicle's nonlinear dynamics and hard limits (gimbal angle, servo-rate, thrust bounds), but is more computationally intensive than proportional integral derivative (PID) control or linear quadratic regulation (LQR). A practical implementation must run at 50 Hz on a Raspberry Pi 5. The transcription method of the resulting nonlinear program (NLP) will affect the speed and accuracy of the control trajectories. Orthogonal collocation on finite elements (Legendre–Radau), Chebyshev pseudospectral collocation, and Runge–Kutta multiple shooting are compared to evaluate mean CPU time and quality of trajectories.

Methodology



Simulations were run using actual specifications from a drone with two stacked brushless motors on a servo-driven gimbal that I built, including experimentally determined mappings of PWM to motor thrust and servo angles.

NLP instances were constructed with 2 second time horizons and identical IPOPT settings and all employed warm starts.
Each method was individually tested to determine optimal time intervals and number of quadrature points for each method.

- MS w/ RK - RK4 integrator; N = 100 uniform intervals of 0.02 (sec). Implemented with CasADi
- Chebyshev PS - 1 interval over full horizon with 6 collocation CGL nodes, dynamics enforced via the spectral differentiation matrix. Implemented with CasADi
- OC w/ LR - 8 elements with 2 Legendre–Radau collocation points per element (degree 2 per element); dynamics constraints and continuity at element boundaries. Implemented using do-mpc library
- Ran multiple test case simulations on the Raspberry Pi 5 with the drone starting in a perturbed state and returning to the goal state to compare methods.

Results

- Orthogonal collocation with Legendre-Radau points and Chebyshev pseudospectral colocation have lower mean CPU time than multiple shooting with Runge-Kutta discretization for NMPC algorithm of a thrust-vector-controlled drone and can be run at 50Hz on Raspberry Pi 5.
- On visual inspection, orthogonal collocation with Legendre-Radau has most reliable trajectories making it the best choice for the TVC drone in practice.

Test Case (200 iterations)	Average CPU time in seconds for NLP solution		
	MS RK	OC LR	CPS
45° rotation about z axis	0.112	0.015	0.008
15° rot about x axis, hop in y dir	0.101	0.012	0.012
hop in x dir, hop in y dir, initial v_x	0.12	0.015	0.014
hover	0.114	0.016	0.009

Next Steps

- Adding other direct methods of transcription to the comparison.
- Adding multiple elements to the Chebyshev PS method. This might improve the quality of the Chebyshev PS trajectories.
- Real world tests on the TVC drone are needed to see if the trajectories observed work in practice.
- Creating a Python library of the Chebyshev PS and multiple shooting RK methods for use in future NMPC projects.

Tools and Skills

- NMPC, state-space modeling
- Numerical methods for ODEs, spectral methods
- Numerical optimization, Ipopt, MUMPS, HSL MA27/57
- 6-DoF dynamics, coordinate frames and transforms (body/world, NED/ENU, quaternions)
- Python, CasADi, do-mpc, NumPy
- Git, GitHub Flow
- Linux, Ubuntu
- MAVLink, I2C/UART/CAN
- Avionics, telemetry, instrumentation
- ROS2, PX4/ArduPilot

Links and Resources

- Github project code
- Technical Report
- Simulation video
- Isidore Mones' personal website