Math 323 (section 05) Binghamton University Fall 2016

Department of Mathematics

Exam 3

Remember to show all of your work. No notes or electronic devices allowed.

(1) Suppose that:

- $f(x, y, z)$ is a scalar function
- $\mathbf{G}(x, y, z)$ and $\mathbf{H}(x, y, z)$ are vector fields
- C is a curve.

Are the following expressions vectors, scalars, scalar functions, vector fields, or nonsense?

(a) $\mathbf{G} \times \mathbf{H}$

(b) $\text{div}(f)$

(c) $\int_C f ds$

(d) $\nabla \times (f\mathbf{G})$

(e) $f + \mathbf{H}$

(2) Use the fundemental theorem of line integrals to calculate

$$\int_C \left\langle y^2z + 2xy, 2xyz + x^2, y^2x + 1 \right\rangle \cdot d\mathbf{r}$$

where C is the curve $\langle x, y, z\rangle = \langle \cos(t), \sin(t), t\rangle$ from $t = 0$ to $t = \pi$.

(3) Use Green's theorem to compute

$$\int_C \langle ye^x, e^x + x \rangle \cdot d\mathbf{r}$$

where C is a triangle with vertices $(0,0), (1,0)$, and $(0,2)$.

(4) Let $\mathbf{F}$ be the vector field

$$\mathbf{F}(x, y, z) = \langle x, \cos(x), yz \rangle$$

Compute

(a) $\nabla \cdot \mathbf{F}$

(b) $\nabla \times \mathbf{F}$

(c) $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where S is the unit sphere $x^2 + y^2 + z^2 = 1$

(d) $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $\langle x, y, z \rangle = \langle \cos(t), \sin(t), 0 \rangle$ on the interval $0 \leq t \leq 2\pi$.

(5) Find all local maxima and minima of the function

$$f(x, y) = x^2 + 2xy + 3y^2 + x$$

on the region defined by

$$2x^2 + y^2 + \frac{2}{3}xy \leq 4.$$

(6) Find, but do not evaluate, an integral which computes the area of the surface

$$z = \sin(x^2 + y^2)$$

inside the region $x^2 + y^2 \leq 1$.

(7) Find the surface integral

$$\int_S \mathbf{F} \cdot d\mathbf{S}$$

where S is the surface defined by $z = x^2 + y^2$ subject to $x^2 + y^2 \leq 1$ and $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$.

(8) Evaluate the integral

$$\int_0^3 \int_0^{\sqrt{9-x^2}} e^{-x^2-y^2}\, dy dx$$

by converting to polar coordinates.