Machine Learning

Time series forecasting

Part 2:
statistical models

Agenda

- we will talk about two **most commonly used** model families:
  - ARIMA (autoregressive)
  - ETS (exponential smoothing)
- lastly, we will cover forecasting based on **regression (ML)**
- other methods are more niche, but very interesting, we'll mention them at the end

ARIMA

ARIMA - introduction

- **family of models:** AR, MA, ARMA, ARIMA, SARIMA, SARIMAX, …
- **idea:**
  - "history repeats itself"
  - recent values directly influence future values
  - model **autocorrelation** in the data
  - **short memory** models - based on recent values
- building blocks:
  - ARIMA = I(d) + ARMA(p,q) = I(d) + AR(p) + MA(q)
  - SARIMA = seasonality + ARIMA
  - SARIMAX = SARIMA + exogenous variables X
  - ARMA, ARMAX, SMAX etc.

Differencing - I(d)

- AR, MA, ARMA models **assume (require)** stationary time series
- first step: differencing of order *d*
- **automated algorithm:**
  - loop, KPSS test, take difference
  - typically $d_{max} = 2$
- you definitely should **manually verify** if possible:
  - trend still present - change detrending method and/or algorithm
  - outliers - various clipping / thresholding / anomaly removal methods
  - seasonality - use SARIMA

Autoregressive AR(p) models

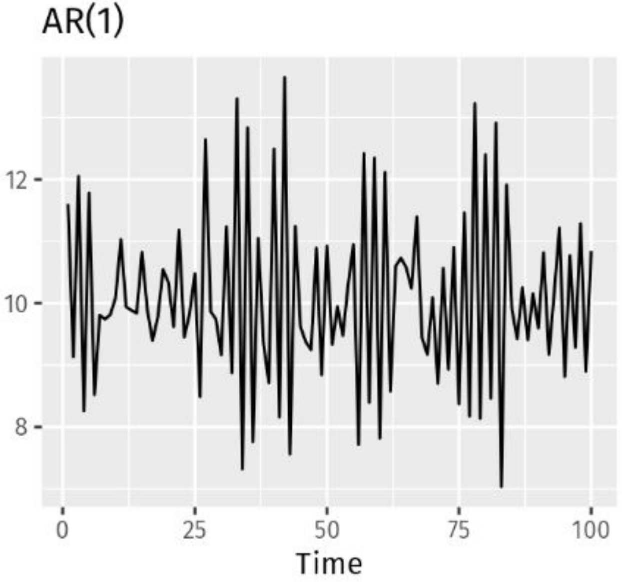
- **idea:** next value depends linearly on the last *p* values
- **linear regression** using last *p* values, known as **lagged features:**

$$\hat{y}_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \epsilon_t$$

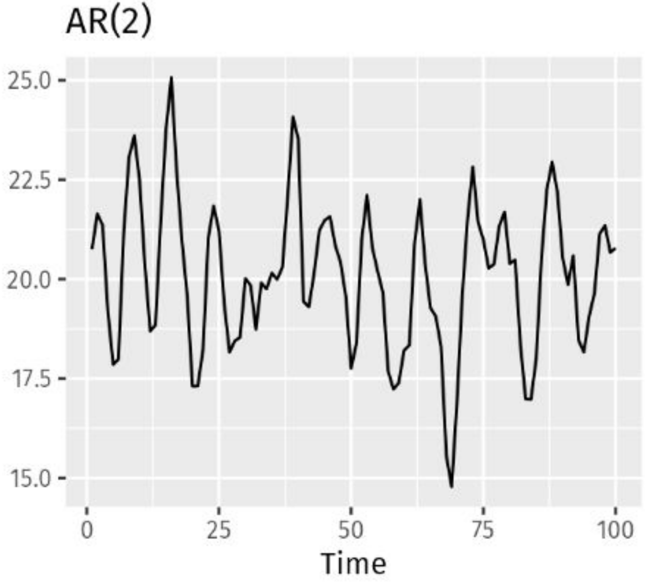
- **training method:** OLS, computed using SVD
- very fast, unique solution, numerically stable
- beginning values: either cut out, or assume $y_0 = 0, y_{-1} = 0, \dots$
- **why such simple model?**
  - closed likelihood formula, MLE estimation
  - can combine with MA(q) to create ARMA and ARIMA models
  - probabilistic forecasts, confidence intervals
  - allows very fast hyperparameter tuning via information criteria

Autoregressive AR(p) processes

$$y_t = 18 - 0.8y_{t-1} + \epsilon_t$$



$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \epsilon_t$$



https://otexts.com/fpp3/AR.html

Moving average MA(q) models

- **be careful:** nothing to do with moving average, nor with exponentially weighted moving average (EWMA) model
- **idea:** next value depends on the last *q* errors

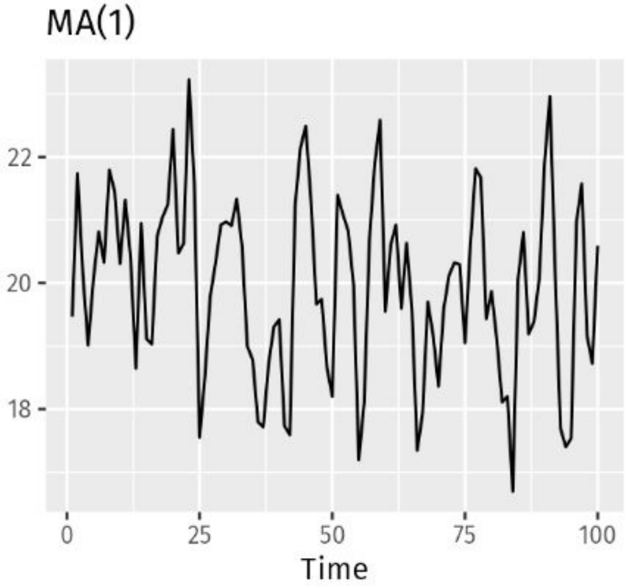
$$\hat{y}_t = c + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_q \epsilon_{t-q} + \epsilon_t$$

- allows for **correction** of the last errors, e.g. when trend direction changes
- assumes that last *q* errors are autocorrelated
- **problem:** those errors are not directly observable, so this is not linear regression
- however, if we **assume normal errors**, we can derive likelihood and do MLE estimation

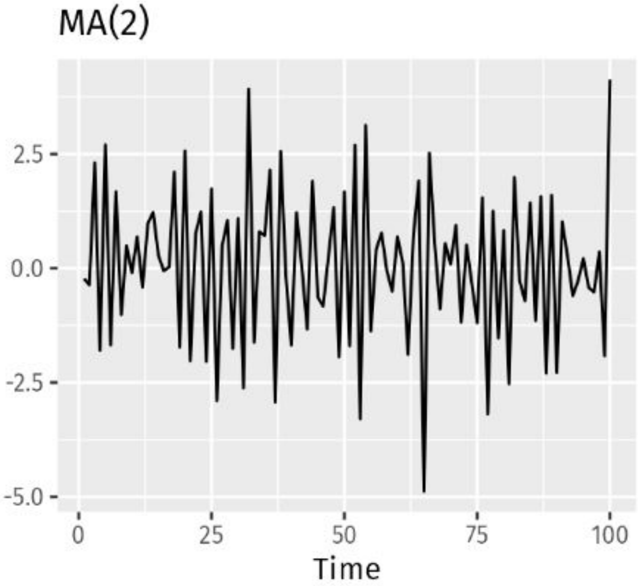
$$\epsilon_t \sim \mathcal{N}(0,\, \sigma^2)$$

Moving average MA(q) processes

$$y_t = 20 + 0.8\epsilon_{t-1} + \epsilon_t$$



$$y_t = -\epsilon_{t-1} + 0.8\epsilon_{t-2} + \epsilon_t$$



https://otexts.com/fpp3/MA.html

ARMA(p,q) models

- **combination** of AR(p) and MA(q) into a single model:

$$\hat{y}_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t$$

- **idea:**
  - AR(p) models values autocorrelation
  - MA(q) corrects mistakes based on errors autocorrelation
- **problem:** since we have MA(q), we can't use OLS
- **training method:** there are a few, but MLE (or hybrid with MLE) is the most common one

- we won't be showing likelihood formulas, since they are quite complex; see additional resources if you're interested

Training ARMA(p,q) models

MLE:

- assume normal errors
- derive likelihood formula, and optimize NLL as cost function
- typically uses L-BFGS quasi-Newton optimizer
- popular: trivial code, strong statistical properties, probabilistic forecasting

Hannan-Rissanen method:

- start with AR(p), train with OLS
- compute errors - when they are known, we can train MA(q) with OLS
- then we have a choice:
  - repeat in a loop, until convergence ("pure" H-R)
  - switch to MLE, which typically rapidly converges (most common)

ARIMA(p,d,q) models

- ARIMA(p,d,q): first differencing I(d), then ARMA(p,q)
- **problem:**
  - how to choose *p*, *d* and *q*?
  - maybe don't use some, e.g. use ARI(p,d)?
  - higher *p* and *q* typically fits training data better, but also has higher risk of overfitting
- grid search CV - possible, but slow
- **AutoARIMA:**
  - *d* - statistical tests in a loop, up to d_{max}
  - *p* and *q* - build a grid and check it stepwise
  - use AIC (or AICc) to measure model quality

ARIMA - additional resources

- D. Childers - "ARIMA"
- J. Li - "ARMA Model"
- P. Cízek et al. - "Estimation of MA(q) and ARMA(p,q) Models"
- R. Nau - "Introduction to ARIMA models"
- M. Zhang - "Time Series: Autoregressive models AR, MA, ARMA, ARIMA"

- Real Statistics - "Calculate ARMA(p,q) coefficients using maximum likelihood"
- Statistics Ninja - "3 ARIMA Models - 3.5.2 Estimation - Maximum Likelihood Estimation"
- Lasse Engbo Christiansen - "02417 Lecture 12 part A: ARMA models on State space form"

Akaike's Information Criterion (AIC)

- **information criteria** measure model quality; AIC specifically is defined as:

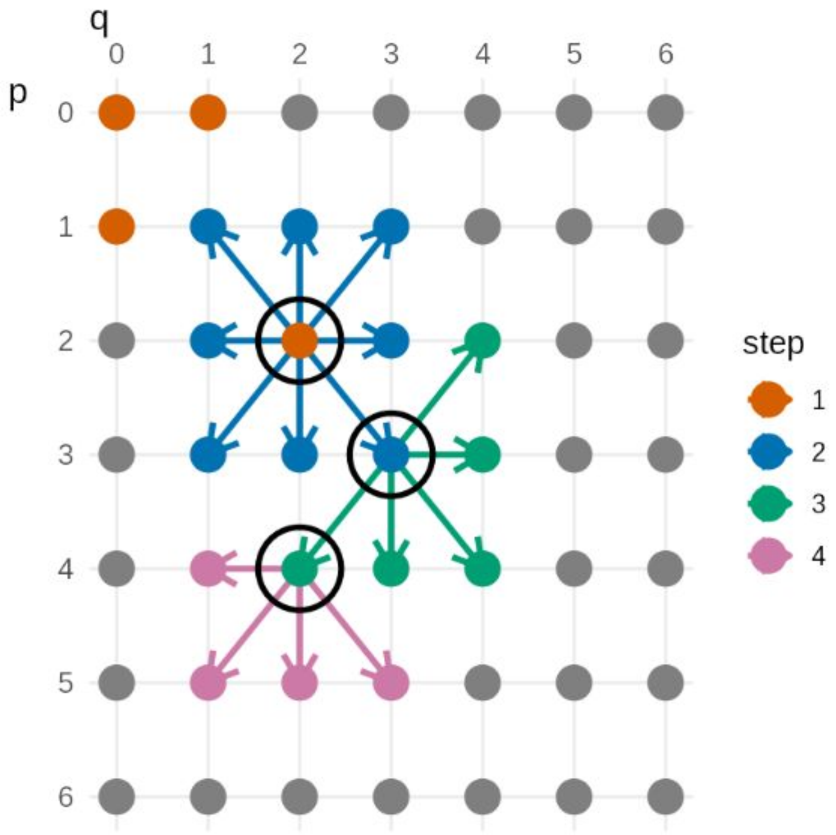
  $$AIC = -2\log(L) + 2k = 2 * NLL + 2k$$

  L - likelihood, how well model fits the data

  k - number of parameters, penalty for model complexity, "regularization"
- for ARIMA, $k = p + q$
- **alternative** to cross-validation score:
  - just calculate AIC using training data, and select the model with the lowest AIC
  - balances data fit and model complexity
- **pros:** speed, uses all data for training
- there are other information criteria, e.g. AICc, BIC, HQIC
- AICc corrects for small samples, useful e.g. in macroeconomics

Stepwise selection

- which *p* and *q* values should we check?
- **grid search:** very expensive, especially for expanding window
- **stepwise selection:**
    - check a few initial models
    - then go in "steps", checking neighboring values of *p* and *q* on the grid
    - typically gives very good results, despite greedy search
- traditionally checks at most 94 models



https://otexts.com/fpp3/arima-r.html

SARIMA

- **SARIMA = Seasonal ARIMA**
- simply add seasonal terms to ARIMA:

$$\hat{y}_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \sum_{i=1}^{P} \eta_i y_{t-i*m} + \sum_{i=1}^{Q} \psi_i \epsilon_{t-i*m} + \epsilon_t$$

  - SARIMA(p,d,q)(P,D,Q) with seasonality *m*
  - seasonal differencing
- **pros:** incorporates seasonality
- **cons:** computational cost, only single seasonality, not suitable for very long seasonal periods
- designed for relatively short seasonalities, e.g. 7, 14, 30 for daily data

Exogenous variables

- **exogenous variables** are additional features, incorporated in order to help the prediction
- e.g. for sales forecasting, variable "is weekend day" or "is a week before Black Friday"
- also known as:
  - auxiliary variables
  - covariates
- can be **static** or **dynamic:**
  - static - do not change with time, e.g. shop location, area in square meters
  - dynamic - change with time, e.g. "is there an ongoing promotion"
- matrix X with features, **known** at the moment of new forecast y_t

ARIMAX

- **ARIMA + exogenous variables**
- variants: ARMAX, ARIMAX, SARIMAX etc.
- they act as an additional linear regression; note that those variables are also differenced!
- ARMAX (for *N* exogenous variables, at time *t*):

$$\hat{y}_t = c + \sum_{i=1}^{N} w_i x_i^t + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t$$

- **pros:** often much better results, features are often quite obvious
- **cons:** computational cost, feature engineering

ARIMA extensions - additional resources

- FPP: Estimation and order selection, ARIMA modelling in fable
- Stack Overflow - Why SARIMA has seasonal limits?
- Rob Hyndman - "Forecasting with long seasonal periods"
- Rob Hyndman - "The ARIMAX model muddle"

ARIMA multi-step forecasts

- ARIMA by default gives formula for **one-step** forecast
- but how to do **multi-step** forecast?
- we just do forecasts one after another *h* times, and assume we are always correct
- last forecast is used as ground truth for the next forecast
- for MA(q) we assume new errors are zeros, e.g. $\epsilon_t = 0$
- model always makes a mistake, so we have **error compounding**
- this is a typical behavior for autoregressive models
- this is why confidence intervals get wider in time!

ARIMA - pros and cons

Pros:

- elastic and powerful
- typically work great for data with strong autocorrelations
- automation, AutoARIMA
- seasonality (SARIMA)
- exogenous variables (ARIMAX)

Cons:

- stationarity assumption (hard to meet)
- do not work well for strong, complex trend and seasonality
- quite large computational cost
- only additive models
- often require complex preprocessing (Box-Cox, seasonal differencing, regular differencing etc.)

Exponential Smoothing (ETS)

ETS models

- ETS = ExponenTial Smoothing / Error-Trend-Seasonality (it's the same)
- **family of models**, which directly model average value, trend, seasonality, and residual error
- they use **exponential weighting** - newest observarions are much more important than past ones
- they generally "smooth out" latest values, hence the name
- **do not** assume stationary data
- work well for **small data samples**, and are also extremely **fast** to train

Moving average smoothing

- a.k.a. **Simple Moving Average (SMA)** model
- **idea:** next value is an average of (last) previous values
- canonical version uses all historical data, and is just "predict average" baseline:

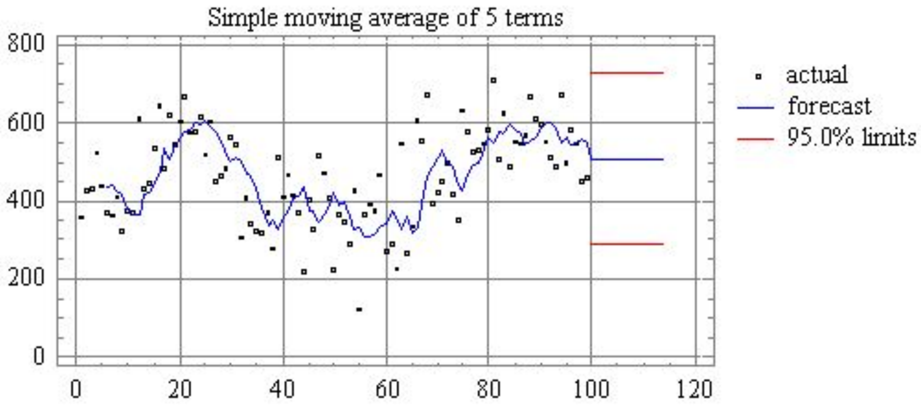
$$\hat{y}_{T+h|T} = \text{avg}(y) = \frac{y_1 + ... + y_T}{T}$$

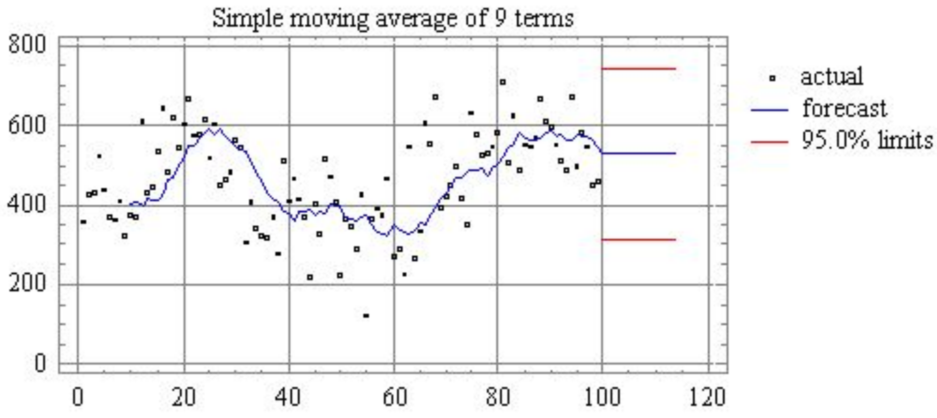
- moving window version uses only last *W* values:

$$\hat{y}_{T+h|T,T-1,...,T-W} = \frac{y_T + y_{T-1} + ... + y_{T-W}}{W}$$

- works surprisingly well if data is slow to change
- prediction is a horizontal line (just like baseline)

Moving average smoothing





https://people.duke.edu/~rnau/411avg.htm

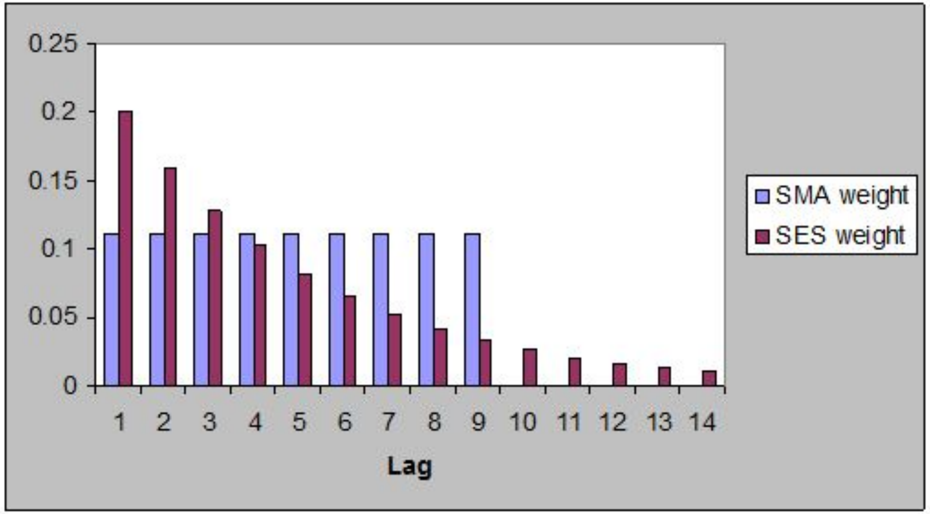
Simple Exponential Smoothing (SES)

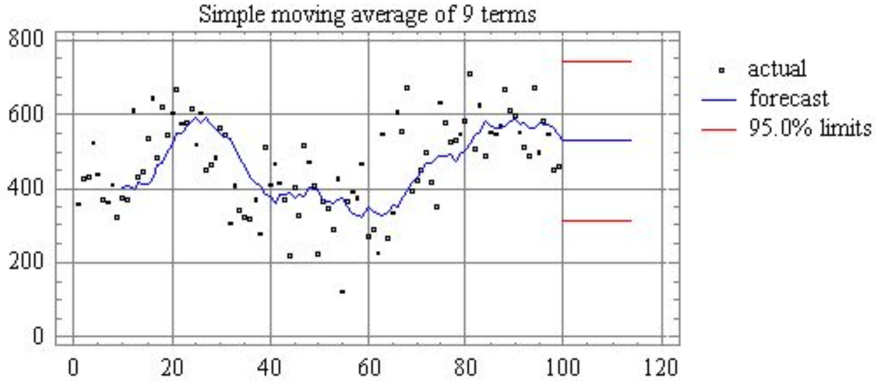
- a.k.a. exponentially weighted moving average (EWMA)
- **idea:** last values are much more important, and weight falls exponentially fast
- weight of last observation is $\alpha \in (0, 1)$, and forecast is:

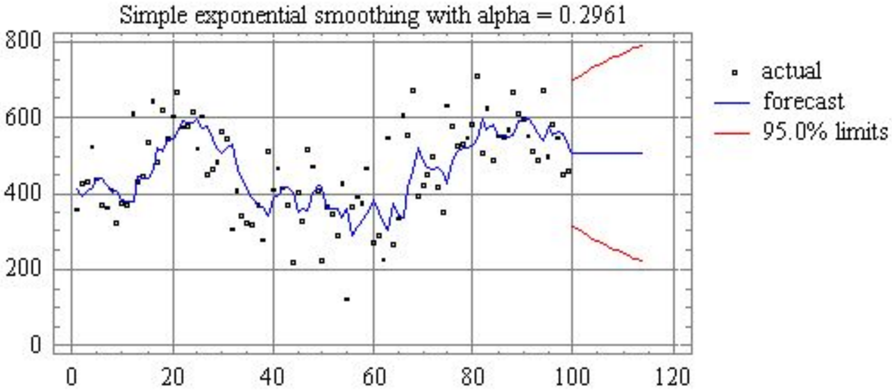
$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha) y_{t-1} + (1 - \alpha)^2 y_{t-2} + \ldots$$

- still predicts a horizontal line, known as **level**
- smoothing, because it "smooths out" last values
- *α* parameter (how much to weight past values) is learned from data

Simple Exponential Smoothing (SES)







https://people.duke.edu/~rnau/411avg.htm

Component form

- to add further elements (trend, seasonality), it's convenient to write SES in so-called **component form**
- SES component formula explicitly models forecast (forecast equation) and level (level equation):

$$\begin{aligned} \hat{y}_{t+h|t} &= l_t \\ l_t &= \alpha y_t + (1-\alpha) l_{t-1} \end{aligned}$$

last forecast
(here, just the level)

l - level

- recurrent formula is convenient for multi-step forecasts
- for SES this is trivial, but further ETS expansions add:
    - further elements to the first formula (forecast equation)
    - further equations below

Holt model (double smoothing)

- a.k.a. Holt linear trend model (author - Charles Holt)
- **idea:** add linear trend, by another smoothing
- estimates trend line with slope b_t (trend equation):

$$
\begin{aligned}
\hat{y}_{t+h|t} &= l_t + b_t * h \\
l_t &= \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1}) \\
b_t &= \beta(l_t - l_{t-1}) + (1-\beta) b_{t-1}
\end{aligned}
$$

line: $y = b + ax$

last forecast is now level + trend

trend = slope = difference in level

- level and trend both use exponential smoothing, hence the name
- equations are still "weight * curr_val + (1 - weight) * prev_val", just now we have 2
- training now learns α and β

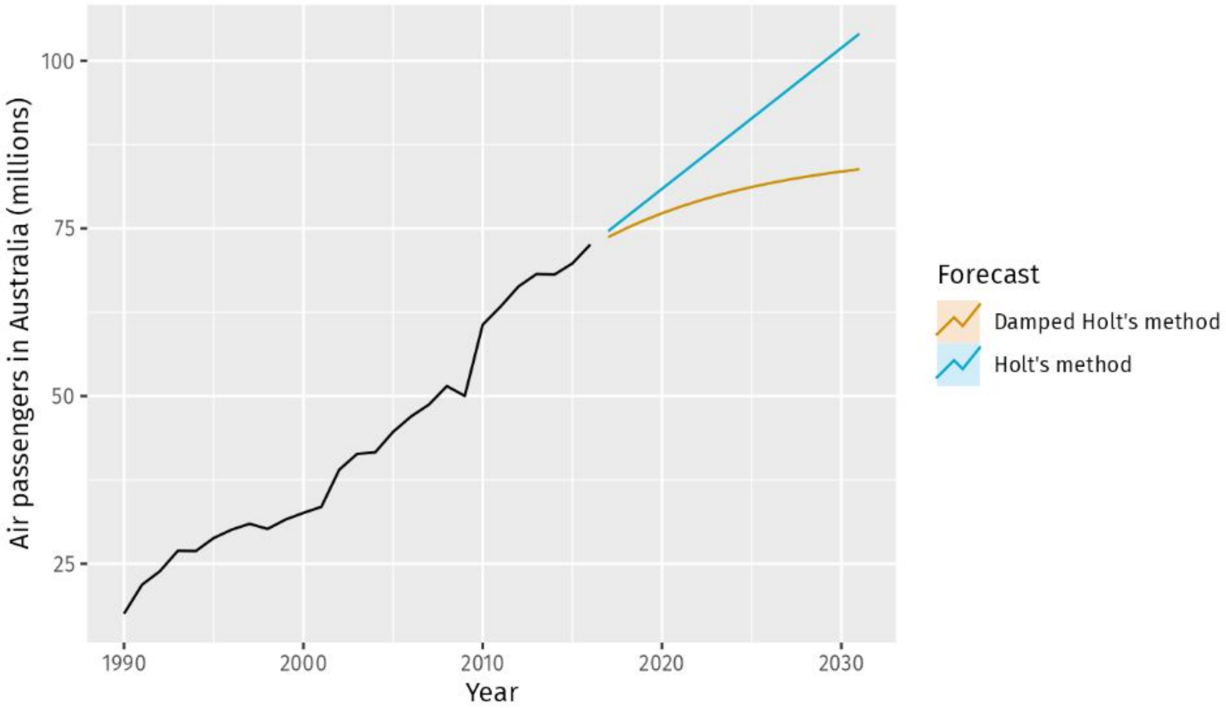
Damped trend

- **problem:** Holt's method trend goes to infinity, which gives unrealistic forecasts for longer horizons
- **damped trend:** multiply the trend in each step by $\phi \in [0, 1]$, which decreases it

$$
\begin{aligned}
\hat{y}_{t+h|t} &= l_t + (\phi + \phi^2 + \ldots + \phi^h) * b_t \\
l_t &= \alpha y_t + (1-\alpha)(l_{t-1} + \phi b_{t-1}) \\
b_t &= \beta(l_t - l_{t-1}) + (1-\beta)\phi b_{t-1}
\end{aligned}
$$

- typically $\phi \in [0.8, 0.98]$
- we'll omit this for readability in the next slides, but can be used for all ETS models
- typically **much better** forecasts, especially for longer horizons
- estimating one extra parameter ϕ is typically cheap

Holt's model (double smoothing)



https://otexts.com/fpp2/holt.html

Holt-Winters model (triple smoothing)

- **idea:** add seasonality with third smoothing
- for given seasonality *m*, add third parameter γ :

same moment from the last season

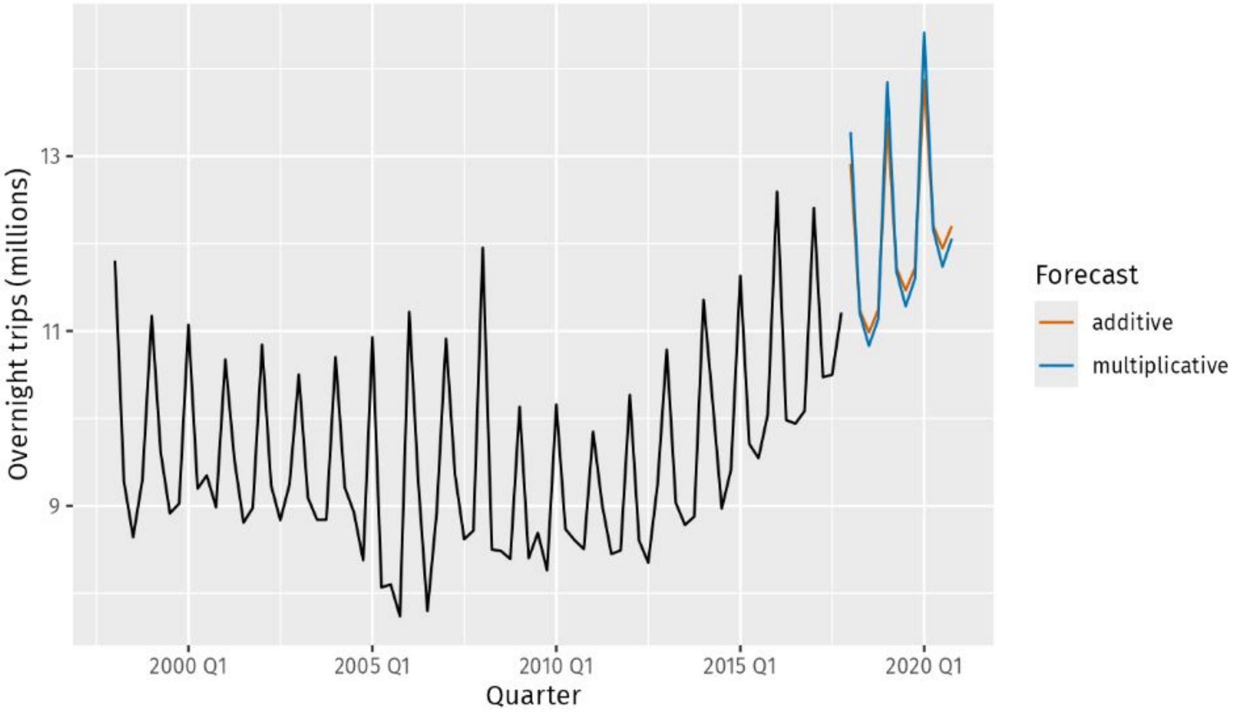
$$
\begin{aligned}
\hat{y}_{t+h|t} &= l_t + b_t * h + s_{t+h-m(k+1)} \\
l_t &= \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1}) \\
b_t &= \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \\
s_t &= \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}
\end{aligned}
$$

$$k = \lfloor \frac{h-1}{m} \rfloor$$

y = level + trend + seasonal --> seasonal = y - level - trend

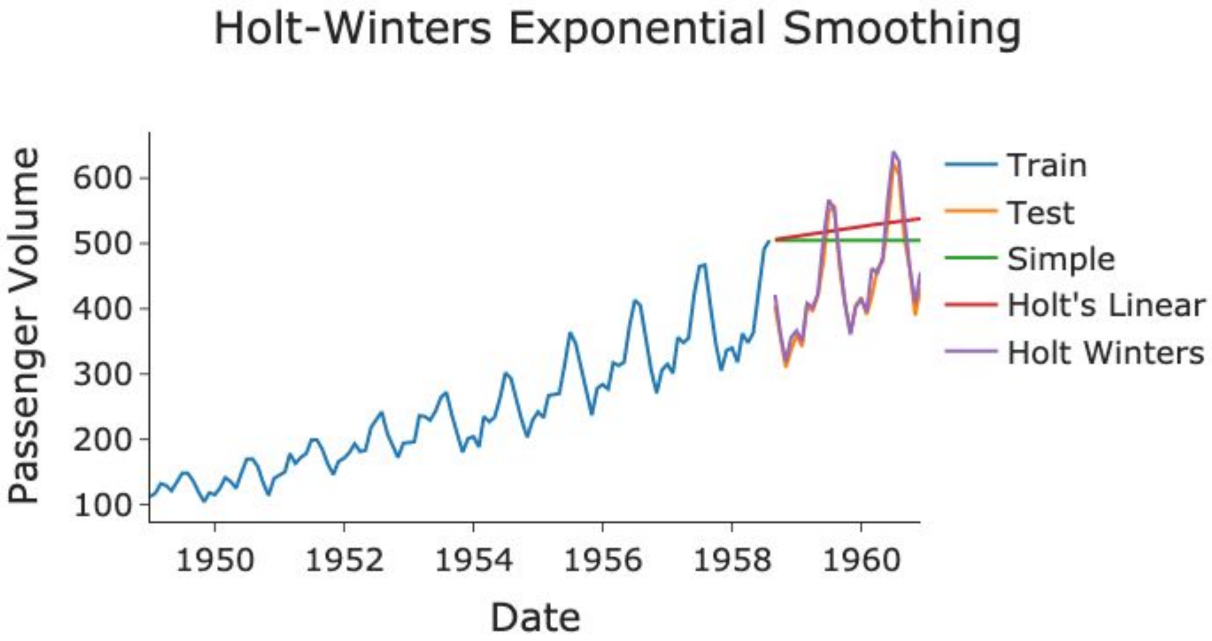
- level equation changes, since we now estimate the level during the given season, so we have to subtract the seasonal part
- assumes additive seasonality (like e.g. STL); there is also a multiplicative variant
- only short seasonality supported, shorter than for ARIMA

Holt-Winters model (triple smoothing)



https://otexts.com/fpp3/holt-winters.html

ETS models comparison



https://freedium.cfd/https://towardsdatascience.com/time-series-forecasting-with-holt-winters-b78ffc322f24

Training ETS models

- **training:** MLE
- **algorithm:** typically L-BFGS
- likelihood equations are complex, and their derivation is **very** complex, so we omit them here
- in practice, training is very fast
- ETS models are very strongly related to state-space models (SSMs) and Kalman filters

ETS models and SSMs - additional resources

- FPP slides - State space models
- "A state space framework for automatic forecasting using exponential smoothing methods" R. Hyndman et al.
- "Forecasting with Exponential Smoothing: The State Space Approach" R. Hyndman et al.
- D. Childers - "State space models"

ETS models and ARIMA

- all ETS models shown were **purely additive:**

  y = level + trend + seasonality + error
- in particular, we assume that prediction errors are additive (sum up with time)
- it can be shown that all purely additive ETS models have ARIMA counterparts
- but this **does not** mean that ETS is just a subset of ARIMA!
  - they don't assume stationarity
  - damped trend variants
  - there are many multiplicative variants, with no ARIMA counterpart
  - much faster training

ETS models family

- full grid of possible ETS models:
  - **E (error):** additive (A), multiplicative (M)
  - **T (trend):** no trend (N), additive (A), additive damped trend (A_d)
  - **S (seasonality):** no seasonality (N), additive (A), multiplicative (M)
- multiplicative trend is typically omitted, due to unstable models and bad forecasts
- additive and multiplicative error give the same point forecasts, but differ in confidence intervals
- if you're interested - full table of ETS models, with formulas

ETS models and ARIMA

Model	ETS formula	ARIMA formula
Simple exponential smoothing	(A,N,N)	(0,1,1)
Holt model	(A,A,N)	(0,2,2)
Holt model with damped trend	(A,A_d,N)	(1,1,2)
Holt-Winters model	(A,A,A)	$(0,1,m+1)(0,1,0)_m$
Holt-Winters with damped trend	(A,A_d,A)	$(1,0,m+1)(0,1,0)_m$
Holt-Winters with multiplicative seasonality	(A,A,M)	-

AutoETS

- **problem:** how to choose ETS model?
- we have MLE = we can compute AIC
- AutoETS works just like AutoARIMA (but typically much faster)
- **be careful:**
  - AIC-based approach is the same, but likelihood has different values, since those are completely different formulas and models!
  - we can't compare AIC between ARIMA and ETS, it can be used only to select models in one family

ETS - additional resources

- FPP: Exponential smoothing
- Crunching the Data - "When to use exponential smoothing models"
- R. Tibshirani - "Exponential Smoothing With Trend and Seasonality"
- I. Svetunkov - "Why you should care about Exponential Smoothing"
- Cross Validated - Holt Winters with exogenous regressors in R

ETS - pros and cons

Pros:

- great results, especially for strong trend and seasonality
- very fast
- good for small data
- many variants (including multiplicative), quite flexible
- can often model longer dependencies than ARIMA

Cons:

- not good when data often has dynamic or drastic changes
- cannot model very complex relations
- typically not good for long forecasting horizons
- only single seasonality
- only relatively short seasonal periods supported
- no exogenous variables support

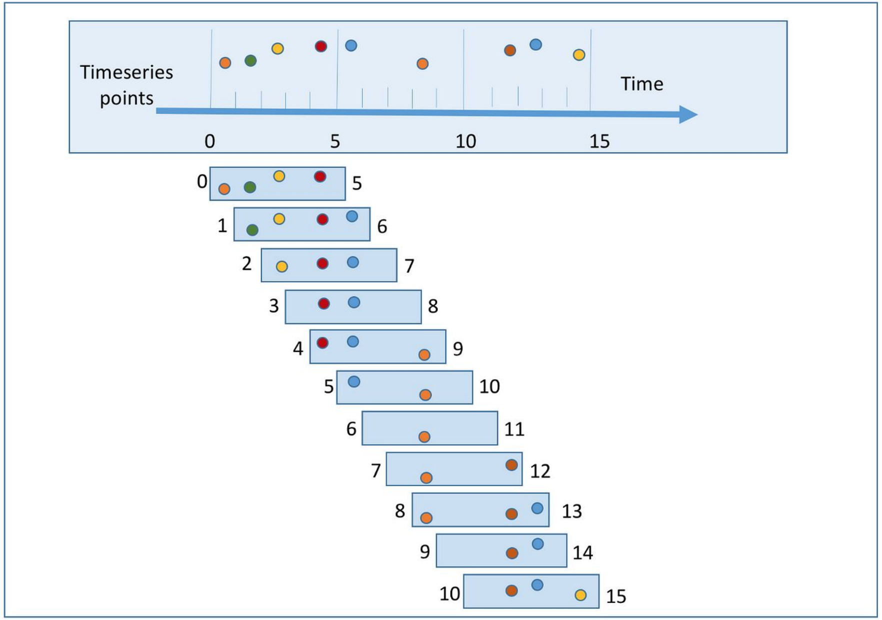
Forecasting as ML regression

Regression-based forecasting

- forecasting = regression + time
- **idea:** treat it exactly like regression problem, using time-based features
- typically called just "ML" in time series literature, and deep learning is always named explicitly
- features can only use data available **at the time of forecast**
- feature engineering relies on past data aggregates and exogenous variables
- **popular regressors:**
    - LightGBM boosting (rarely other ones)
    - linear models, in particular robust models (quantile / LAD, Huber)
- typically **state-of-the-art (SOTA)** in forecasting competitions, and often in business applications

Moving window features

- a. k. a. sliding window, rolling window
- **statistics** of the last N values, typically multiple ones
- e.g. mean, sum, min, max, stddev, quantiles
- **moving average** measures latest trend, e.g. average sales in the last 7, 14, 30 days
- **standard deviation** measures volatility, i.e. strength of shocks and changes
- **expanding window** can also be used, with values from the start of the time series; this captures more global properties



https://freedium.cfd/https://medium.com/@thedatabeast/time-series-part-4-determining-the-window-size-for-moving-averages-a07c5cfcfac9

Date / event features

- exogenous variables describing **important dates** throughout the time period
- also called "event features", since they often describe whether a special event takes place
- application- and domain-dependent
- examples:
    - **quarter** - sales often drop in 4th quarter
    - **is a Black Friday week** - a lot of promos and higher sales
    - **is December / January** - much higher sales before holidays, drastic fall after
    - **days in month** - 28 vs 31 days impact sales
    - **is weekend / is Sunday / is a holiday** - high impact on business days, also shops can be closed

Encoding date and time

- including **date and time features** is not straightforward:
  - we have a "wraparound", e.g. hours 1 and 23 are far as numbers, but close in reality
  - seasonality can have similar impact, e.g. similar weather each July
  - same for other cases, e.g. days in month (1st and 31st), months
  - formally called **cyclical** nature of time
- **naive** options:
  - ordinal encoding (just integers), one-hot encoding (independent binary features)
  - very simple, sometimes work, but don't encode the cyclicity
  - **pros:** works well when particular value is important, e.g. given month
  - **cons:** high dimensionality, sparse data, doesn't work well if data ranges are important (e.g. afternoon hours)

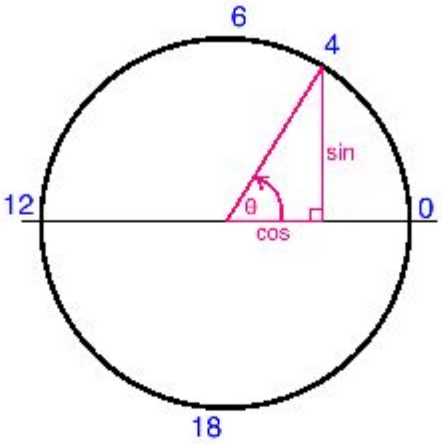
Cyclical encoding

- **idea:** encode timestamp using a **trigonometric basis**, as a point on a unit circle
- uses sine and cosine parts, so 2 features, with **radians** as unit:

$$x(a) = \sin\left(\frac{a * 2\pi}{\max(a)}\right) \qquad y(a) = \cos\left(\frac{a * 2\pi}{\max(a)}\right)$$

  a - timestamp as integer, e.g. hour, month

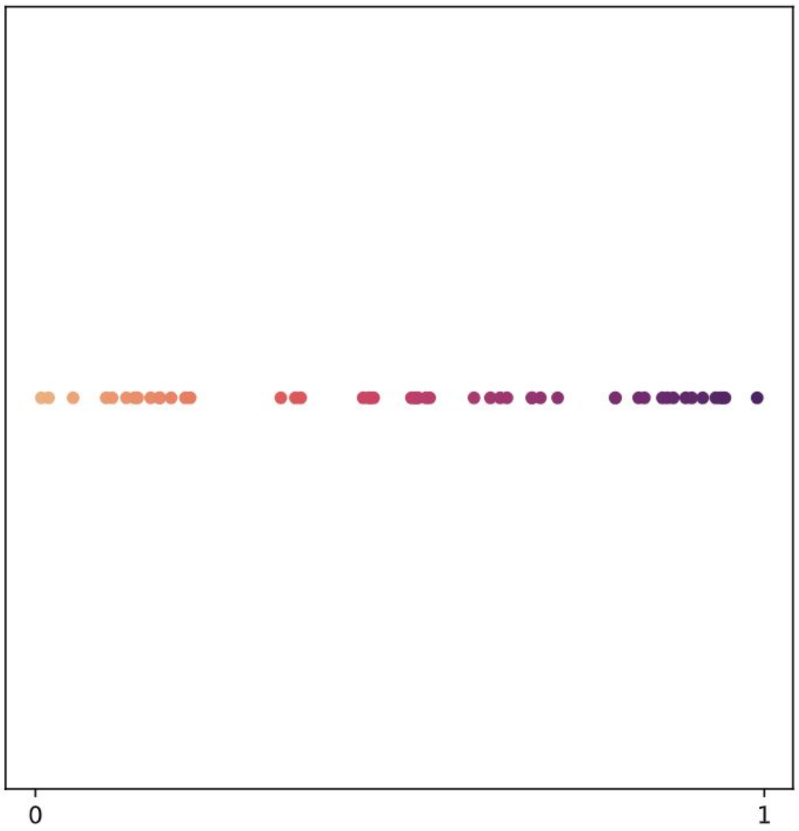
  max(a) - maximal period value, e.g. 24 for hours, 12 for months

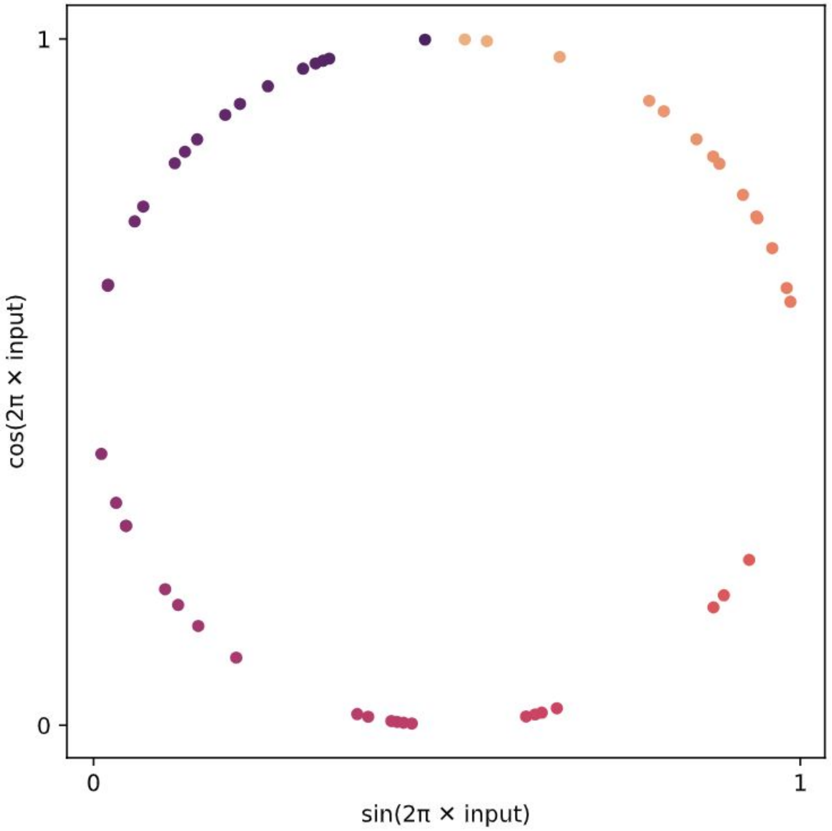


- this way we have a natural wraparound, like on a clock - large values are close (in radians) to small ones
- **pros:** encodes cyclicity, works well for time periods
- **cons:** 2 separate features, which are not directly connected (tree-based models split on single feature)

https://blog.davidkaleko.com/feature-engineering-cyclical-features.html

Cyclical encoding - visualization





https://harrisonpim.com/blog/the-best-way-to-encode-dates-times-and-other-cyclical-features

Cyclical encoding - additional resources

- "Cyclical Encoding: An Alternative to One-Hot Encoding for Time Series Features" P. Hayden
- "Encoding Cyclical Features for Deep Learning" A. Van Wyk
- "The best way to encode dates, times, and other cyclical features" H. Pim
- "Why We Need Encoding Cyclical Features" A. Kud
- "Understand the capabilities of cyclic encoding" S. Matsumoto
- "Feature Engineering - Handling Cyclical Features" D. Kaleko
- "Handling cyclical features, such as hours in a day, for machine learning pipelines with Python example" A. Sefidian

Fourier features

- **idea:** model seasonality with sines and cosines with varying frequency - **Fourier terms**
- also called **harmonic regression**
- seasonality is **periodic** by definition, so Fourier approximation works really well
- for time series with period *m*, it creates pairs of features:

$$x_k(t) = \sin\left(\frac{2k\pi t}{m}\right) \qquad x_{k+1}(t) = \cos\left(\frac{2k\pi t}{m}\right)$$

$$x_1(t) = \sin\left(\frac{2\pi t}{m}\right), x_2(t) = \cos\left(\frac{2\pi t}{m}\right), x_3(t) = \sin\left(\frac{4\pi t}{m}\right), x_4(t) = \cos\left(\frac{4\pi t}{m}\right), \ldots$$

- more terms = better approximation, but often **just a few** suffice
- K is a hyperparameter, also feature selection can be used
- K has max value *m*/2, when it's equal to one-hot encoding

Fourier features

- pure harmonic regression:

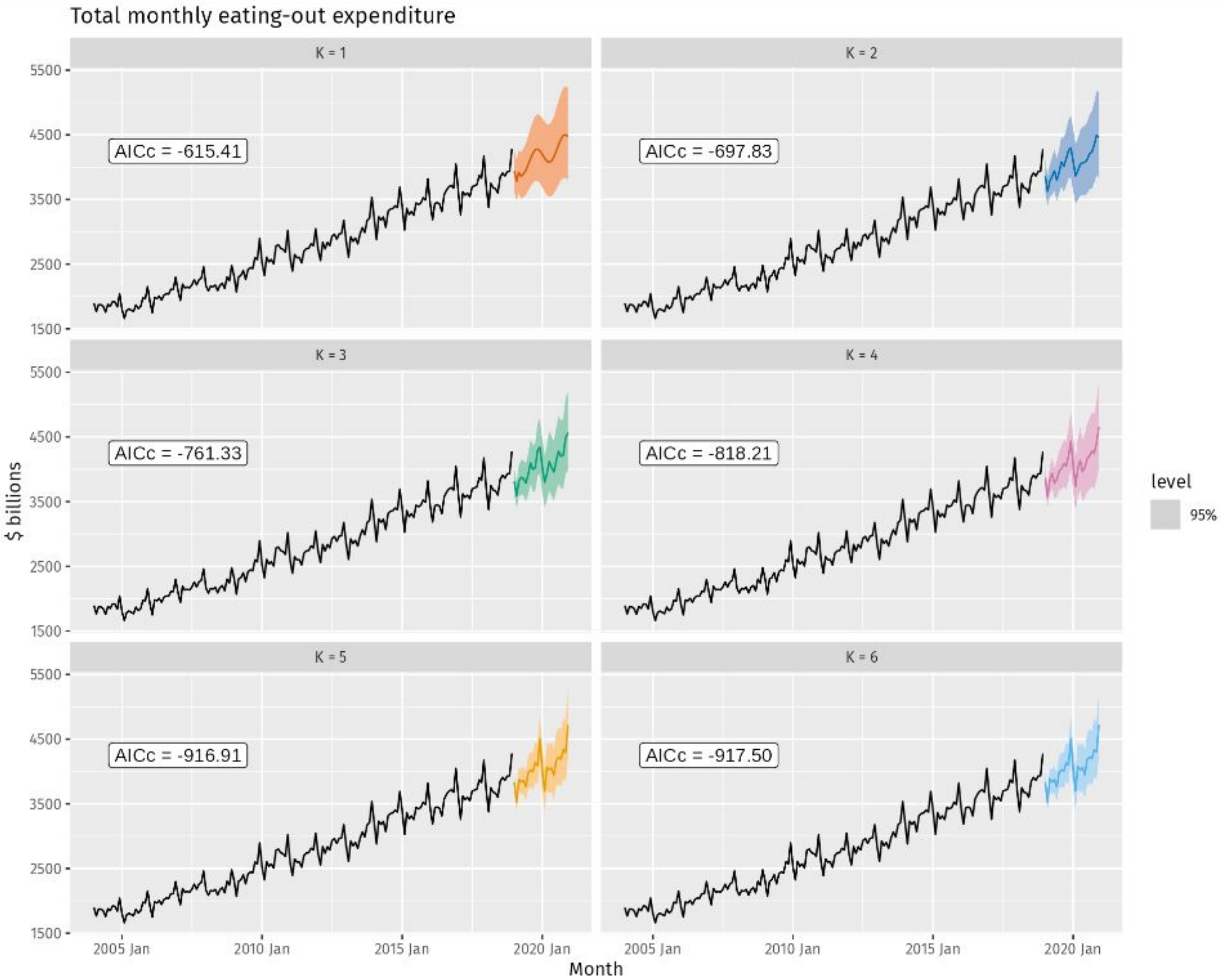
$$\hat{y}(t) = \sum_{k=1}^{K} \left(a_k \sin\left(\frac{2k\pi t}{m}\right) + b_k \sin\left(\frac{2k\pi t}{m}\right)\right)$$

  a_k, b_k - weights

- **pros:**
  - models **long periods** well - for low K the patterns are smooth and change slowly
  - models **non-integer periods**
  - useful for high frequency and long seasonality

Harmonic regression example

- K=1, 2, …, 6
- larger K = learns more complex patterns



https://otexts.com/fpp3/dhr.html

Fourier features - additional resources

- FPP: Fourier features
- "Forecasting with long seasonal periods" R. Hyndman
- "Forecasting with daily data" R. Hyndman
- "Forecasting with weekly data" R. Hyndman
- Sktime docs: Fourier features
- Cross Validated - Importance of Fourier terms in time series forecasting
- Cross Validated - Fourier terms to model seasonality in ARIMA models
- "Modeling variable seasonal features with the Fourier transform" F. Andrei

Time series features - additional resources

- FPP: Some useful predictors
- "Must-Know Base Tips for Feature Engineering With Time Series Data" V. Shkulov
- scikit-learn docs: Time-related feature engineering
- "Practical Guide for Feature Engineering of Time Series Data" J. Gordon
- "Feature-based time-series analysis" B. Fulcher

Combining everything

- those features can be used **in conjunction** with statistical models, as exogenous variables
- using ML regression is not required
- you can even combine the two, in so-called **dynamic regression:**
  - feature engineering + ML-based regression
  - subtract forecast from time series
  - model and forecast the residuals with ARIMA
  - if using Fourier terms, known as **dynamic harmonic regression (DHR)**
- alternatively, ARIMA / ETS forecasts can be input features for ML regression in a form of **stacking** (a kind of ensemble learning)
- those combinations are particularly useful for long seasonal periods and long forecasts

Dynamic regression - additional resources

- FPP: Dynamic regression models (whole book chapter)
- "The ARIMAX model muddle" R. Hyndman

Regression-based forecasting - pros and cons

Pros:

- can encode rich additional data
- works great, typically SOTA with good feature engineering
- can learn complex patterns, long time dependencies etc.
- many algorithms to choose from
- explainability
- LightGBM pros, e.g. fast, widely used, well researched

Cons:

- have to do feature engineering
- MLOps complexity, have to build and maintain the whole pipeline
- underperforms for univariate time series
- LightGBM cons, e.g. quite easily overfits, requires a lot of hyperparameter tuning

Why not deep learning?

Deep learning is hard

- deep learning often **underperforms**, especially for:
  - univariate time series
  - small data
  - short forecasting horizons
  - high frequency data
  - important exogenous variables (they typically can't be used)
- many other **problems:**
  - slow and costly to train
  - hard to train: unstable, overfit easily, many hyperparameters to tune
  - high hardware requirements for inference

Statistical methods still work well

- statistical models have **a lot of advantages:**
  - great for univariate data
  - automated variants, work out-of-the-box
  - fast training and inference
- they often **give good results:**
  - frequently not exactly SOTA, but very close
  - good enough for most practical use cases
  - M4 competition (2018): all pure ML models failed to outperform statistical methods
  - M5 competition (2020): 92.5% of submissions, including ML, deep learning and hybrids, did not outperform AutoETS

Boosting-based ML is still SOTA

- LightGBM **typically wins** forecasting competitions:
  - great for multivariate time series
  - gains a lot from feature engineering
  - works surprisingly well for learning cross-series information
  - very fast training (a lot of hyperparameter tuning, though)
- **disclaimer:** competitions are often unrealistic, e.g. lots of data, overoptimization for test set, typically huge unstable ensembles win
- but LightGBM is **always** in top 3, typically wins, so it says a lot
- in M5 competition, winner and 4 out of 5 top solutions used LightGBM ensembles
- a lot of other examples and analysis: "Kaggle forecasting competitions: An overlooked learning opportunity" C. Bojer, J. Meldgaard

Further reading

- J. Dancker "Why You (Currently) Do Not Need Deep Learning for Time Series Forecasting"
- State of Competitive Machine Learning 2023 - Time Series Forecasting
- "Statistical, machine learning and deep learning forecasting methods: Comparisons and ways forward" S. Makridakis et al.
- "M5 accuracy competition: Results, findings, and conclusions" S. Makridakis
- "Statistical vs Deep Learning forecasting methods" - Nixtla
- "Deep Learning Is Not What You Need" V. Manokhin

Questions?

Other forecasting algorithms

ARFIMA

- **FI = fractional differencing**, with order *d* in range (0,1)
- keeps the shape between original data and stationary one, still partially incorporating the trend
- ARFIMA models have long memory, keeping long dependencies in the data
- particularly useful in macroeconomics and finance, which have
- Wikipedia - ARFIMA
- "ARIMA and ARFIMA models" C. Baum
- "AutoRegressive Fractionally Integrated Moving Average (ARFIMA) model" J. Tanaka

BATS and TBATS

- BATS and TBATS models are arguably the most modern statistical models for time series
- evolution combining most previous important building blocks:
    - **T**rigonometric seasonality - Fourier features
    - **B**ox-Cox transformation
    - **A**RMA errors
    - **T**rend and **S**easonal components
- trend and seasonality are modelled like in ETS, Box-Cox and ARMA errors like in ARIMA
- "Forecasting time series with complex seasonal patterns using exponential smoothing" A. De Livera et al.
- "Forecasting Time Series with Multiple Seasonalities using TBATS in Python" G. Skorupa

ARCH and GARCH

- ARCH = autoregressive conditional heteroskedasticity
- ARCH and GARCH explicitly **model the autocorrelation of variance**
- ARCH uses AR(p), and GARCH uses ARMA(p,q)
- useful in finance in econometric, where shocks, sudden changes etc. are particularly relevant and can have long influence
- so-called volatility and volatility clustering
- R. Engle and C. Granger won Nobel's Prize in economics for inventing the ARCH model
- Wikipedia - ARCH and GARCH
- "Time Series Model(s)—ARCH and GARCH" R. Kumar

Other interesting models

- Theta
  - Theta: remove seasonality, model with OLS regression and SES, combine
  - expanded by Four Theta and Optimized Theta methods
- Croston method
  - for intermittent time series (lots of zeros) and count data
  - 2 SES models: one to forecast non-zero period, second for amount for next period
- vector autoregression (VAR)
  - extension of ARIMA family for multivariate time series
  - FPP: VAR
  - "Vector AutoRegressive Moving Average Models: A Review" M.C. Düker et al.

Prophet

- "Forecasting at scale" S. Taylor, B. Letham
- this is **not** an interesting model, but rather a dangerous one!
- Generalized Additive Model (GAM):

  y = trend + seasonality + holidays + noise
- Bayesian MCMC estimation, with probabilistic forecasts
- at its heart, this is just **curve fitting** to historical data, and it assumes constant variance
- both too simple and too complicated to be really useful

Prophet

- "Zillow, Prophet, Time Series, & Prices"
- "Facebook Prophet, Covid and why I don't trust the Prophet" S. Seitz
- "Is Facebook's "Prophet" the Time-Series Messiah, or Just a Very Naughty Boy?"
- "Shortcomings of Facebook Prophet for Time Series Analysis"
- "Comparing Prophet and Deep Learning to ARIMA in Forecasting Wholesale Food Prices" L. Menculini et al.
- "A Worrying Analysis of Probabilistic Time-series Models for Sales Forecasting" S. Jung et al.
- "Cash flow prediction: MLP and LSTM compared to ARIMA and Prophet" H. Weytjens