Lab7

2023-04-05

```
#Demonstrating convergence in incomes across racial groups using all-race/gender model
#set seed
HUID <- 21519588
set.seed(HUID)

#Demonstrating model across two generations
#Gen 1-2
parents_rank <- 57.9
kids_rank <- 33.31 + 0.351 * parents_rank
kids_rank
```

```
## [1] 53.6329
```

```
#Gen 2-3
parents_rank = kids_rank
kids_rank = 33.31 + 0.351 * parents_rank
kids_rank
```

```
## [1] 52.13515
```

```
#Iterating across multiple generations
generations <- seq(1,7,1)

parents_rank_white = 57.9
parents_rank_black = 32.7

#white gen for loop
for(i in generations){
  kids_rank <- 33.31 + 0.351 * parents_rank_white
  print(paste0("In generation ", i, ", parent_rank = ", parents_rank_white, ", child_rank = ", kids_ranl
  parents_rank_white <- kids_rank
}
```

```
## [1] "In generation 1, parent_rank = 57.9, child_rank = 53.6329"
## [1] "In generation 2, parent_rank = 53.6329, child_rank = 52.1351479"
## [1] "In generation 3, parent_rank = 52.1351479, child_rank = 51.6094369129"
## [1] "In generation 4, parent_rank = 51.6094369129, child_rank = 51.4249123564279"
## [1] "In generation 5, parent_rank = 51.4249123564279, child_rank = 51.3601442371062"
## [1] "In generation 6, parent_rank = 51.3601442371062, child_rank = 51.3374106272243"
## [1] "In generation 7, parent_rank = 51.3374106272243, child_rank = 51.3294311301557"
```

```
#black gen for loop
for(i in generations){
  kids_rank <- 33.31 + 0.351 * parents_rank_black
  print(paste0("In generation ", i, ", parent_rank = ", parents_rank_black, ", child_rank = ", kids_ranl
  parents_rank_black <- kids_rank
}
```

```
## [1] "In generation 1, parent_rank = 32.7, child_rank = 44.7877"
## [1] "In generation 2, parent_rank = 44.7877, child_rank = 49.0304827"
## [1] "In generation 3, parent_rank = 49.0304827, child_rank = 50.5196994277"
## [1] "In generation 4, parent_rank = 50.5196994277, child_rank = 51.0424144991227"
## [1] "In generation 5, parent_rank = 51.0424144991227, child_rank = 51.2258874891921"
## [1] "In generation 6, parent_rank = 51.2258874891921, child_rank = 51.2902865087064"
## [1] "In generation 7, parent_rank = 51.2902865087064, child_rank = 51.312890564556"
```

Using the all-race/gender model, white and black inter-generational mobility outcomes converge around gen 7 at a rank of about 51.3. But we know that this is incorrect; let's find the steady state prediction for Black and Hispanic children using their respective rank-rank models:

```
#Steady state for Black children
generations <- seq(1,7,1)
parents_rank_black = 32.7

#black gen for loop
for(i in generations){
  kids_rank <- 25.4 + 0.28 * parents_rank_black
  print(paste0("In generation ", i, ", parent_rank = ", parents_rank_black, ", child_rank = ", kids_rank
  parents_rank_black <- kids_rank
}
```

```
## [1] "In generation 1, parent_rank = 32.7, child_rank = 34.556"
## [1] "In generation 2, parent_rank = 34.556, child_rank = 35.07568"
## [1] "In generation 3, parent_rank = 35.07568, child_rank = 35.2211904"
## [1] "In generation 4, parent_rank = 35.2211904, child_rank = 35.261933312"
## [1] "In generation 5, parent_rank = 35.261933312, child_rank = 35.27334132736"
## [1] "In generation 6, parent_rank = 35.27334132736, child_rank = 35.2765355716608"
## [1] "In generation 7, parent_rank = 35.2765355716608, child_rank = 35.277429960065"
```

```
#Steady state for Hispanic children
parents_rank_hisp = 36.17
for(i in generations){
  kids_rank <- 36.14 + 0.26 * parents_rank_hisp
  print(paste0("In generation ", i, ", parent_rank = ", parents_rank_hisp, ", child_rank = ", kids_rank)
  parents_rank_hisp <- kids_rank
}
```

```
## [1] "In generation 1, parent_rank = 36.17, child_rank = 45.5442"
## [1] "In generation 2, parent_rank = 45.5442, child_rank = 47.981492"
## [1] "In generation 3, parent_rank = 47.981492, child_rank = 48.61518792"
## [1] "In generation 4, parent_rank = 48.61518792, child_rank = 48.7799488592"
## [1] "In generation 5, parent_rank = 48.7799488592, child_rank = 48.822786703392"
## [1] "In generation 6, parent_rank = 48.822786703392, child_rank = 48.8339245428819"
## [1] "In generation 7, parent_rank = 48.8339245428819, child_rank = 48.8368203811493"
```

The steady state prediction for Black children is around 35.27 and for Hispanic children, 48.83.

Question 2 Cross-validation helps us avoid the overfit problem by addressing the bias-variance tradeoff in machine learning models. More complex models will eventually fit the noise of the training data, which causes the overfit problem. Cross-validation addresses that by evaluating a model's performance with different sets of training data taken from the original dataset. We can cross-validate a portion of the training data to find the optimal model complexity that minimizes RMSPE and over-fitting.

```
#Implementing 5-fold cross-validation using two predictors

#Store predictor variables which all start with P_*
```

```r
vars <- colnames(training[,grep("^[P_]", names(training))])
vars
```

```
##   [1] "P_1"   "P_2"   "P_3"   "P_4"   "P_5"   "P_6"   "P_7"   "P_8"   "P_9"
##  [10] "P_10"  "P_11"  "P_12"  "P_13"  "P_14"  "P_15"  "P_16"  "P_17"  "P_18"
##  [19] "P_19"  "P_20"  "P_21"  "P_22"  "P_23"  "P_24"  "P_25"  "P_26"  "P_27"
##  [28] "P_28"  "P_29"  "P_30"  "P_31"  "P_32"  "P_33"  "P_34"  "P_35"  "P_36"
##  [37] "P_37"  "P_38"  "P_39"  "P_40"  "P_41"  "P_42"  "P_43"  "P_44"  "P_45"
##  [46] "P_46"  "P_47"  "P_48"  "P_49"  "P_50"  "P_51"  "P_52"  "P_53"  "P_54"
##  [55] "P_55"  "P_56"  "P_57"  "P_58"  "P_59"  "P_60"  "P_61"  "P_62"  "P_63"
##  [64] "P_64"  "P_65"  "P_66"  "P_67"  "P_68"  "P_69"  "P_70"  "P_71"  "P_72"
##  [73] "P_73"  "P_74"  "P_75"  "P_76"  "P_77"  "P_78"  "P_79"  "P_80"  "P_81"
##  [82] "P_82"  "P_83"  "P_84"  "P_85"  "P_86"  "P_87"  "P_88"  "P_89"  "P_90"
##  [91] "P_91"  "P_92"  "P_93"  "P_94"  "P_95"  "P_96"  "P_97"  "P_98"  "P_99"
## [100] "P_100" "P_101" "P_102" "P_103" "P_104" "P_105" "P_106" "P_107" "P_108"
## [109] "P_109" "P_110" "P_111" "P_112" "P_113" "P_114" "P_115" "P_116" "P_117"
## [118] "P_118" "P_119" "P_120" "P_121"
```

```r
#Create a training data frame with just predictors P_* and kfr_pooled_pooled_p25
training_subset <- subset(training, training==1, vars)
training_subset$kfr_pooled_pooled_p25 <- training[training$training==1,]$kfr_pooled_pooled_p25

#cross-validation
n <- nrow(training_subset)
K <- 5
B <- seq(1,20,1)

cv <- training_subset
cv$foldid <- rep(1:K,each=ceiling(n/K))[sample(1:n)]
OOS <- data.frame(fold=rep(NA,K*length(B) ),
                  squarederror=rep(NA,K*length(B) ),
                  maxdepth=rep(NA,K*length(B) ))

row <- 0

for(i in B){

  for(k in 1:K){

    row <- row + 1

    cvtrain <- subset(cv, foldid != k)

    cvfold <- subset(cv, foldid == k)

    cvtree <- rpart(kfr_pooled_pooled_p25 ~ P_12 + P_80,
                    data=cvtrain,
                    maxdepth = c(i),
                    cp=0)


    predfull <- predict(cvtree, newdata=cvfold)

    OOS$squarederror[row] <- sum((cvfold$kfr_pooled_pooled_p25 - predfull)^2)
```

```r
    OOS$maxdepth[row] <- i

    OOS$fold[row] <- k

  }

}

OOS
```

```
##     fold squarederror maxdepth
## 1      1     5384.794        1
## 2      2     4641.916        1
## 3      3     5746.422        1
## 4      4     5128.230        1
## 5      5     4337.194        1
## 6      1     4538.326        2
## 7      2     4154.131        2
## 8      3     5252.789        2
## 9      4     4591.880        2
## 10     5     4012.419        2
## 11     1     4400.510        3
## 12     2     3856.398        3
## 13     3     4755.413        3
## 14     4     4365.298        3
## 15     5     3910.760        3
## 16     1     3550.421        4
## 17     2     3854.592        4
## 18     3     4807.080        4
## 19     4     4273.982        4
## 20     5     3718.511        4
## 21     1     3481.735        5
## 22     2     4023.202        5
## 23     3     4583.083        5
## 24     4     4588.044        5
## 25     5     3525.753        5
## 26     1     3489.742        6
## 27     2     4071.722        6
## 28     3     4595.772        6
## 29     4     4537.064        6
## 30     5     3670.873        6
## 31     1     3490.964        7
## 32     2     4166.760        7
## 33     3     4584.770        7
## 34     4     4578.060        7
## 35     5     3918.333        7
## 36     1     3610.763        8
## 37     2     4198.802        8
## 38     3     4747.887        8
## 39     4     4864.464        8
## 40     5     4078.479        8
## 41     1     3681.679        9
## 42     2     4281.377        9
```

```
## 43     3     4973.341          9
## 44     4     4947.999          9
## 45     5     4128.395          9
## 46     1     3768.974         10
## 47     2     4300.370         10
## 48     3     5020.607         10
## 49     4     4993.543         10
## 50     5     4209.944         10
## 51     1     3797.818         11
## 52     2     4387.460         11
## 53     3     5022.392         11
## 54     4     4996.704         11
## 55     5     4205.684         11
## 56     1     3786.890         12
## 57     2     4391.268         12
## 58     3     5058.183         12
## 59     4     5000.416         12
## 60     5     4212.717         12
## 61     1     3791.447         13
## 62     2     4410.442         13
## 63     3     5060.055         13
## 64     4     5000.416         13
## 65     5     4221.388         13
## 66     1     3805.139         14
## 67     2     4410.442         14
## 68     3     5068.832         14
## 69     4     5000.416         14
## 70     5     4221.388         14
## 71     1     3805.139         15
## 72     2     4410.442         15
## 73     3     5082.219         15
## 74     4     5000.416         15
## 75     5     4221.388         15
## 76     1     3805.139         16
## 77     2     4410.442         16
## 78     3     5082.219         16
## 79     4     5000.416         16
## 80     5     4221.388         16
## 81     1     3805.139         17
## 82     2     4410.442         17
## 83     3     5082.219         17
## 84     4     5000.416         17
## 85     5     4221.388         17
## 86     1     3805.139         18
## 87     2     4410.442         18
## 88     3     5082.219         18
## 89     4     5000.416         18
## 90     5     4221.388         18
## 91     1     3805.139         19
## 92     2     4410.442         19
## 93     3     5082.219         19
## 94     4     5000.416         19
## 95     5     4221.388         19
## 96     1     3805.139         20
```

```
## 97     2     4410.442       20
## 98     3     5082.219       20
## 99     4     5000.416       20
## 100    5     4221.388       20
```

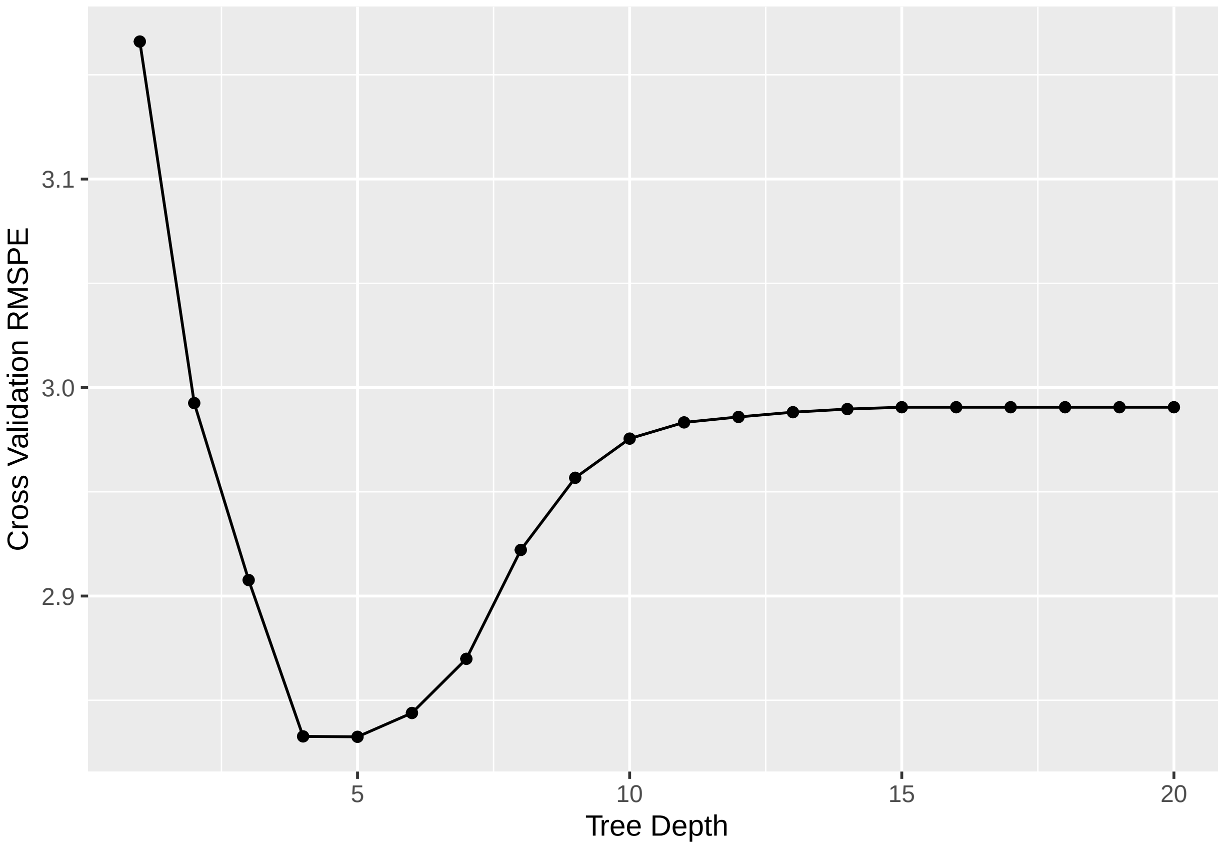
```
summary(OOS)
```

```
##       fold    squarederror     maxdepth
##  Min.   :1   Min.   :3482   Min.   : 1.00
##  1st Qu.:2   1st Qu.:4021   1st Qu.: 5.75
##  Median :3   Median :4396   Median :10.50
##  Mean   :3   Mean   :4414   Mean   :10.50
##  3rd Qu.:4   3rd Qu.:4994   3rd Qu.:15.25
##  Max.   :5   Max.   :5746   Max.   :20.00
```

```
ssr <- tapply(OOS$squarederror, OOS$maxdepth, sum)
ssr <- as.data.frame(ssr)
ssr$maxdepth <- seq(1,20,1)
ssr
```

```
##         ssr maxdepth
## 1  25238.56        1
## 2  22549.55        2
## 3  21288.38        3
## 4  20204.59        4
## 5  20201.82        5
## 6  20365.17        6
## 7  20738.89        7
## 8  21500.40        8
## 9  22012.79        9
## 10 22293.44       10
## 11 22410.06       11
## 12 22449.47       12
## 13 22483.75       13
## 14 22506.22       14
## 15 22519.60       15
## 16 22519.60       16
## 17 22519.60       17
## 18 22519.60       18
## 19 22519.60       19
## 20 22519.60       20
```

```
ssr$rmse <- sqrt(ssr$ssr / nrow(training))

ggplot(ssr, aes(x=maxdepth,y=rmse)) +
  geom_point() +
  geom_line() +
  labs(y = "Cross Validation RMSPE",
       x = "Tree Depth")
```



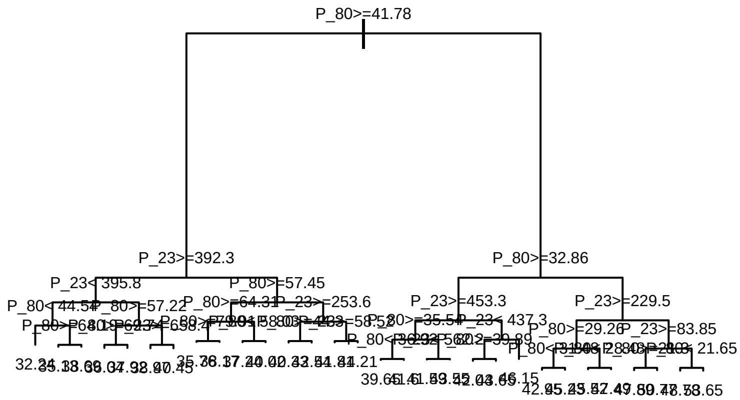

```
cv_optimal_depth = ssr$maxdepth[which.min(ssr$rmse)]
cv_optimal_depth
```

```
## [1] 5
```

Question 3b The optimal tree depth for this training dataset is 5

Question 3c I am using the following two predictors: P_12 (Total Violent and Property Crimes Rate) and P_80 (Percent of Children Eligible for Free Lunch (Persons < 18 Years)).

```
#Using full training dataset to estimate tree of depth 5

tree <- rpart(kfr_pooled_pooled_p25 ~ P_23 + P_80,
                      data=training_subset,
                      maxdepth = cv_optimal_depth,
                      cp=0)

#visualize tree
plot(tree, margin = 0.2)
text(tree, cex = 0.5)
```




```
#Calculate predictions for all rows in training sample
y_train_predictions_tree <- predict(tree, newdata=training_subset)
```

Question 4 Random forests improve upon decision trees in two distinct ways. First, they apply bagging to build a series of trees which are each trained on a subset of the original data. The average of each series' RMPSE is taken to determine the most accurate prediction model. Bagging averages across a large number of trees to cancel out the training data noise and left with real signal instruction. The other way is through input randomization. This reduces the correlation between trees, which improves model accuracy.

```
#Question 5
#Random forest with at least 1000 trees bootstrap with P_12 and P_80
smallforest <- randomForest(kfr_pooled_pooled_p25 ~ P_12 + P_80,
                            ntree=1000,
                            mtry=2,
                            data=training_subset)
smallforest
```

```
##
## Call:
##  randomForest(formula = kfr_pooled_pooled_p25 ~ P_12 + P_80, data = training_subset,      ntree = 10
##                Type of random forest: regression
##                      Number of trees: 1000
## No. of variables tried at each split: 2
##
##           Mean of squared residuals: 16.18565
##                     % Var explained: 39.41
```

```
y_train_predictions_smallforest <- predict(smallforest, newdata=training_subset, type="response")
```

```
#Question 6
#Random forest with at least 1000 trees bootstrap with all predictor variables
mobilityforest <- randomForest(kfr_pooled_pooled_p25 ~ .,
                               ntree=1000,
                               mtry=40,
                               importance=TRUE,
                               data=training_subset)
mobilityforest
```

```
##
## Call:
##  randomForest(formula = kfr_pooled_pooled_p25 ~ ., data = training_subset,      ntree = 1000, mtry =
##                Type of random forest: regression
##                      Number of trees: 1000
```

```
## No. of variables tried at each split: 40
##
##           Mean of squared residuals: 4.718369
##                     % Var explained: 82.34
```

```r
y_train_predictions_forest  <- predict(mobilityforest, newdata=training_subset, type="response")
```

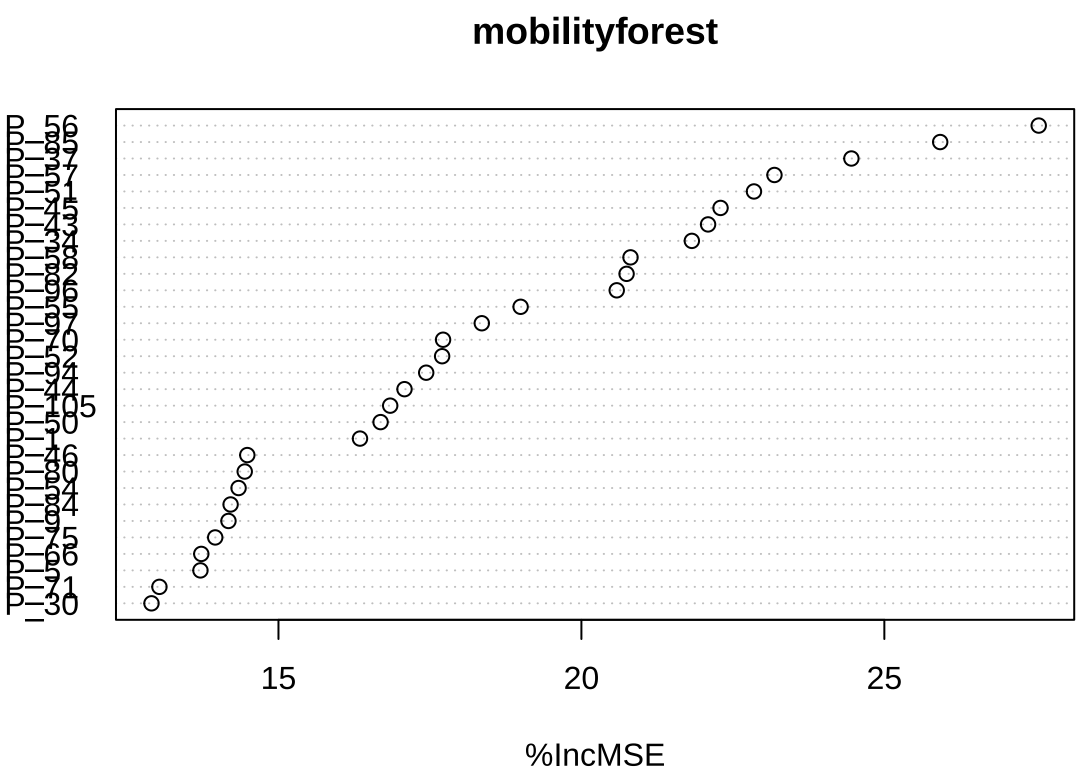
```r
#Determing the importance of each predictor

importance(mobilityforest)
```

```
##          %IncMSE IncNodePurity
## P_1   16.3455820    199.328248
## P_2   10.2712926     98.255430
## P_3   10.7000979    111.589622
## P_4   11.9545339    117.956467
## P_5   13.7120456    167.582547
## P_6   11.3247682    145.369966
## P_7   12.6905886    164.179476
## P_8   12.4903481    201.905775
## P_9   14.1747572    183.092895
## P_10   9.7535234    195.409940
## P_11   7.0492571     57.157426
## P_12  10.6984586     85.047655
## P_13   1.9037341     31.051537
## P_14   6.4691321     68.309649
## P_15   9.2092723     76.530012
## P_16   2.0323752     46.423382
## P_17  11.1078861     98.847945
## P_18  12.0176255    113.243239
## P_19   6.6996761     68.906102
## P_20   7.1606517     55.984769
## P_21   6.0141348     62.331337
## P_22   1.2096611     44.140021
## P_23   6.8896174     75.028373
## P_24   9.3991699     86.399224
## P_25   9.1638963    110.570345
## P_26   5.8513214     43.771991
## P_27   3.8512346     82.507938
## P_28   7.7010223     59.470377
## P_29  12.1864395    505.522446
## P_30  12.9041849    139.771784
## P_31  12.2946057    566.147284
## P_32   6.8245607     41.875939
## P_33  10.4216864     65.745224
## P_34  21.8229028    852.005131
## P_35   7.8854770    280.229248
## P_36   8.1330322    120.645117
## P_37  24.4560730   3260.297576
## P_38  11.5378112    155.084261
## P_39  10.0126801     79.081082
## P_40   6.8379078     56.245509
## P_41   6.2448714     55.379599
## P_42  12.6360548    193.512936
## P_43  22.0908189    896.694381
```

```
## P_44  17.0801853    375.667811
## P_45  22.2962168    708.077680
## P_46  14.4854301    290.176913
## P_47   8.6636183     78.103496
## P_48   7.4610831    177.550122
## P_49   3.3609602    108.757577
## P_50  16.6855067    542.233355
## P_51  22.8481462    530.139749
## P_52  17.7014392    479.754581
## P_53   8.0868271    137.231557
## P_54  14.3416741    210.534959
## P_55  18.9966812   1877.724623
## P_56  27.5480675   2401.875465
## P_57  23.1860215   3654.376898
## P_58  20.8103725   2008.048796
## P_59   2.4509265     45.981814
## P_60   4.7913864     51.962092
## P_61   5.8592338     49.897462
## P_62   8.1091302     89.615013
## P_63  10.0993867     83.404715
## P_64   8.3501297    101.014595
## P_65   7.5318321     51.391941
## P_66  13.7261989    416.846229
## P_67   7.0185078     64.374985
## P_68   8.1576331    167.826569
## P_69  10.2890179     93.921106
## P_70  17.7167059    609.727261
## P_71  13.0352532    162.786895
## P_72   7.8260484     83.274592
## P_73   9.0191148    113.003081
## P_74  12.5400266    210.481363
## P_75  13.9559948    153.454602
## P_76   5.1557610     60.892367
## P_77   8.0690111     66.445584
## P_78   5.8265648     53.108854
## P_79   8.9319417     48.603761
## P_80  14.4434829    638.477179
## P_81  10.9282742    107.782896
## P_82  20.7454715    359.528370
## P_83   9.7760588     80.012078
## P_84  14.2099207   1253.362983
## P_85  25.9219072    422.458410
## P_86   4.1380489     15.661462
## P_87   4.6818619     47.792573
## P_88   0.9655265      4.716139
## P_89  -1.7431782      1.657015
## P_90   1.2461441      2.609570
## P_91   1.8610989     14.554408
## P_92   4.3732460     43.378518
## P_93   8.0931759     96.399640
## P_94  17.4382970    313.627200
## P_95  10.2962768     97.426696
## P_96  20.5832828   1917.421913
## P_97  18.3559435    230.073404
```

```
## P_98    2.2779578      8.484016
## P_99    2.7802871     31.575275
## P_100   0.3652998      4.231020
## P_101   0.3145750      1.955011
## P_102   0.8873717      2.401153
## P_103   1.6979183     10.303829
## P_104   2.4239752     41.708710
## P_105  16.8442488    421.951795
## P_106   1.5577063      6.692724
## P_107   1.1590978      1.254908
## P_108   3.6988947     20.685840
## P_109   4.8012498     40.555115
## P_110   0.9138843      1.770115
## P_111   4.9866219     18.599150
## P_112   3.1169550     12.126711
## P_113  11.1505137    395.846777
## P_114   2.1147394     11.720815
## P_115   5.7758946     91.124191
## P_116   2.2432942     15.057878
## P_117   8.2728463     64.903436
## P_118   1.8307240      8.058542
## P_119   2.1597017     12.971350
## P_120   2.5461377     15.652504
## P_121   2.2753544     11.534708
```

```
varImpPlot(mobilityforest, type=1)
```




```
#type   is either 1 or 2, specifying the type of importance measure
#(1=mean decrease in accuracy, 2=mean decrease in node impurity)
```

```r
as.data.frame(importance(mobilityforest)) %>%
arrange(desc(`%IncMSE`)) %>%
head(10)
```

```
##       %IncMSE IncNodePurity
## P_56 27.54807     2401.8755
## P_85 25.92191      422.4584
## P_37 24.45607     3260.2976
## P_57 23.18602     3654.3769
## P_51 22.84815      530.1397
## P_45 22.29622      708.0777
## P_43 22.09082      896.6944
## P_34 21.82290      852.0051
## P_58 20.81037     2008.0488
## P_82 20.74547      359.5284
```

The most important predictors are P_56 (Mentally Unhealthy Days per Month (Persons 18 Years and Over)), P_37 (black share of the population in 2000), and P_85 (percentage of the population Roman Catholic)

```r
#Question 8
#Dtermining the best model by RMSPE for each of the three models

p <- 3
RMSPE <- matrix(0, p, 1)
RMSPE[1] <- sqrt(mean((training_subset$kfr_pooled_pooled_p25 - y_train_predictions_tree)^2, na.rm=TRUE)
RMSPE[2] <- sqrt(mean((training_subset$kfr_pooled_pooled_p25 - y_train_predictions_smallforest)^2, na.r
RMSPE[3] <- sqrt(mean((training_subset$kfr_pooled_pooled_p25 - y_train_predictions_forest)^2, na.rm=TRU

#Display a table of the results
data.frame(RMSPE, method = c("Tree", "Small RF", "Large RF"))
```

```
##       RMSPE   method
## 1 3.5917785     Tree
## 2 2.0008860 Small RF
## 3 0.8685554 Large RF
```

The large random forest model (of 1000 trees and including all 120+ predictor variables) performed the best, as it has the lowest RMSPE.

```r
# Question9
#Applying models to lockbox data - which model perfoms the best with actual social mobility data?


#Merge with truth to evaluate predictions.
atlas <- left_join(lockbox, training , by="geoid")

#Separate test data set as a separate data frame
test <- subset(atlas, training==0)

#Get predictions for test data
y_test_predictions_tree <- predict(tree, newdata=test)
y_test_predictions_smallforest <- predict(smallforest, newdata=test, type="response")
y_test_predictions_forest  <- predict(mobilityforest, newdata=test, type="response")

#Calculate RMSPE for test data
p <- 3
```

```r
OOS_RMSPE <- matrix(0, p, 1)
OOS_RMSPE[1] <- sqrt(mean((test$kfr_actual - y_test_predictions_tree)^2, na.rm=TRUE))
OOS_RMSPE[2] <- sqrt(mean((test$kfr_actual - y_test_predictions_smallforest)^2, na.rm=TRUE))
OOS_RMSPE[3] <- sqrt(mean((test$kfr_actual - y_test_predictions_forest)^2, na.rm=TRUE))

# Display table of results
data.frame(OOS_RMSPE, method = c("Tree", "Small RF", "Large RF"))
```

```
##   OOS_RMSPE   method
## 1  3.872614     Tree
## 2  3.835932 Small RF
## 3  2.192877 Large RF
```

Once again, the large random forest model performs the best, albeit with a higher error than when using it with the training data.