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Exercise sheet 2 - Topics in Topology

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1. Draw the attaching sphere, attaching region, belt sphere, core and cocore for the attachment of a 3-dimensional 2-handle.

2. What does attaching an n-dimensional 0-handle consist of? For what $k = 0, 1, \ldots, n$ can an n-dimensional k-handle be attached to the empty set?

3. Let $K_0, K_1 : S^1 \hookrightarrow S^3$ be two smooth knots. We say that they are *isotopic* if there is a map $H : S^1 \times I \to S^3$ such that H_t is an embedding for all $t \in I$ and $H_i = K_i$ for $i = 0, 1$. We say that they are *ambient isotopic* if there is a map $F : S^3 \times I \to S^3$ such that F_t is a diffeomorphism for all $t \in I$, $F_0 = id$ and $K_1 = F_1 \circ K_0$.

   Show that two knots are isotopic if and only if they are ambient isotopic.

4. Show that the complement of an open disc in $\mathbb{RP}^2$ is diffeomorphic to a Möbius strip.

5. Consider handle decompositions for S^n, T^2 and $\mathbb{RP}^2$ and describe their dual decompositions.

6. Draw what a cancelling pair and a handle slide look like for $n = 3$ and $k = 2$.

7. Show that the argument given in the lectures to obtain a unique 0-handle in a handle decomposition is an example of stabilisation according to the Cerf theorem.

8. Give a handle decomposition for the 3-manifold $S^1 \times S^1 \times S^1$.