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Exercise sheet 6 - Topics in Topology

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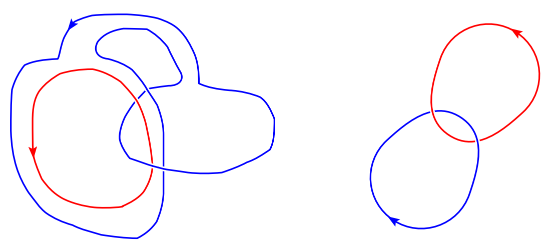
1. Convince yourself that the boundary of the 4-manifold obtained by attaching a 4-dimensional 2-handle along a framed knot is the 3-manifold produced by surgery on that framed knot.

2. Show that viewing the framings as integers, if n_i is the framing for K_i, $i = 1, 2$, then the framing n_1' for the new component $K_1' = K_1 \#_b \pm \lambda_2$ is given by

$$n_1' = n_1 + n_2 \pm 2\ lk(K_1, K_2)$$

where $\pm$ depends on whether $\pm\lambda_2$ is used.

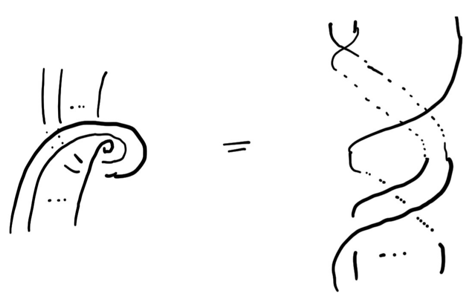
3. Show that the following two links are isotopic.



(a) (b)

4. Show that surgery on the Hopf link with the 2 components 0-framed produces S^3.

5. Show that the right/left full twist of a set of $r > 0$ strands is isotopic to the braid below:



6. Show that any knot K can be turned into the unknot by changing the sign of some of its crossings.

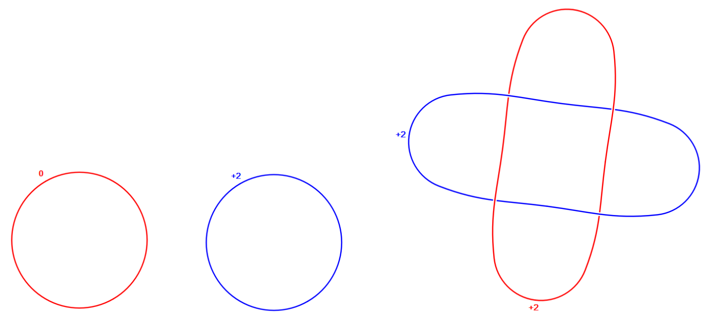
7. Show the last corollary of the lecture: given a framed link $L = U \cup K_1 \cup K_2 \cup \cdots$ where U is the unknot with framing 0 with K_1 piercing the disc bounding U exactly once (and not any

other component), then surgery on L produces the same 3-manifold as the link obtained from L by removing the components U and K_1.

Hint: Show first that changing crossings we can unknot any two components of a link.

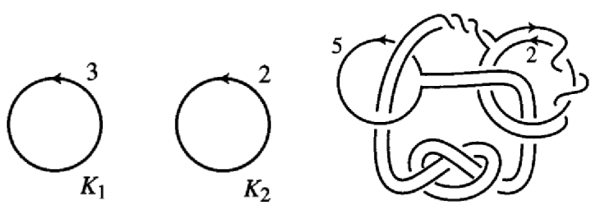
8. Show that the following two framed links produce the same 3-manifold.

   Hint: Use a band with a single full twists.



(a) (b)

9. Show that the following two framed links produce the same 3-manifold:



(a) (b)