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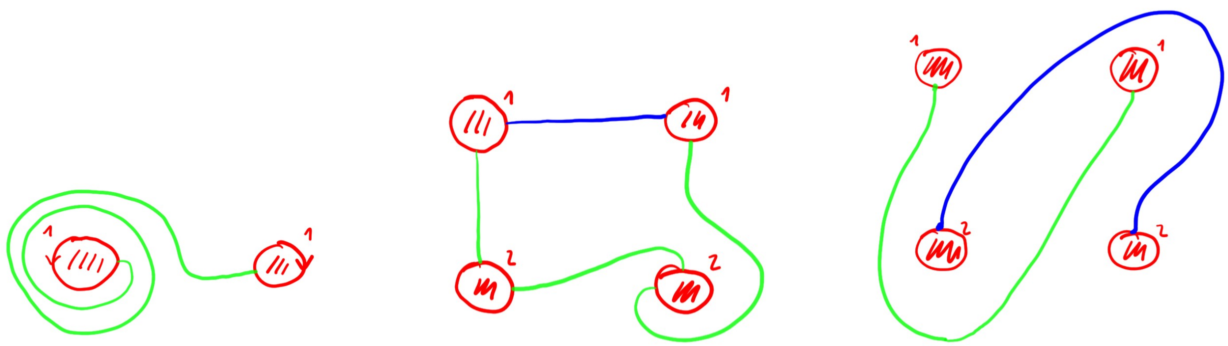
Exercise sheet 5 - Topics in Topology

March 8, 2022

1. Understand the planar Heegaard diagram for $L(5, 2)$ from the extra document on the course website. In general, what does a planar Heegaard diagram for $L(p, q)$ look like?

2. What are the closed, connected, orientable 3-manifold determined by each of the following planar Heegaard diagrams?



(a) (b) (c)

3. Give a planar Heegaard diagram for the connected sum of two closed, connected, orientable 3-manifolds out of planar Heegaard diagrams of the two 3-manifolds.

   Hint: Mimic the argument that Maurits gave in his talk for surfaces.

4. Give examples of the following concepts: homotopic maps, homotopy equivalence of spaces, deformation retraction (go to Hatcher if you have not learned those yet!).

5. Use the van Kampen theorem to show that if $X = S^1 \vee \overset{n}{\cdots} \vee S^1$ denotes the wedge sum of n spheres (a flower with n petals), then $\pi_1(X) \cong \langle x_1, \ldots, x_n | \rangle$, the free group on n elements. You are allowed to use that for $\pi_1(S^1) \cong \mathbb{Z} \cong \langle x | \rangle$.

6. Use the van Kampen theorem to show that $\pi_1(S^2) \cong 0$.

7. Let $G \cong \langle x_1, \ldots, x_n | r_1, \ldots, r_m \rangle$ be a presentation of a group G. Use van Kampen to show that if X_G is the 2-dimensional CW complex built in the lectures, from a bouquet of n spheres with the 2-cells attached according to the relations r_i, then $\pi_1(X_G) \cong G$.

8. Let M be a 4 -manifold with boundary. Show that to especify an embedding $S^1 \times D^2 \hookrightarrow \partial M$, up to isotopy, is sufficient to especify a simple closed curve on ∂M and an integer $n \in \mathbb{Z}$.