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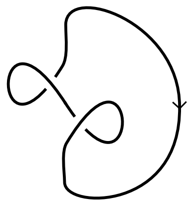
# Homework 1 - Topics in Topology

9 February 2023

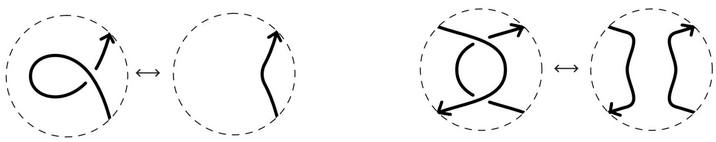
*Please return Thursday 16 February 2022.*

1. Show that the knot below is isotopic to the unknot by giving a sequence of only Reidemeister $\Omega 2$ and $\Omega 3$ moves:



2. In this exercise you will be required to compute integral knot and link invariants.
   (a) Draw two different diagrams for the Whitehead link (look it up) and compute the linking number of its two components for each of the diagrams.
   (b) Draw two different diagrams for the trefoil knot and compute the Casson invariant for each of the diagrams.

3. The goal of this exercise is to give a partial proof of the isotopy invariance of the Casson invariant.
   (a) For each of the following Reidemeister $\Omega 1$ and $\Omega 2$ moves, draw and explain the corresponding moves in terms of Gauss diagrams. Only draw arrows for the relevant crossings for each move:



   (b) Using the previous item, show that the Casson invariant is invariant under the above-depicted Reidemeister moves.

4. Compute the Jones polynomial of the figure-of-eight knot. For this you can choose your preferred diagram.

5. Let $L$ be a link in $\mathbb{R}^3$. The *orientation-reversal* of $L$ is the link $-L$ obtained by reversing the orientation of every component. Let $m : \mathbb{R}^3 \to \mathbb{R}^3$ be the map $m(x, y, z) := (x, y, -z)$. The *mirror image* of $L$ is the link $\overline{L} := m(L)$.
   (a) Show that if $D$ is a link diagram for $L$, then a diagram for $\overline{L}$ is obtained by turning every positive crossing into a negative crossing and the other way around.

(b) Show that

$$J_{-L}(t) = J_L(t)$$

(c) Show that

$$J_{\overline{L}}(t) = J_L(t^{-1}).$$

(*Hint:* How do the writhe and the underlying unoriented diagrams change between $L, -L$ and $\overline{L}$?