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Exercise sheet 1 - Topics in Topology

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1. Prove or give a counterexample of the following statements:
   (a) If L is a 2-component link $L = K_1 \cup K_2$, then L is completely determined by the linking number $lk(K_1, K_2)$.
   (b) If $K_1 = K_2 =$ unknot, then the previous statement holds.
2. Show that the linking number is independent of the isotopy class of a link.
3. For any $n \in \mathbb{Z}$, give an example of a 2-component link $L = K_1 \cup K_2$ with $lk(K_1, K_2) = n$.
4. Can you give infinitely many 2-component links $L = K_1 \cup K_2$ such that $lk(K_1, K_2) = 0$ such that there is no ball $D^3 \subset S^3$ satisfying $D^3 \cap K_i = K_i$?
5. For a path-connected space X, is it true that if $\pi_1(X) \cong 0$ then $H_1(X; \mathbb{Z}) \cong 0$? What about the converse: if $H_1(X; \mathbb{Z}) \cong 0$ then $\pi_1(X) \cong 0$?
6. Show that the figure-of-eight knot is amphicheiral, ie, it is isotopic to its mirror image - obtained by changing the sign of all crossings.

7. Show that the following knot is the trivial knot.