Group de travail on (non)-semisimple TQFTs

2025-2026

The goal of this groupe de travail is to learn different semisimple and non-semisimple TQFTs, to explain the usual constructions and to learn the relations between them.

1 Overview?

General overview of the topics to treat, this will be rather informal.

2 Basics on TQFTs

Motivation, definition and basic properties. Classification of 1-d and 2-d TQFTs. A very good reference is [CR18].

3 The BHMV TQFT

[BHMV95]

4 Categorical background

Monoidal, braided categories, rigidity, pivotal structures, twists, ribbon structures. Spherical structures, the Drinfeld center, modular categories.

Fusion categories, modular fusion categories. S-matrix. Relation with $SL(S, \mathbb{Z})$.

5 Examples: quantum groups

Ribbon Hopf algebras give rise to ribbon categories.

The Drinfeld double. If A is a finite dimensional semisimple Hopf algebra, then $D(A)$-mod is a modular fusion category [EG97].

The quantum group $U_q(sl_2)$, for generic q and q a root of unity. The finite-dimensional small quantum groups $u_q(sl_2)$. A comment about other semisimple Lie algebras.

The topological ribbon Hopf algebra $U_h(sl_2)$. The unrolled quantum group $\bar{U}_\zeta^H(sl_2)$.

6 The RT 3d TQFT

Turaev's book, Bakalov-Kirillov

7 The TV TQFT

6j symbols,... Patureau-Mirand - Geer's book

8 Relation between RT and TV

If $\mathcal{C}$ is spherical fusion, then $Z_{\mathcal{C}}^{TV} \simeq Z_{Z(\mathcal{C})}^{RT}$.

9 Extended TQFTs

Thesis Marco de Renzi

10 Luybashenko TQFT

Non-ss.

Semisimple:

- BHMV TQFT
- RT TQFT (modular fusion categories)
- TV TQFT (spherical fusion categories)
- Link between the two
- CY (Crane-Yetter) invertible 4d TQFT

Non-semisimple:

- Luybashenko
- Modified traces
- CGP invariants of 3-manifolds

More advanced topics:

- Factorisation homology
- Skein categories. Relation with factorisation homology (Cooke).
- Classification of anomalous free, extended 3d TQFTs [BDSV]

References:

- MAIN REFERENCE: Book by Geer and Patureau-Mirand.
- Runkel's https://arxiv.org/abs/1705.05734
- Guide: https://sites.google.com/site/psafronov/notes/non-semisimple-tqfts
- 3D TQFTS AND 3-MANIFOLD INVARIANTS, https://arxiv.org/pdf/2401.10587 (survey)
- Turaev's book
- Turaev-Virelizier's book
- Bakalov-Kirillov's book
- Book Patureau-Mirand - under request
- BDSV MODULAR CATEGORIES AS REPRESENTATIONS OF THE 3-DIMENSIONAL BORDISM 2-CATEGORY, https://arxiv.org/pdf/1509.06811

References

[BHMV95] C. Blanchet, N. Habegger, G. Masbaum, and P. Vogel, *Topological quantum field theories derived from the Kauffman bracket*, Topology **34** (1995), no. 4, 883–927.

[CR18] Nils Carqueville and Ingo Runkel, *Introductory lectures on topological quantum field theory*, Banach Center Publications **114** (2018), 9–47.

[EG97] Pavel Etingof and Shlomo Gelaki, *Some properties of finite-dimensional semisimple hopf algebras*, 1997.