



Algorithms: Design  
and Analysis, Part II

# Minimum Spanning Trees

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## Correctness of Prim's Algorithm (Part II)

# Correctness of Prim's Algorithm

Theorem: Prim's algorithm always outputs a minimum-cost spanning tree.

Key Question: When is it "safe" to include an edge in the tree so-far?

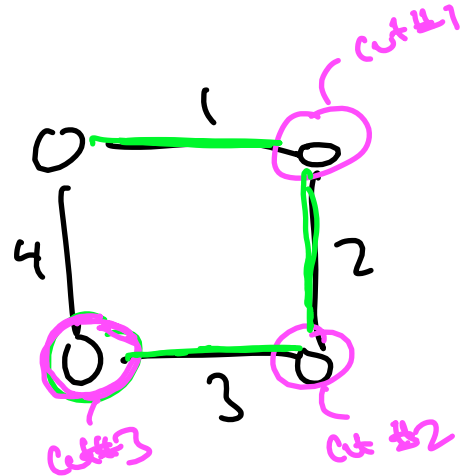
# The Cut Property

**CUT PROPERTY:** consider an edge  $e$  of  $G$ .

Suppose there is a cut  $(A, B)$  such that  $e$  is the cheapest edge of  $G$  that crosses it.

Then  $e$  belongs to the MST of  $G$ .

turns out MST  
is unique if edge  
costs are distinct



# Cut Property Implies Correctness

Claim: Cut Property  $\Rightarrow$  Prim's algorithm is correct.

Proof: By previous video, Prim's algorithm outputs a spanning tree  $T^*$ .

Key point: every edge  $e \in T^*$  is explicitly justified by the Cut Property.

$\Rightarrow T^*$  is a subset of the MST

$\Rightarrow$  Since  $T^*$  is already a spanning tree, it must be the MST

QED!