Department of Engineering Mathematics

EMAT20920: Numerical Methods in MATLAB

COURSEWORK ASSESSMENT
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All figures in this report have been saved using `saveFigPDF` function as it automatically resizes the paper to the correct size.

```
function saveFigPDF(fileName)
    % saveFigPDF saves open figure as a PDF file
    %
    % Inputs:
    %   fileName = File name to save figure as
    % Usage:
    %   saveFigPDF("polynomial") -> Saves current figure as polynomial.pdf

    % Get current figure handle
    figureHandle = gcf;
    % Resize paper
    set(figureHandle,'PaperPosition',3*[0 0 6 4]);
    set(figureHandle,'PaperSize',3*[6 4]);
    set(figureHandle,'PaperUnits','centimeters');

    print(fileName,'-dpdf');
end
```

Question 1: Root-finding

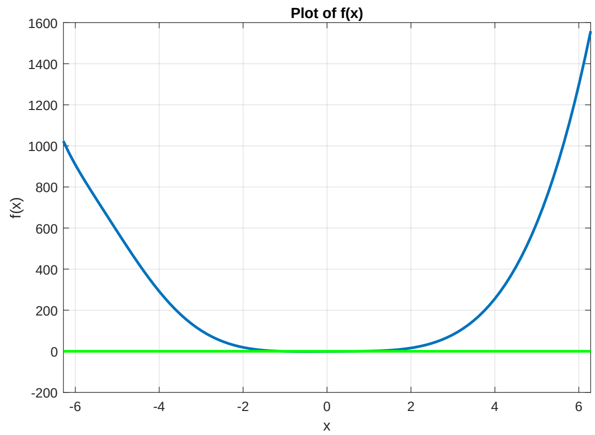
(a) To find how many solutions each equation has in the given domain I will rearrange all the equations to be equal to zero and then looks for the zeros of the rearranged equations. As a corollary to the intermediate value theorem, if a function is continuous and changes sign in a bracket then that bracket must contain a zero. So I will plot each of the rearranged equation and I look for appropriate brackets. I will use the `pltFunc` function to plot the functions as it removes values outside a defined limit which prevents MATLAB plotting discontinuous functions as continuous. The limits can then be changed using the property explorer to give a more useful plot.

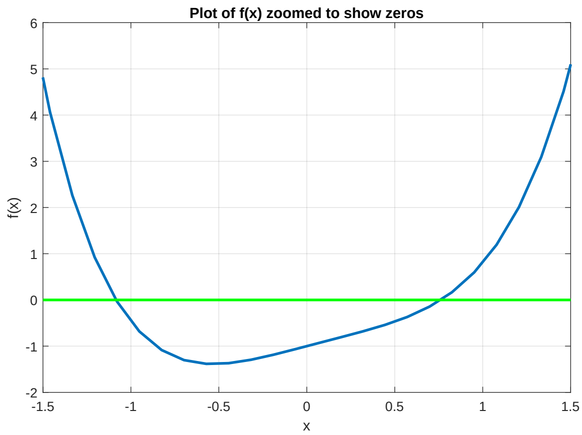
```
function pltFunc(f, xLimits, discontLim)
    %pltFunc plots function f between values of xLim removing any values
    % that are greater than discontLim to prevent MATLAB plotting
    % discontinuous functions as continous and plots a line of x = 0 to
    % help make any zeros clear
    %
    % Input:
    %   f = function handle to plot
    %   xLimits = domain of function to plot
    %   discountLim = absolute values of the function greater than this are
    %   changed to NaN. Setting to inf will plot all values of the function
    %
    % Usage:
    %   pltFunc(@(x) 1./x, [-10 10], 5) -> Plots 1/x between -10 and 10
    %   changing the values where |1/x|>5 to Nan
```

```
    % Check xLim is the correct dimensions
    assert(isequal(size(xLimits), [1 2]), "xLim must be a 1x2 array")

    %% Generate values to plot
    x = linspace(xLimits(1), xLimits(2));
    y = f(x);

    % Remove large values of y to prevent MATLAB plotting discontinuous
    % functions as continuous
    y(abs(y)>discontLim) = NaN;

    %% Plot function and line x = 0
    plot(x, y, [min(x) max(x)], [0 0], "g-", "LineWidth", 2);
    xlabel("x");
    ylabel("f(x)");
    xlim(xLimits);
    title("Plot of f(x)");
    grid on;
end
```

(i) Rearranging $x^4 = e^{-x}\cos(x)$ gives $f(x) = x^4 - e^{-x}\cos(x)$.

```
f = @(x) x.^4 - exp(-x).*cos(x);
pltFunc(f, [-2*pi 2*pi], inf);
```

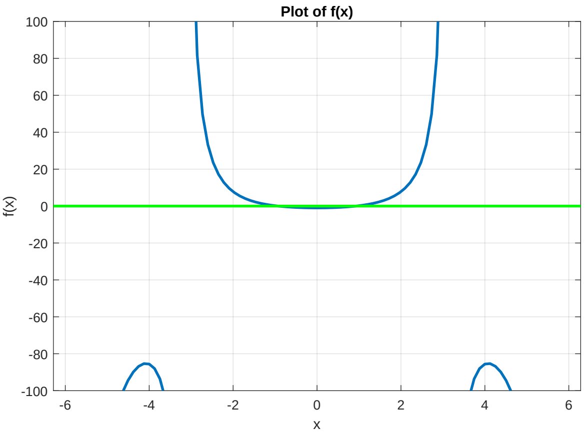


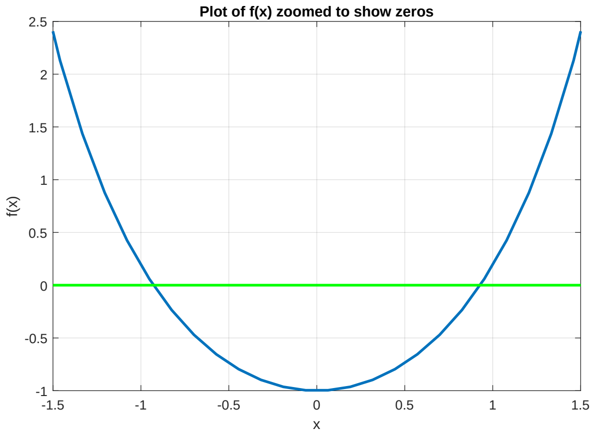


The second zoomed in plot shows there are two zeros in the given domain. The first zero can be bracketed by $[-1.5, -1]$ as $f(-1.5) = 4.7455$ and $f(-1) = -0.4687$ so since the function is continuous and there is a change of sign this bracket must contain a zero. Like wise the second root can be bracketed by $[0.5, 1]$ as $f(0.5) = -0.4698$ and $f(1) = 0.8012$. So the original equation has two solutions.

(ii) Setting $f(x) = \frac{x^3}{\sin(x)} - 1$.

```
f = @(x) (x.^3)./sin(x) - 1;
pltFunc(f, [-2*pi 2*pi], 500);
```

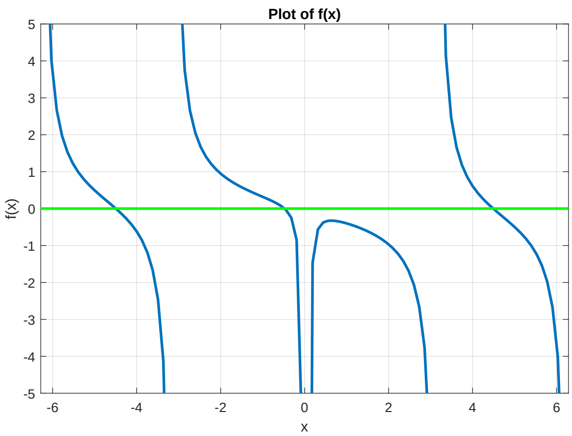




The first root can be bracketed by $[-1, -0.5]$ as $f(-1) = 0.1884$ and $f(-0.5) = -0.7393$ and $f(x)$ is continuous in this bracket. Likewise, the second root can be bracketed by $[0.5, 1]$ as $f(0.5) = -0.7393$ and $f(1) = 0.1884$. So the original equation has two solutions.

(iii) Rearranging $\cot(x) = \frac{25}{25x-1}$ gives $f(x) = \cot(x) - \frac{25}{25x-1}$.

```
f = @(x) cot(x) - 25./(25*x - 1);
pltFunc(f, [-2*pi 2*pi], 30);
```



(iv) Rearrnaging $4e^{-x^2/5} = cos(5x) + 2$ gives $f(x) = 4e^{-x^2/5} - cos(5x) - 2$.

```
f = @(x) 4*exp(-x.^2/5) - cos(5*x) - 2;
pltFunc(f, [-2*pi 2*pi], inf);
```