

**EMAT20920: Numerical Methods in MATLAB**

**COURSEWORK ASSESSMENT**

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All figures in this report have been saved using `saveFigPDF` function as it automatically resizes the paper to the correct size.

```
function saveFigPDF(fileName)
    % saveFigPDF saves open figure as a PDF file
    %
    % Inputs:
    %   fileName = File name to save figure as
    % Usage:
    %   saveFigPDF("polynomial") -> Saves current figure as polynomial.pdf

    % Get current figure handle
    figureHandle = gcf;
    % Resize paper
    set(figureHandle, 'PaperPosition', 3*[0 0 6 4]);
    set(figureHandle, 'PaperSize', 3*[6 4]);
    set(figureHandle, 'PaperUnits', 'centimeters');

    print(fileName, '-dpdf');
end
```

**Question 1: Root-finding**

- (a) To find how many solutions each equation has in the given domain I will rearrange all the equations to be equal to zero and then look for the zeros of the rearranged equations. As a corollary to the intermediate value theorem, if a function is continuous and changes sign in a bracket then that bracket must contain a zero. So I will plot each of the rearranged equation and I look for appropriate brackets. I will use the `pltFunc` function to plot the functions as it removes values outside a defined limit which prevents MATLAB plotting discontinuous functions as continuous. The limits can then be changed using the property explorer to give a more useful plot.

```
function pltFunc(f, domain, discountLim)
    %pltFunc plots function f between values of xLim removing any values
    % that are greater than discountLim to prevent MATLAB plotting
    % discontinuous functions as continuous and plots a line of x = 0 to
    % help make any zeros clear
    %
    % Input:
    %   f = function handle to plot
    %   domain = 1x2 vector containing the lower and upper bound of the
    %   domain of f
    %   discountLim = absolute values of the function greater than this are
    %   changed to NaN. Setting to inf will plot all values of the function
    %
    % Usage:
    %   pltFunc(@(x) 1./x, [-10 10], 5) -> Plots 1/x between -10 and 10
    %   changing the values where |1/x|>5 to NaN
```

```

% Check xLim is the correct dimensions
assert(isequal(size(domain), [1 2]), "xLim must be a 1x2 vector")

%% Generate values to plot
x = linspace(domain(1), domain(2));
y = f(x);

% Remove large values of y to prevent MATLAB plotting discontinuous
% functions as continuous
y(abs(y)>discontLim) = NaN;

%% Plot function and line x = 0
plot(x, y, [min(x) max(x)], [0 0], "g-", "LineWidth", 2);
xlabel("x");
ylabel("f(x)");
xlim(domain);
title("Plot of f(x)");
grid on;
end

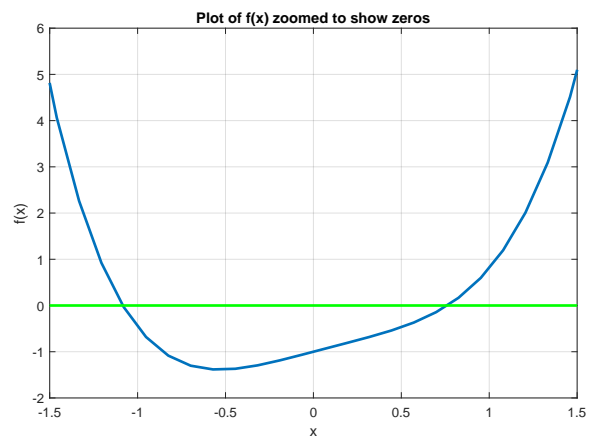
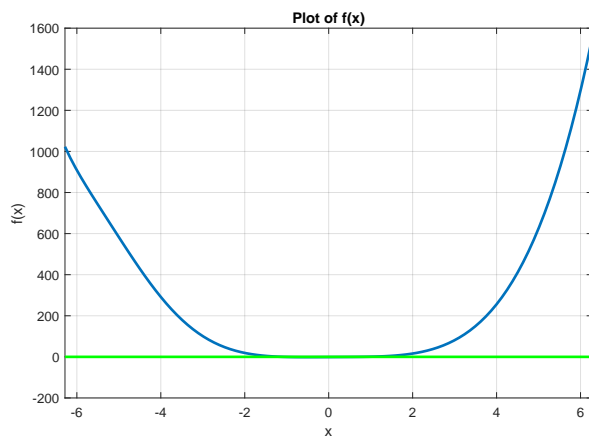
```

- (i) Rearranging  $x^4 = e^{-x} \cos(x)$  gives  $f(x) = x^4 - e^{-x} \cos(x)$ .

```

f = @(x) x.^4 - exp(-x).*cos(x);
pltFunc(f, [-2*pi 2*pi], inf);

```



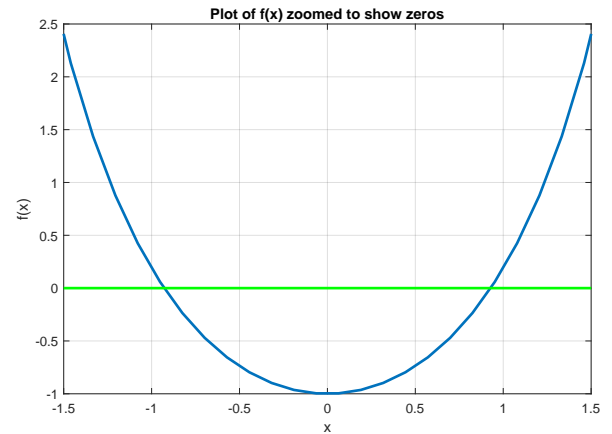
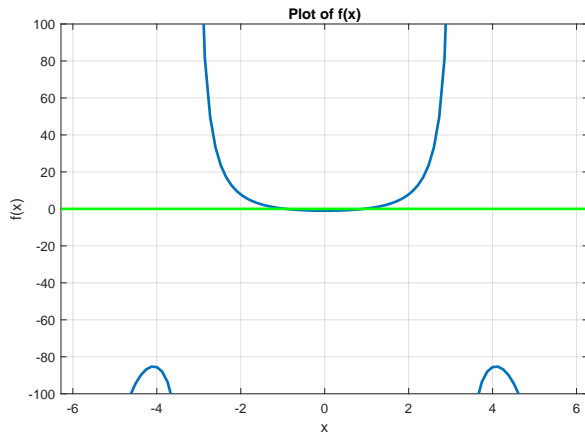
The second zoomed in plot shows there are two zeros in the given domain. The first zero can be bracketed by the interval  $[-1.5, -1]$  as  $f(-1.5) = 4.7455$  and  $f(-1) = -0.4687$  so since the function is continuous and there is a change of sign this bracket must contain a zero. Like wise the second root can be bracketed by the interval  $[0.5, 1]$  as  $f(0.5) = -0.4698$  and  $f(1) = 0.8012$ .

- (ii) Setting  $f(x) = \frac{x^3}{\sin(x)} - 1$ .

```

f = @(x) (x.^3)./sin(x) - 1;
pltFunc(f, [-2*pi 2*pi], 500);

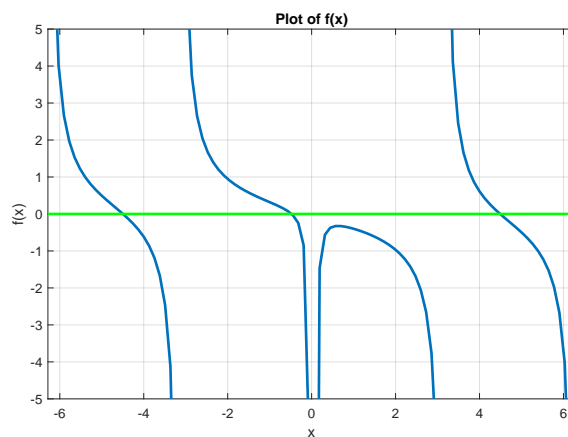
```



The second plot shows there are two roots. The first root can be bracketed by the interval  $[-1, -0.5]$  as  $f(-1) = 0.1884$  and  $f(-0.5) = -0.7393$  and  $f(x)$  is continuous in this bracket. Likewise, the second root can be bracketed by the interval  $[0.5, 1]$  as  $f(0.5) = -0.7393$  and  $f(1) = 0.1884$ .

- (iii) Rearranging  $\cot(x) = \frac{25}{25x-1}$  gives  $f(x) = \cot(x) - \frac{25}{25x-1}$ .

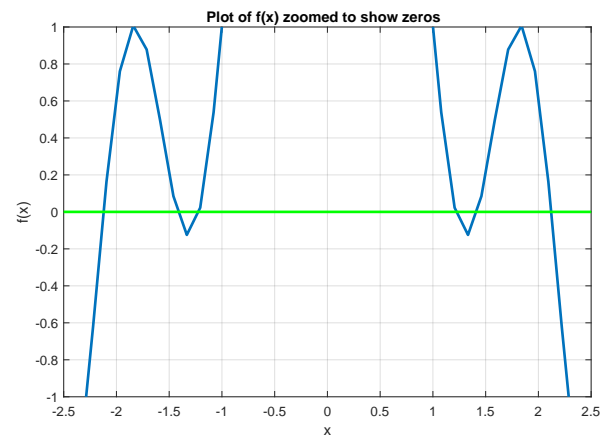
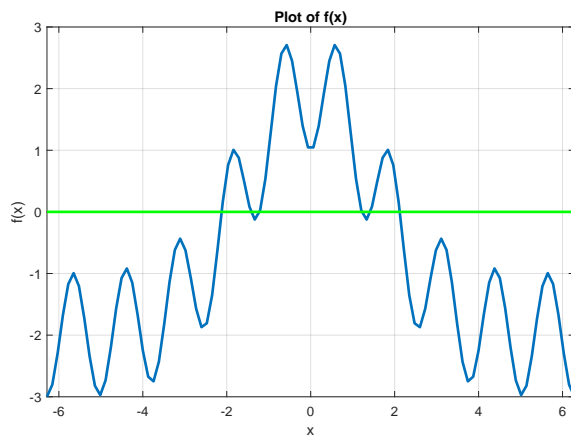
```
f = @(x) cot(x) - 25./(25*x - 1);
pltFunc(f, [-2*pi 2*pi], 30);
```



The plot shows that the equation has three solutions. The first can be bracketed by the interval  $[-5, -4]$  as  $f(-5) = 0.4942$  and  $f(-4) = -0.6162$ . The second solution can be bracketed by the interval  $[-1, -0.1]$  as  $f(-1) = 0.3194$  and  $f(-0.1) = -2.8238$ . The third solution can be bracketed by the interval  $[4, 5]$  as  $f(4) = 0.6112$  and  $f(5) = -0.4974$ .  $f(x)$  is continuous in each of the bracketing intervals.

- (iv) Rearranging  $4e^{-x^2/5} = \cos(5x) + 2$  gives  $f(x) = 4e^{-x^2/5} - \cos(5x) - 2$ .

```
f = @(x) 4*exp(-x.^2/5) - cos(5*x) - 2;
pltFunc(f, [-2*pi 2*pi], inf);
```



The second plot shows that the equation has 6 solutions. The bracketing intervals are shown in the table below.

$[a, b]$	$f(a)$	$f(b)$
$[-2.5, -2]$	-1.8518	0.6364
$[-1.5, -1.25]$	0.2039	-0.0730
$[-1.25, -1]$	-0.0730	0.9913
$[1, 1.25]$	0.9913	-0.0730
$[1.25, 1.5]$	-0.0730	0.2039
$[2, 2.5]$	0.6364	-1.8518

(b) The bisection method is used by calling the the `bisectRoot` function.

```
function [sol, i, err] = bisectRoot(f, a, b, tol)
    %bisectRoot Use the bisection method to find roots of the function f
    % bracketed within the intervals [a, b].
    %
    %Inputs:
    % f = function handle to function whose root is to be found
    % a = 1*n array containing all the lower ends of the brackets
    % where n is the number of roots
    % b = 1*n array containing all the lower ends of the brackets
    % where n is the number of roots
    % tol = absolute error tolerance with which to find the root;
    % Iteration terminates when the root is known to within +/- tol
    %
    %Usage:
    % [r, i, err] = bisect(f,a,b,tol) -> returns the approximation to a root
    % within [a, b], the number of iterations require to find the root
    % and the final absolute error

    % check if all intervals are correctly defined
    assert(isequal(size(a), size(b)), "");

    % check whether f changes sign
    assert(all(sign(f(a)) ~= sign(f(b))),...
        'f(a) and f(b) should have opposite sign');

    % initialise variables
    % iteration counter
```

```

i = zeros(size(a));
% current solution estimate
sol = (a + b)/2;
% previous solution estimate
sol_old = Inf;
% absolute error
err = Inf;
withinTol = zeros(size(a));

% bisection algorithm:
% at each iteration, find the half-interval that contains a sign change
% and relabel the endpoints appropriately
while any(~withinTol)
    i(~withinTol) = i(~withinTol) + 1;
    sol_old = sol;
    mid = (a + b)/2;

    % mid point is a root
    exactRoot = f(mid) == 0;
    sol(exactRoot) = mid(exactRoot);
    err(exactRoot) = 0;
    withinTol(exactRoot) = true;

    % solution is in first half of interval and mid point not a root
    firstHalf = (sign(f(a)) ~= sign(f(mid))) & ~exactRoot;
    b(firstHalf) = mid(firstHalf);

    % solution is in second half of interval and mid point not a root
    secondHalf = (sign(f(a)) == sign(f(mid))) & ~exactRoot;
    a(secondHalf) = mid(secondHalf);

    % update solutions and errors values that aren't within tolerance
    sol(~withinTol) = (a(~withinTol) + b(~withinTol))/2;
    err(~withinTol) = abs(sol(~withinTol) - sol_old(~withinTol));
    withinTol(err < tol) = true;
end
end

```

- (i) Solutions to  $f(x) = x^4 - e^{-x} \cos(x) = 0 \quad x \in [-2\pi, 2\pi]$ .

```

f = @(x) x.^4 - exp(-x).*cos(x);
a = [-1.5 0.5];
b = [-1 1];
[r, i, err] = bisectRoot(f, a, b, [5e-8 5e-9])

```

Note the two different tolerances since one root is an order of magnitude larger so requires one less decimal place of accuracy to be accurate to 8 significant figures.

$[a, b]$	Root	# Iterations
$[-1.5, -1]$	-1.0843597	23
$[0.5, 1]$	0.76221107	26

- (ii) Solutions to  $f(x) = \frac{x^3}{\sin(x)} - 1 = 0 \quad x \in [-2\pi, 2\pi]$ .

```

f = @(x) (x.^3)./sin(x) - 1;
a = [-1 0.5];
b = [-0.5 1];
[r, i, err] = bisectRoot(f, a, b, 5e-9)

```

$[a, b]$	Root	# Iterations
$[-1, -0.5]$	-0.92862631	26
$[0.5, 1]$	0.92862631	26

(iii) Solutions to  $f(x) = \cot(x) - \frac{25}{25x-1} = 0 \quad x \in [-2\pi, 2\pi]$ .

```
f = @(x) cot(x) - 25./(25*x - 1);
a = [-5 -1 4];
b = [-4 -0.1 5];
[r, i, err] = bisectRoot(f, a, b, [5e-8 5e-9 5e-8])
```

$[a, b]$	Root	# Iterations
$[-5, -4]$	-4.4953722	24
$[-1, -0.1]$	-0.47773376	27
$[4, 5]$	4.4914097	24

(iv) Solutions to  $f(x) = 4e^{-x^2/5} - \cos(5x) - 2 = 0 \quad x \in [-2\pi, 2\pi]$ .

```
f = @(x) 4*exp(-x.^2/5) - cos(5*x) - 2;
a = [-2.5 -1.5 -1.25 1 1.25 2];
b = [-2 -1.25 -1 1.25 1.5 2.5];
[r, i, err] = bisectRoot(f, a, b, 5e-8)
```

$[a, b]$	Root	# Iterations
$[-2.5, -2]$	-2.1222382	23
$[-1.5, -1.25]$	-1.4255432	22
$[-1.25, -1]$	-1.2145933	22
$[1, 1.25]$	1.2145933	22
$[1.25, 1.5]$	1.4255432	22
$[2, 2.5]$	2.1222382	23