Control Introduction

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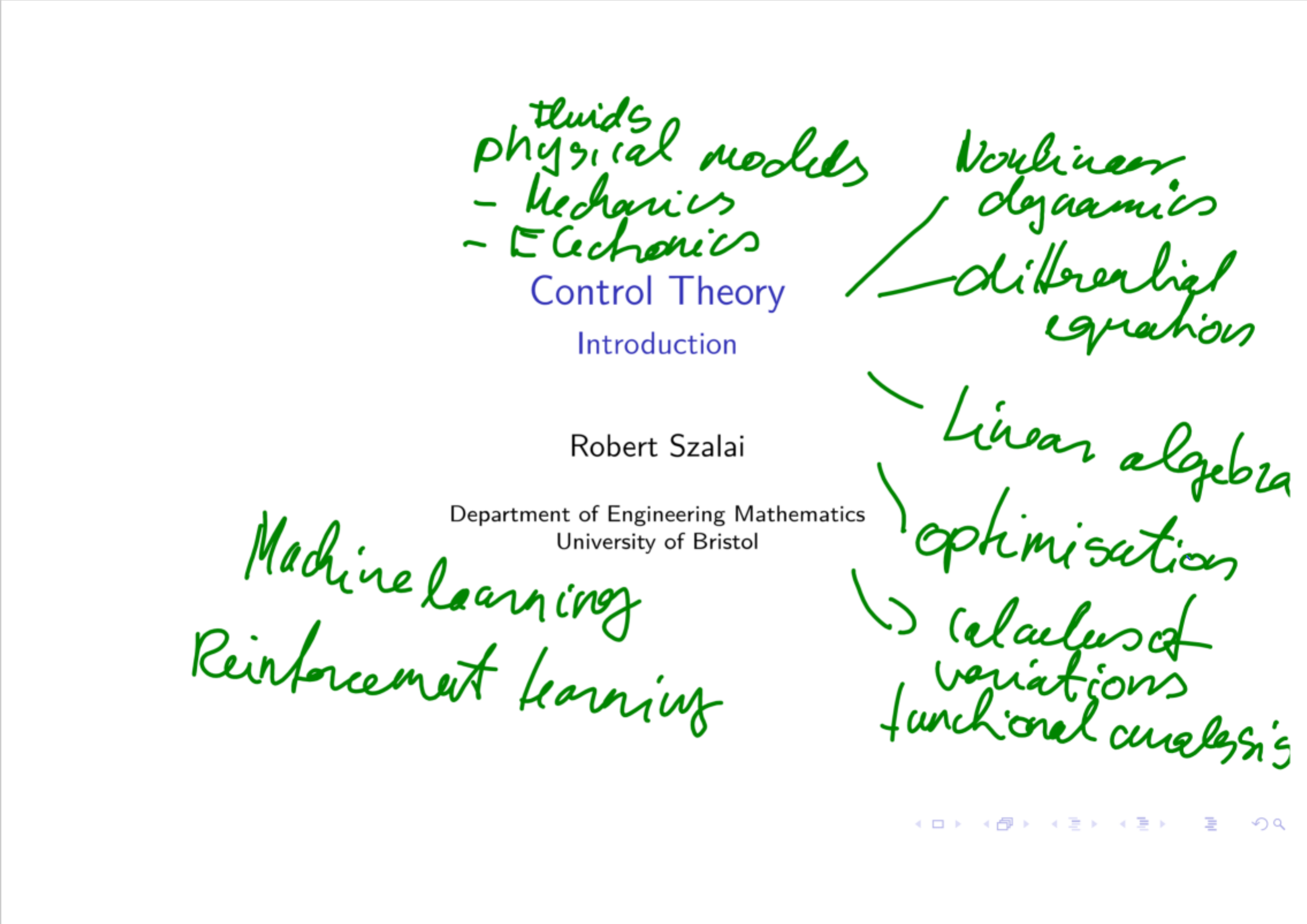


Introduction

- Overview of topics
- What is control thory?
- Terminology

Outline

1. System modelling and analysis
2. Stability
3. Controllability and Observability
4. State-feedback control (designing an observer)
5. Optimal control (Pontryagin)

dual definition

Bellman + dynamic programming

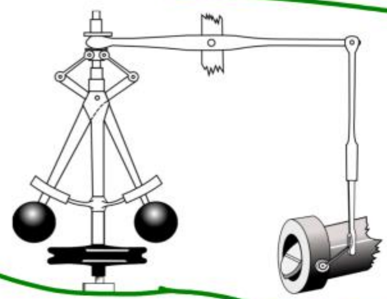
List of books:

- K.J. Aström & R.M. Murray, Feedback Systems, 2018 (available online)
- D.E. Kirk, Optimal Control Theory: An Introduction, 2012

Assessment: 100% exam in the summer
There are past exam papers, though some questions will not apply.

What problems does it solve?

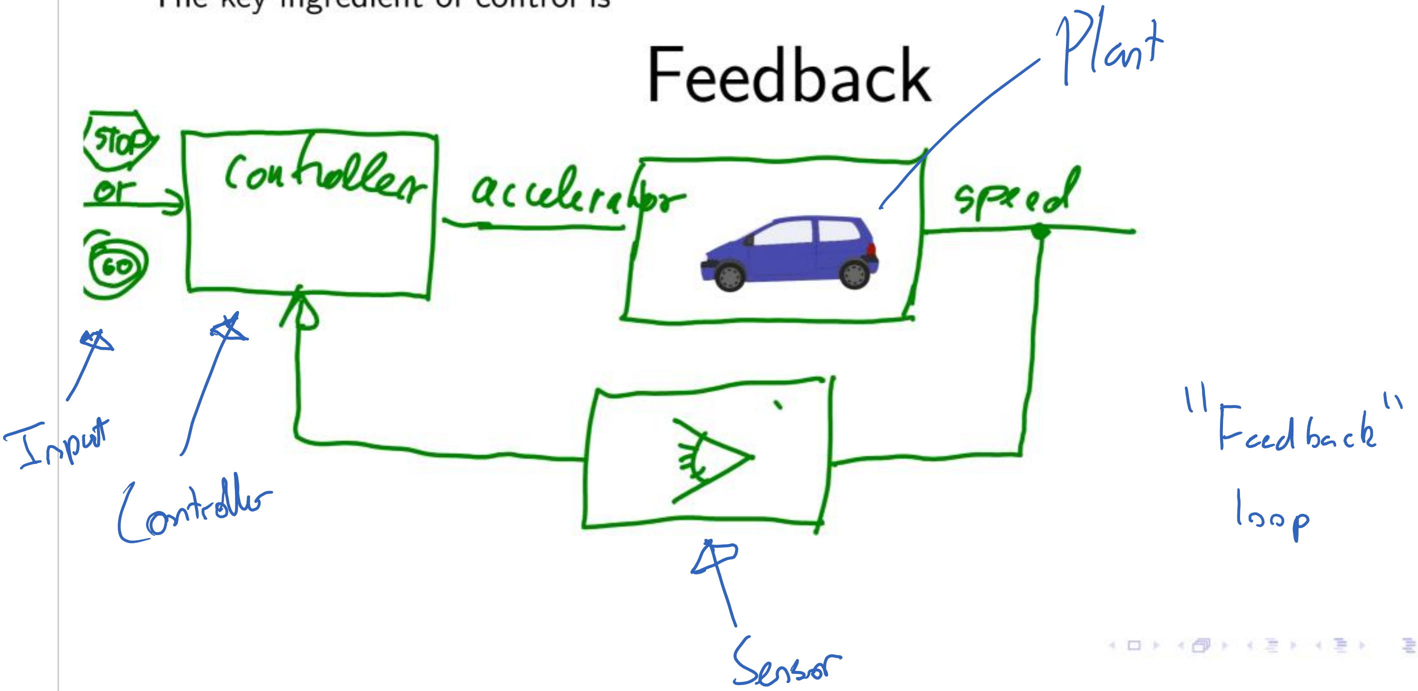
- Automate production, e.g., $\rightarrow$ Watt's regulator
- Cruise control, ABS,
- Engine performance (see diesel car emissions scandal)
- Autopilot, missile guidance (iron dome)
- Spacecraft navigation, robotics
- Reinforcement learning (DeepMind)
- Economics, game theory, automatic trading
- Pacemakers, ventillators, life support...



Feedback control

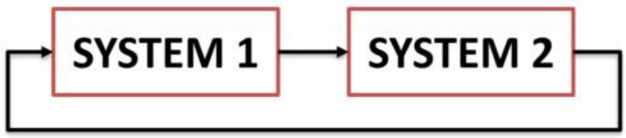
The key ingredient of control is

Feedback



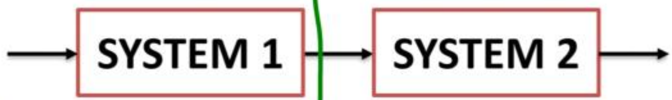
The two alternatives

- Closed Loop



- Open Loop control

Controller



Don't care about observations

Terminology

Control = Sensing + Computation + Actuation

Goals

- Stability: system maintains desired operating point (hold steady speed)
- Robustness: system tolerates perturbations in dynamics (mass, drag, etc)
- Performance: system responds rapidly to changes (accelerate to 6 m/s)

Design of a control system

Assume we have a **technical specification** of the system we need to take three fundamental steps:

1. Create a mathematical model of the system
2. Study the properties of the model
3. Design the controller

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$t = 1, 2, 3, 4, 5$ [ms]

What to do?

- **Step 1.** Use ODEs or discrete time systems to describe the change of variables
- **Step 2.** Check stability, do bifurcation analysis, check controllability and observability
- **Step 3. The main point of this unit.** Answer the question: how can we make the system do what we want? $\rightarrow$

Formally

$t = k_i = 1, 2 \ldots$

T period of sampling

Discrete-time

$$\mathbf{x}_{k+1} = \hat{\mathbf{f}}(\mathbf{x}_k, \mathbf{u}_k), \ \mathbf{x} \in \mathbb{R}^n$$
$$(u_{k+1}) \quad \mathbf{u}_k = \hat{\mathbf{h}}(\mathbf{x}_k, \mathbf{u}_k), \ \mathbf{u} \in \mathbb{R}^p$$

Continuous-time (ODEs)

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \ \mathbf{x} \in \mathbb{R}^n$$
$$\mathbf{u}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t)), \ \mathbf{u} \in \mathbb{R}^p$$

where $\mathbf{x}$ is the **vector of system states** and $\mathbf{u}$ is the **vector of control inputs**.

X : Dynamics

U : Controller

n dimensions

p parameters

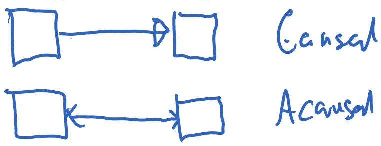
How to do modelling

There are things you can revise:

- **Engineering Physics 2** for creating mechanical models
- Differential equations from **Eng Maths 1**, and **Applied Linear Algebra**

There are software tools

- The Julia Modelling Toolkit (free)
- Modelica (an industry standard) based tools:
  - OpenModelica (free)
  - MapleSim (from MapleSoft)
  - Wolfram System Modeler Mathematica
  - ...
- Modia.jl (free)
- MATLAB/Simulink (probably obsolete)



The End

Cruise control case study

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$$m\dot{v} = F = u - cv - \delta$$ outside disturbance

$$\Rightarrow \dot{v} = \frac{u}{m} - \frac{cv}{m} - \frac{\delta}{m}$$

Step 2: Analysis $\quad \delta = 0$

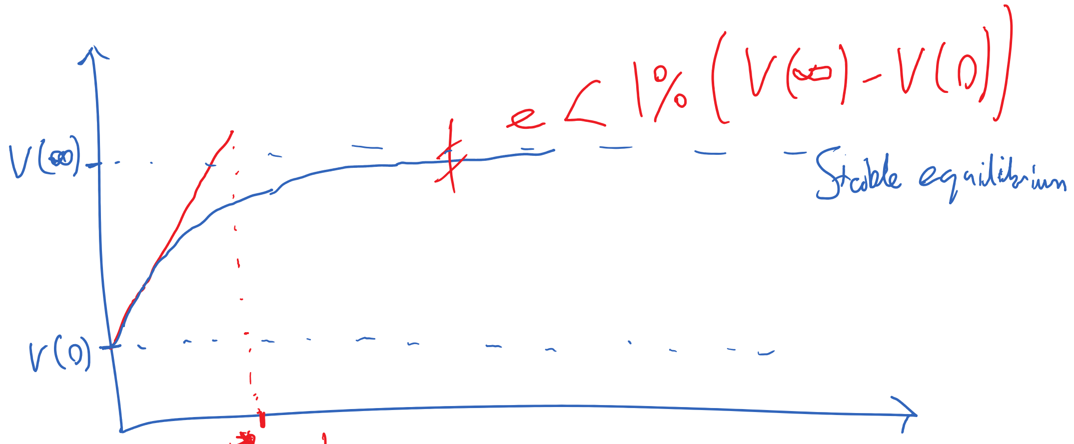
$$t \to \infty \qquad v = a + b e^{\lambda t} \qquad \lambda = -\frac{c}{m}$$

$$a + b = V(0) = V_0$$

$$a = V(\infty) = \lim_{t \to \infty} V(t)$$

$$\dot{v} = 0 \qquad \frac{u}{m} - \frac{c}{m} V(\infty) = 0$$

$$V(\infty) = \frac{u}{c} = a$$



$$t^* = \frac{1}{\lambda} = -\frac{c}{m}$$

Decay time

$$\underbrace{t > 5t^*}_{\text{Decay time}} = -\frac{5c}{m}$$

$$\mathrm{Re}\,\lambda < 0 \to V(\infty) \text{ is stable}$$

Step 3: Control Design

$$\dot v = -\frac{c}{m}v + \frac{u}{m} - \frac{\delta}{m} \qquad \lim_{t\to\infty} v(t) = \bar v$$

$$u = k(\bar v - v)$$

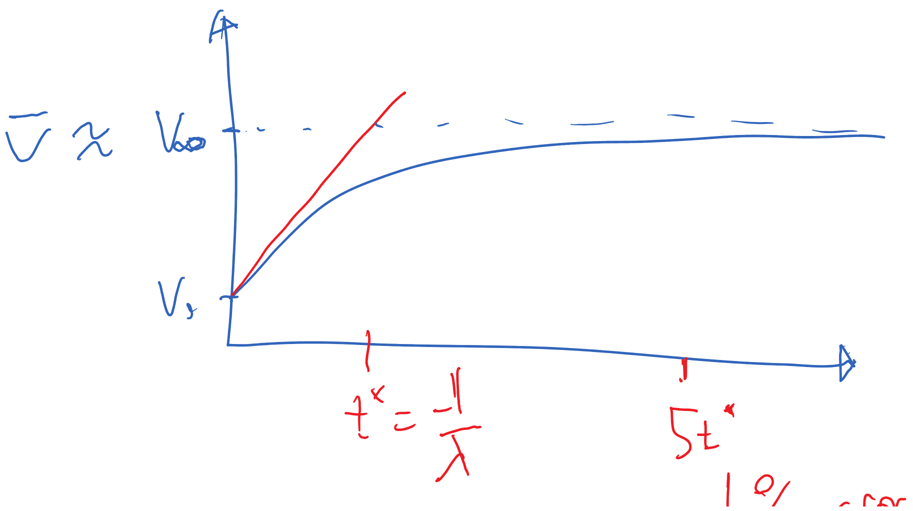
$$\dot v = \underbrace{-\frac{c}{m}v + \frac{kv}{m}}_{-\left(\frac{c}{m}+\frac{k}{m}\right)v} + \frac{k\bar v}{m} - \frac{\delta}{m} \qquad t\to\infty$$

$$V_\infty = \underbrace{\frac{k}{k+c}}_{\approx 1}\bar v - \underbrace{\frac{1}{k+c}}_{\approx 0}\delta$$

$$k \gg 1$$

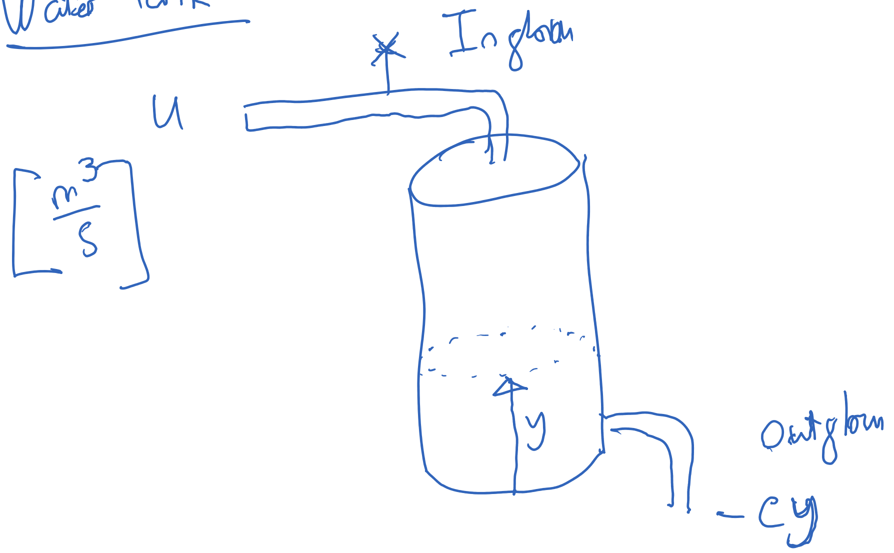
$$V(t) = a + be^{\lambda t} \qquad a+b = V(0)$$

$$a = V(\infty)$$



1% error

Water Tank



$t^* \approx 20\text{mins}$

$A = 2m^2$