Control Introduction

21 January 2023 16:08

fluids
physical models
- Mechanics
- Electronics

Control Theory

Introduction

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Nonlinear dynamics

differential equations

Linear algebra

optimisation

calculus of variations

functional analysis

Machine learning

Reinforcement learning

Introduction

- Overview of topics
- What is control thory?
- Terminology

Outline

1. System modelling and analysis
2. Stability — dual definition
3. Controllability and Observability
4. State-feedback control (designing an observer)
5. Optimal control (Pontryagin) Bellman + dynamic programming

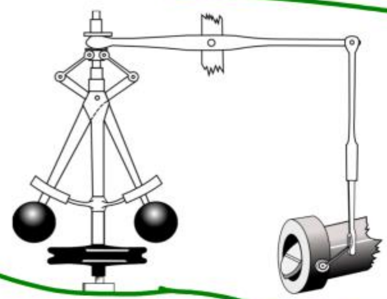
List of books:

- K.J. Aström & R.M. Murray, Feedback Systems, 2018 (available online)
- D.E. Kirk, Optimal Control Theory: An Introduction, 2012

Assessment: 100% exam in the summer
There are past exam papers, though some questions will not apply.

What problems does it solve?

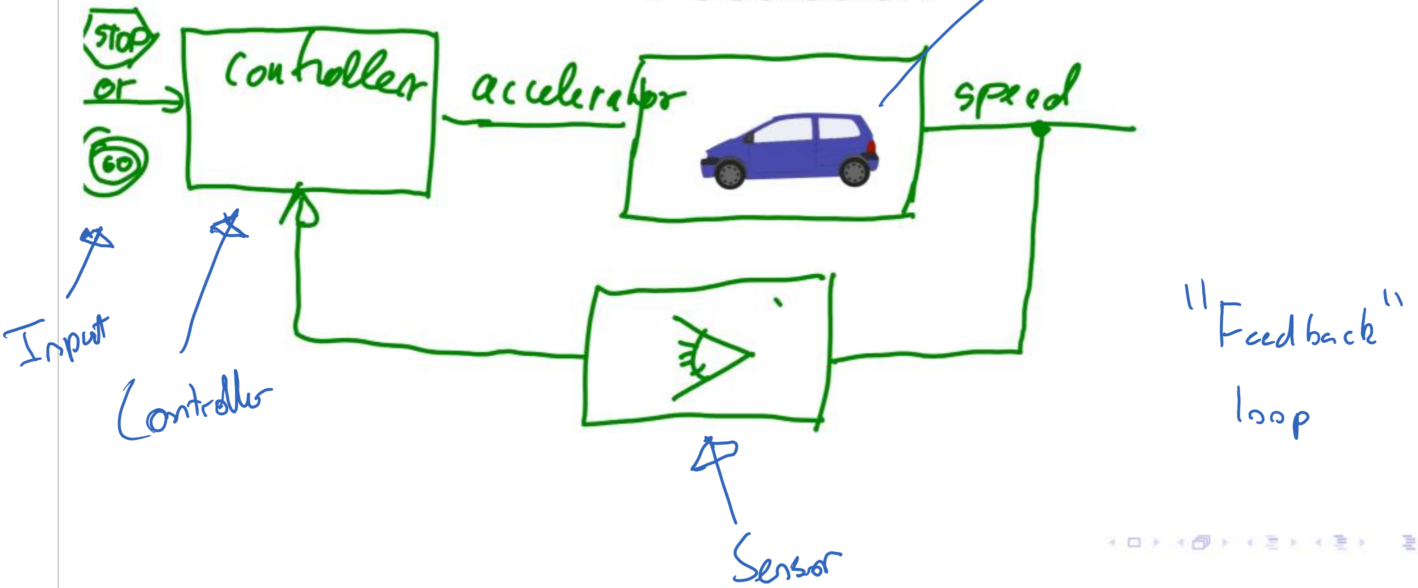
- Automate production, e.g., → Watt's regulator
- Cruise control, ABS,
- Engine performance (see diesel car emissions scandal)
- Autopilot, missile guidance (iron dome)
- Spacecraft navigation, robotics
- Reinforcement learning (DeepMind)
- Economics, game theory, automatic trading
- Pacemakers, ventillators, life support...



Feedback control

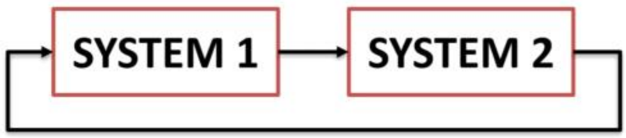
The key ingredient of control is

Feedback



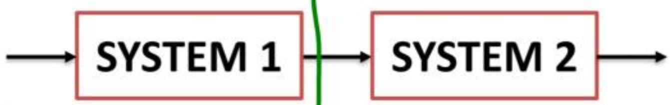
The two alternatives

- Closed Loop



- Open Loop control

Controller



↳ Don't care about observations

Terminology

Control = Sensing + Computation + Actuation

Goals

- Stability: system maintains desired operating point (hold steady speed)
- Robustness: system tolerates perturbations in dynamics (mass, drag, etc)
- Performance: system responds rapidly to changes (accelerate to 6 m/s)

Design of a control system

Assume we have a **technical specification** of the system we need to take three fundamental steps:

1. Create a mathematical model of the system
2. Study the properties of the model
3. Design the controller

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$t = 1, 2, 3, 4, 5$ [ms]

What to do?

- **Step 1.** Use ODEs or discrete time systems to describe the change of variables
- **Step 2.** Check stability, do bifurcation analysis, check controllability and observability
- **Step 3. The main point of this unit.** Answer the question: how can we make the system do what we want? $\rightarrow$

Formally

$t = k,\ = 1, 2 \ldots$

T period of sampling

Discrete-time

$$\left[\begin{aligned} \boldsymbol{x}_{k+1} &= \hat{\boldsymbol{f}}(\boldsymbol{x}_k, \boldsymbol{u}_k), \ \boldsymbol{x} \in \mathbb{R}^n \\ (\boldsymbol{u}_{k+1}) \quad \boldsymbol{u}_k &= \hat{\boldsymbol{h}}(\boldsymbol{x}_k, \boldsymbol{u}_k), \ \boldsymbol{u} \in \mathbb{R}^p \end{aligned}\right.$$

Continuous-time (ODEs)

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)), \ \boldsymbol{x} \in \mathbb{R}^n \\ \boldsymbol{u}(t) &= \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t)), \ \boldsymbol{u} \in \mathbb{R}^p \end{aligned}\right\}$$

where $\boldsymbol{x}$ is the **vector of system states** and $\boldsymbol{u}$ is the **vector of control inputs**.

X : Dynamics

U : Controller

n dimensions

p parameters

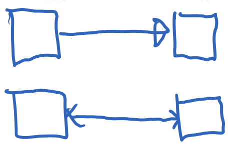
How to do modelling

There are things you can revise:

- **Engineering Physics 2** for creating mechanical models
- Differential equations from **Eng Maths 1**, and **Applied Linear Algebra**

There are software tools

- The Julia Modelling Toolkit (free)
- Modelica (an industry standard) based tools:
  - OpenModelica (free)
  - MapleSim (from MapleSoft)
  - Wolfram System Modeler ← Mathematica
  - . . .
- Modia.jl (free)
- MATLAB/Simulink (probably obsolete)



Causal

Acausal

The End

Cruise control case study

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$$m\dot{v} = F = u - cv - \delta$$ outside disturbance

$$\Rightarrow \dot{v} = \frac{u}{m} - \frac{cv}{m} - \frac{\delta}{m}$$

Step 2: Analysis $\delta = 0$

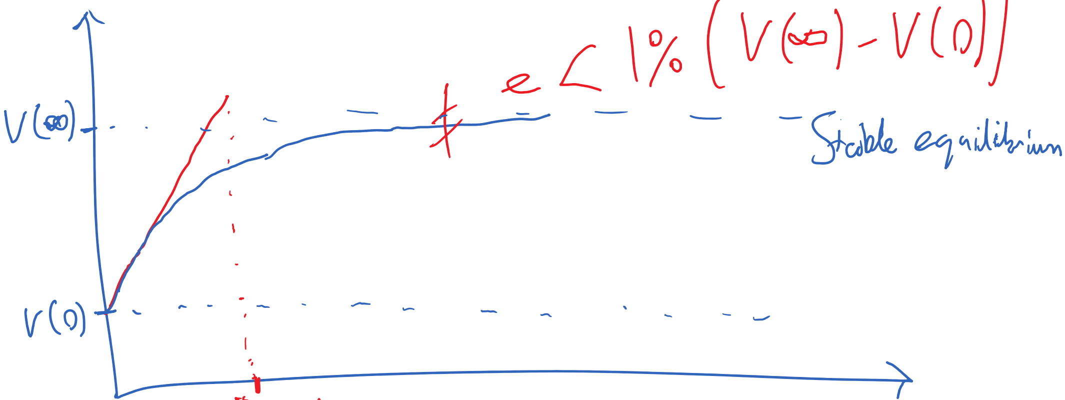
$t \to \infty$ $\quad V = a + be^{\lambda t}$ $\quad \lambda = -\frac{c}{m}$

$$a + b = V(0) = V_0$$

$$a = V(\infty) = \lim_{t \to \infty} V(t)$$

$\dot{v} = 0$ $\quad \frac{u}{m} - \frac{c}{m} V(\infty) = 0$

$$V(\infty) = \frac{u}{c} = a$$



$e < 1\% \left(V(\infty) - V(0) \right)$

Decay time

$\overbrace{t > 5t^*}$

$= -\frac{5c}{m}$

$t^* = \frac{1}{\lambda} = -\frac{c}{m}$

$\operatorname{Re} \lambda < 0 \to V(\infty)$ is stable

Step 3: Control Design

$$\dot v = -\frac{c}{m}v + \frac{u}{m} - \frac{\delta}{m} \qquad \lim_{t\to\infty} v(t) = \bar v$$

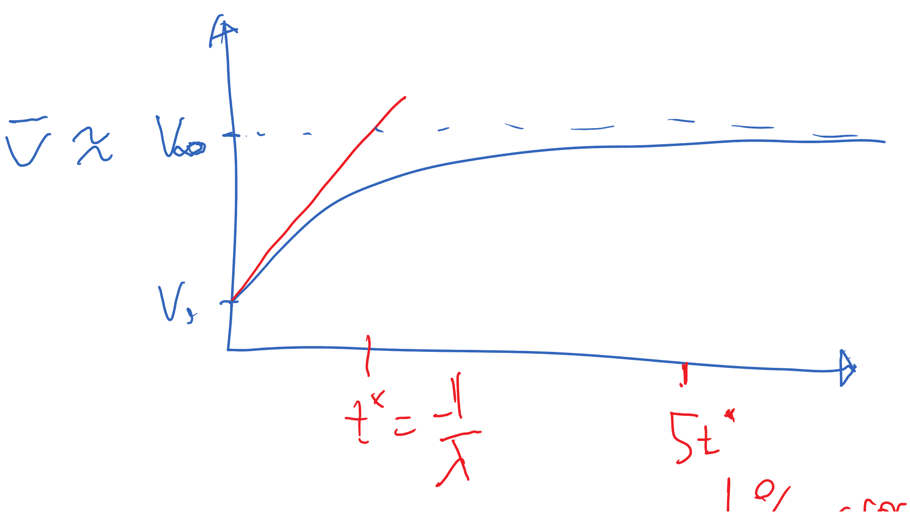
$$u = k(\bar v - v)$$

$$\dot v = \underbrace{-\frac{c}{m}v + \frac{kv}{m}}_{-\left(\frac{c}{m}+\frac{k}{m}\right)v} + \frac{k\bar v}{m} - \frac{\delta}{m} \qquad t \to \infty$$

$$V_\infty = \underbrace{\frac{k}{k+c}}_{\approx 1}\bar v - \underbrace{\frac{1}{k+c}}_{\approx 0}\delta$$

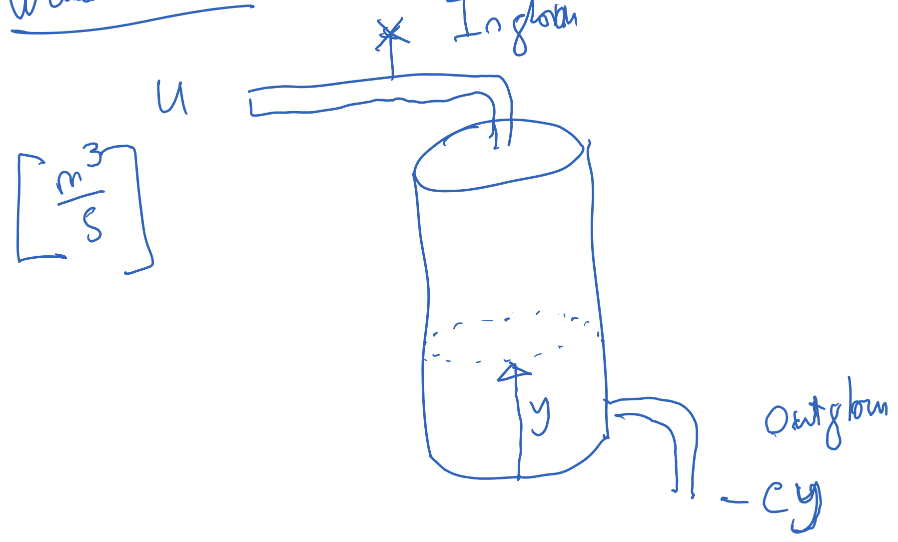
$$k \gg 1$$

$$V(t) = a + be^{\lambda t} \qquad \begin{aligned} a+b &= V(0) \\ a &= V(\infty) \end{aligned}$$



1% error

Water Tank



$t^* \approx 20$ mins

$A = 2m^2$

$$2\dot{y} = u - cy$$

$$\dot{y} = \frac{1}{2}(u - cy)$$

$$\int \frac{2}{u-cy}\,dy = \int dt$$

$$\Rightarrow -\frac{2}{c}\ln(u-cy) = t + c$$

$$\Rightarrow \ln(u-cy) = -\frac{c}{2}(t+c)$$

$$\Rightarrow u - cy = e^{-\frac{c}{2}(t+c)}$$

$$= Ae^{\frac{c}{2}t}$$

$$y = \frac{1}{c}\left(u - Ae^{-\frac{c}{2}t}\right)$$