Stability of Linear Systems

$$\frac{dx}{dt} = Ax \qquad x(0) = x_0 \qquad A \in \mathbb{R}^{n \times n}$$

$$\lambda(A) = \{ s \in \mathbb{C} \mid \det(sI - A) = 0 \}$$

Characteristic polynomial

Origin is always an equillibrium for a linear system

Equillibrium point $x_e = 0$ is stable if $\mathrm{Re}\,\lambda_i \leq 0$

and asymptotically stable if $\mathrm{Re}\,\lambda_i < 0$

Lyapunov Stability Analysis

A Lyapunov Function $V : \mathbb{R}^n \to \mathbb{R}$ is an energy like function.

$$\dot{V} = \frac{\partial V}{\partial x} \frac{dx}{dt} = \frac{\partial V}{\partial x} F(x)$$

If there exists an $r > 0$ such that V is positive definite and $\dot{V}$ is negative semidefinite the $x = 0$ is locally stable. If V is positive definite and $\dot{V}$ negative definite $x = 0$ is asymptotically stable.

$V(x) = x^T P x \qquad P \in \mathbb{R}^{n \times n} \quad \text{s.t.} \quad P = P^T$