

Week 17: Continuum

$$1. \quad \underbrace{u_t + v u_x}_{\text{Drift}} = \underbrace{a u - b u^2}_{\text{Reaction}}$$

$$u(x, 0) = f(x)$$

$$u = F(x - ct) = F(z)$$

$$u_t = -c F' \quad u_x = F'$$

$$-c F' + v F' = a F - b F^2$$

$$F'(v - c) = a F - b F^2$$

$$\frac{dF}{dz} \rightarrow F' = \frac{a F - b F^2}{v - c} \quad \text{ODE in } F$$

$$\int \frac{1}{a F - b F^2} dF = \int \frac{1}{v - c} dz$$

$$2. \quad u_t + u u_x = 0 \quad u(x, 0) = \phi(x)$$

$$\frac{dt}{ds} = 1 \quad t(r, s) = s + c_1 \quad t(r, 0) = c_1 = 0$$

$$t(r, s) = s$$

$$\frac{du}{ds} = 0 \quad u(r, s) = c_2 \quad u(r, 0) = \phi(r)$$

$$u(r, s) = \phi(r)$$

$$\frac{dx}{ds} = u = \phi(r) \quad x(r, 0) = r$$

$$x(r, s) = \phi(r)s + c_3$$

$$x(r, 0) = c_3 = r \quad \Rightarrow c_3 = r$$

$$t = s \quad x = ut + r$$

$$u(x, t) = \phi(x - ut)$$