PDEs Week 1 in person

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}, \quad x \in [-\infty, \infty] \qquad (1)$$

$$\mathcal{F}(u(x,t)) = \int_{-\infty}^{\infty} u(x,t)\, e^{-ikx}\, dx = \hat{u}(k,t)$$

Fourier Transform of (1):

$$\int_{-\infty}^{\infty} \frac{\partial u}{\partial t} e^{-ikx}\, dx = \int_{-\infty}^{\infty} D\frac{\partial^2 u}{\partial x^2} e^{-ikx}\, dx$$

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty} u e^{-ikx}\, dx = D\int_{-\infty}^{\infty} \frac{\partial^2 u}{\partial x^2} e^{-ikx}\, dx$$

$$\hat{u}_t = D(ik)^2 \hat{u}$$
$$= -k^2 D\hat{u} \qquad \text{ODE in } t$$

$$\hat{u}(t) = \hat{u}_0 e^{-k^2 D t}$$