1. $u_t + x u_x = 0$

$$u(x,0) = \phi(x)$$

$\frac{dt}{ds} = 1$ $\qquad t(r,0) = 0$

$\frac{dx}{ds} = x$ $\qquad x(r,0) = r$

$\frac{du}{du} = 0$ $\qquad u(r,0) = \phi(r)$

$t(r,s) = s + c_1$ $\qquad t(r,0) = c_1 = 0$

$\boxed{t(r,s) = s}$ ①

$x(r,s) = c_2 e^s$ $\qquad x(r,0) = c_2 = r$

$\boxed{x(r,s) = re^s}$ ②

$u(r,s) = c_3$ $\qquad u(r,0) = c_3 = \phi(r)$

$\boxed{u(r,s) = \phi(r)}$ ③

① into ② $\quad x = re^t \Rightarrow r = xe^{-t}$

$$\Rightarrow \boxed{u(x,t) = \phi(xe^{-t})}$$

2. $u_t + (x+t)\, x_x = t$

$u(x,0) = \phi(x)$

$\frac{dt}{ds} = 1 \qquad t(r,0) = 0$

$t(r,s) = s + c_1 \qquad t(r,0) = c_1 = 0$

$\boxed{t(r,s) = s}$

$\frac{dx}{ds} - x = s \qquad x(r,0) = r$

$x = e^{\lambda s}$

$\lambda - 1 = 0 \Rightarrow \quad x_H = c_1 e^{s}$

$x_p = c_2 s + c_3$

$\frac{dx_p}{ds} = c_2$

$c_2 - c_2 s - c_3 = s$

$-c_2 = 1 \quad \Rightarrow c_2 = -1$

$c_2 - c_3 = 0 \quad \Rightarrow c_3 = -1$

$x(s) = c_1 e^{s} - s - 1$

$x(r,0) = c_1 - 1 = r \quad \Rightarrow c_1 = r + 1$

$$\boxed{x(r,s) = (r+1)e^{s} - s - 1}$$

$$\frac{du}{ds} = s \qquad u(r,0) = \phi$$

$$u(r,s) = \frac{s^2}{2} + \phi(r)$$

$$x = (r+1)e^{s} - s - 1$$

$$\frac{x+s+1}{e^{s}} - 1 = r$$

$$r(t) = \frac{x+t+1}{e^{s}} - 1$$

$$u(x,t) = \frac{t^2}{2} + \phi\left(\frac{x+t+1}{e^{s}} - 1\right)$$