Chapter 1 - Introduction

August 27, 2022

Theorem 1.1 [2-steps]

If an operation consists of two steps, of which the first can be done in n_1 ways and for each of these the second can be done in n_2 ways, then the whole operation can be done in $n_1 \cdot n_2$ ways.

Theorem 1.2 [k-steps]

If an operation consists of k steps, of which the first can be done in n_1 ways, for each of these the second step can be done in n_2 ways, for each of the first two the third step can be done in n_3 ways, and so forth, then the whole operation can be done in $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ ways.

Theorem 1.3 [# of permutations]

The number of permutations of n distinct objects is $n!$

Theorem 1.4 [# of permutations, r at a time]

The number of permutations of n distinct objects taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}$$

for $r = 0, 1, 2, \ldots, n$.

Proof

The formula $_nP_r = n(n-1) \cdot \ldots \cdot (n-r+1)$ cannot be used for $r = 0$, but we do have

$$_nP_0 = \frac{n!}{(n-0)!} = 1$$

For $r = 1, 2, \ldots, n$, we have

$$
\begin{aligned}
{}_nP_r &= n(n-1)(n-2)\cdot\ldots\cdot(n-r-1) &(1)\\
&= \frac{n(n-1)(n-2)\cdot\ldots\cdot(n-r-1)(n-r)!}{(n-r)!} &(2)\\
&= \frac{n!}{(n-r)!} &(3)
\end{aligned}
$$

Theorem 1.5 [Circular Permutations]

The number of permutations of n distinct objects arranged in a circle is $(n-1)!$

Theorem 1.6 [Permutations of n objects]

The number of permutations of n objects of which n_1 are of one kind, n_2 are of a second kind, $\ldots, n_k$ are of a kth kind, and $n_1 + n_2 + \cdots + n_k = n$ is

$$\frac{n!}{n_1! \cdot n_2! \cdot \ldots \cdot n_k!}$$