# A Comparison Between Semi-Physical and Black-Box Neural Net Modeling: A Case Study

**Peter Lindskog and Jonas Sjöberg**

Department of Electrical Engineering, Linköping University, S-581 83 Linköping, Sweden
✉: name@isy.liu.se

**Abstract:** This paper considers identification of a solar-heated house. Using prior physical knowledge and a semi-physical modeling procedure, a set of physically motivated regressors are determined. With these as inputs a reasonable neural network model of the plant is estimated.

**Keywords:** system identification, neural networks, semi-physical modeling

## 1. Introduction

System identification methods are excellent and engineering appealing tools for designing mathematical models of dynamical systems. In brevity, the problem can be divided into two parts: *model structure identification* followed by *parameter estimation*. While different least-squares type of techniques are predominant for parameter estimation, one has several model structure approaches to choose between.

Physically parameterized modeling (where all the physical insight about the plant is built into the model) is a rather time-consuming procedure which demands a lot of prior that can be hard, or virtually impossible, to gain. However, such an approach often leads to models which are sparse in the number of parameters – something which is highly desired in system identification.

On the other extreme we find the *black-box* approach where the model is searched for in a sufficiently flexible model set. Instead of incorporating prior knowledge the model contains many parameters so that the unknown function can be approximated without too large a bias. This approach demands much less engineering time but is heavily dependent on the information contained in the data. *Neural networks* is one out of many possible choices within this category.

Between these model structure selection paradigms there is a large zone where important physical knowledge and common sense reasoning are used in the identification process, but not to the extent that a fully physically parameterized model is constructed. The basis functions used here are often the result of physical reasoning, whereas the parameters to be estimated typically have little or no direct physical meaning. This middle zone is frequently labeled *semi-physical modeling*.

In this contribution we will apply semi-physical and black-box neural net modeling in order to describe a solar-heated house, depicted in Figure 1. With this application as a departure point, what are the advantages and disadvantages with the obtained models? Topics such as parameter estimation, model understanding and model quality will be discussed. It will also be discussed how these approaches can be combined and benefit from each other.

## 2. The Solar-Heated House

The oil crises of the seventies triggered an increasing search for alternative and more environmental friendly energy sources. The solar-heated house of Figure 1 was one project initiated in this spirit [2]. Functionally, the heating idea is very simple: The sun heats the air in the solar cells, whereupon the heated air is transported to the heat storage, which is an isolated box filled with pebbles. Later, the stored heat is used to heat the house. The modeling aim is to investigate how the solar radiation $I(t)$ and the pump speed $u(t)$ affect the storage inlet temperature $y(t)$.

The measured inputs, $I(t)$ and $u(t)$, and the output $y(t)$ are shown in Figure 2. These signals were measured every tenth minute over a period of 48 hours. The first 120 samples (the first 20

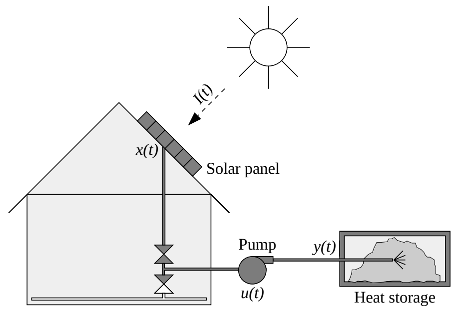


**Figure 1.** Sketch over the solar-heated house.

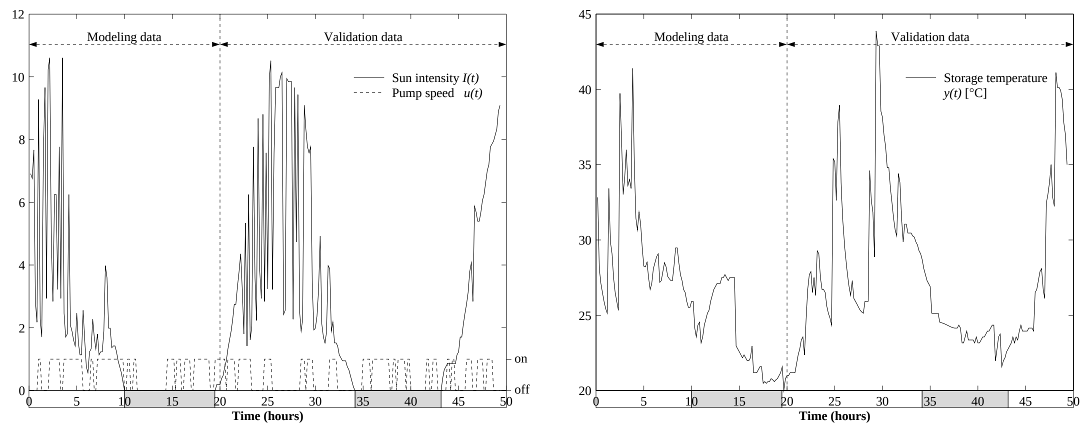


**Figure 2.** Left: Sun intensity $I(t)$ and pump speed $u(t)$ (input signals). Right: Inlet storage temperature $y(t)$ (output signal). Grey time slots indicate darkness during the night.

hours) were used for identification, while the remaining data were saved for validation purposes.

## 3. Linear Modeling

Before pursuing any nonlinear kind of model structure identification it is of interest to see how an ordinary linear model structure, such as

$$y(t) = \boldsymbol{\theta}^T \boldsymbol{\varphi}(t) = \theta_1 y(t-1) + \theta_2 y(t-2) + \theta_3 u(t-1) + \theta_4 u(t-2) + \theta_5 I(t-1) + \theta_6 I(t-2) \quad (1)$$

would perform on this data. After removing mean values and focusing on the second day-time period, it is from the simulation detailed in Figure 3 (left) clear that the linear least-squares fitted model has severe difficulties explaining the heating dynamics. In fact, staying within the linear framework does not lead to a much improved fit – the same kind of discrepancy between measured and simulated output is still there. This observation indicates that the system has a major nonlinear behavior, and thus such a model structure should be searched for.

## 4. Semi-Physical Modeling

By semi-physical modeling we mean the process to take simple physical insight about the behavior of the system into account, to use that insight to find nonlinear transformations of the raw measurements so that the new variables – the new inputs and outputs – stand a better chance to describe the true system when subjected to standard model structures (typically linear in the new

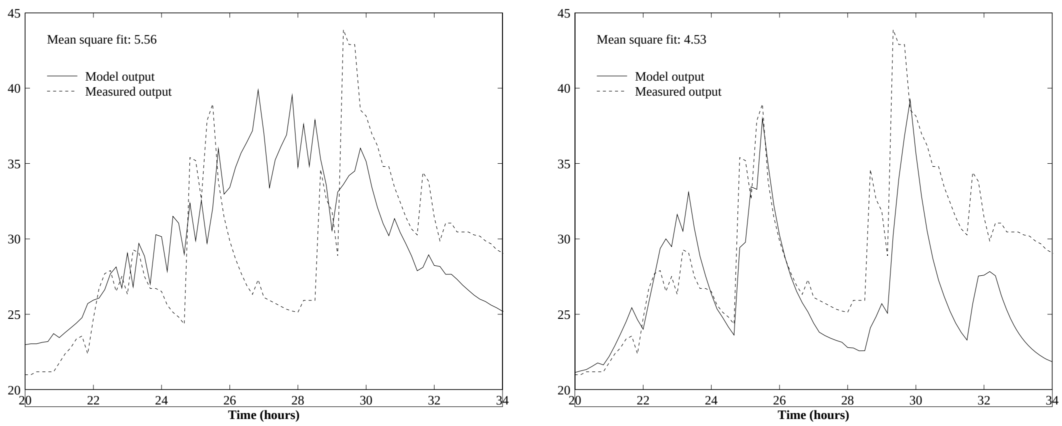


**Figure 3.** Measured storage temperature compared with the simulated output of the linear ARX model (1) (left) and compared with the second order nonlinear semi-physical model (5) (right).

variables). Using the law of conservation of energy, a simple model of the solar house [1] is

$$x(t+1) - x(t) = \eta_1 I(t) - \eta_2 x(t) - \eta_3 x(t)u(t) \tag{2}$$
$$y(t+1) - y(t) = \eta_3 x(t)u(t) - \eta_4 y(t), \tag{3}$$

where $x(t)$ is the mean solar panel temperature and $\eta_1, \ldots, \eta_4$ are unknown parameters. Since $x(t)$ cannot be measured, the natural next step is to eliminate this variable, which yields a structure of the form $y(t) = \boldsymbol{g}(\boldsymbol{\eta})^T \boldsymbol{\varphi}(t)$. In this case, the regression vector $\boldsymbol{\varphi}(t)$ contains 6 entries, all being combinations of measured signals only. Notice that the parameters $\boldsymbol{\eta}$ enter the structure in a complicated nonlinear fashion, thus meaning that iterative parameter estimation schemes must be employed. To avoid this, we can reparameterize the model:

$$y(t) = \sum_{i=1}^{6} g_i(\boldsymbol{\eta})\varphi_i(t) = \sum_{i=1}^{6} \theta_i \varphi_i(t) = \boldsymbol{\theta}^T \boldsymbol{\varphi}(t), \tag{4}$$

which means that the new parameters $\boldsymbol{\theta}$ can be estimated using the least-squares algorithm. By finally adopting conventional statistical hypothesis tests it turns out that only 2 out of these 6 regressors are really important, namely

$$y(t) = \theta_1 \varphi_1(t) + \theta_2 \varphi_2(t) = \theta_1 y(t-1) + \theta_2 u(t-1)I(t-2). \tag{5}$$

As can be seen in Figure 3 (right) this second order nonlinear least-squares fitted model performs much better than the previously discussed linear one. Notice also that the model has a nice physical interpretation. The sun intensity cannot affect the storage temperature much when the pump is off. Instead, we should expect a multiplicative relationship between the available input signals, which is exactly what the second regressor of (5) expresses. Furthermore, the difference in time shift between the sun intensity and the pump speed states that it takes one time unit to transport the energy from the solar panel to the pump and another time unit to transport it to the storage.

## 5. Combination of Black-Box Neural Net and Semi-Physical Modeling

The black-box model selection problem can be regarded as composed of two design questions: the choice of regressor $\boldsymbol{\varphi}(t)$ and the choice of model structure $g$, to get

$$y(t) = g(\boldsymbol{\theta}, \boldsymbol{\varphi}(t)). \tag{6}$$

Although it is known that the model structure $g$ is nonlinear it can often be worthwhile to start the modeling work by considering linear models. The reason is mainly that it is easier to play around

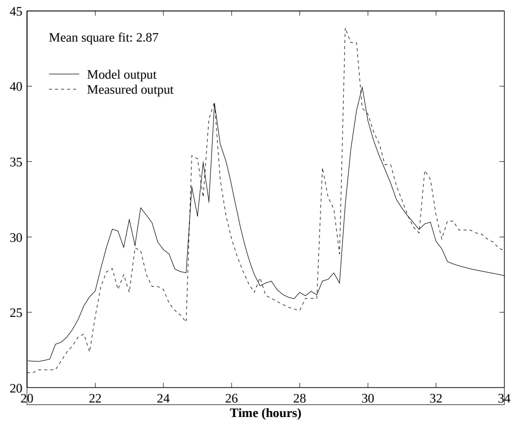


**Figure 4.** Measured storage temperature compared with the simulated output of the NARX model with three hidden units (6).

and try different $\varphi(t)$. The knowledge gained from these experiments can then, hopefully, be used when nonlinear structures $g$ are considered. In this example, however, linear modeling gives no clue on how to choose $\varphi(t)$ which easily can be understood from the semi-physical model. Since the true relationship is likely to contain a product it cannot be approximated by a linear model.

Keeping a pure black-box approach would here mean the testing of different choices of $\varphi(t)$ in the model structure (6). This, however, is not a pleasant situation. The iterative numerical search for the parameters may take quite some time, especially as the search has to be done several times with different initial guesses due to local minima.

To avoid such an *ad hoc* search, it is appealing to use the physical insight from the plant. The semi-physical modeling procedure indicated that three regressors were especially important, namely $y(t-1)$, $u(t-1)$ and $I(t-2)$. Thus, let this knowledge guide us in the choice of regressors

$$\varphi(t) = [y(t-1)\ u(t-1)\ I(t-2)]^T \tag{7}$$

when trying neural network modeling. More precisely, this $\varphi(t)$ is fed into a NARX model [3], *i.e.*, $g$ in (6) is a sigmoidal neural network model and $\boldsymbol{\theta}$ are the parameters (or weights) to be estimated.

The neural network model so obtained will have more parameters than the semi-physical model (5). These additional parameters may cause overfitting, but they also give the model some freedom to adapt to relationships not encountered in the semi-physical model.

Best performance is achieved with a neural net with three hidden units *i.e.*, a model having 16 parameters. The simulation of this model is shown in Figure 4. As can be seen, the fit is better than what is obtained with the semi-physical model, especially at late hours.

## 6. Conclusions

We have seen an example of how physical insight and semi-physical modeling can be successfully combined with black-box neural network modeling. The regressors fed into the neural network could be chosen from this insight, which saved a lot of time trying different possible choices.

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