Libraries and Iterative Methods

This homework explores the use of scientific libraries. In the homework you will build a matrix matrix multiplication routine using the blas3 libraries. You will time the speed of blas code and compare it to your hand written matrix matrix multiply from the previous homework.

In the second part of the homework you will use LAPACK and blas routines to build and solve a large system of equations. You will time the factorization and solution steps for matrix sizes similar to those used in the previous homework.

Finally you will write a Jacobi iterative solver to solve the linear system from part 2. You will compare the time it takes to solve the linear system iteratively against the time for direct solution.

The point of the exercise is to familiarize you with the use of scientific libraries (LAPACK in particular) and for you to investigate the relative merits of iterative vs direct solution.

1. Create a program that uses the level 3 blas routine sgemm to multiply two matrices together. Time this routine and compare the times to the hand coded application you wrote for the previous homework.
    a. Reuse the code you already have.
    b. Perform the exercise for the same matrix sizes you used in the previous homework.
    c. Add rows to your table of previous results showing new times for sgemm.
2. Create a program that solves a linear system of equations Ax=b using LAPACK functions. Use the program to solve two linear systems of the size used in the previous homework.
    a. Build a coefficient matrix A that is diagonally dominant. Use the same matrix sizes from previous homework.
        i. Diagonal dominance means the absolute value of the diagonal element is greater than or equal to the sum of the absolute values of the of diagonal elements in that row. $|A_{i,i}| \geq \sum_{j,i \neq j} |A_{i,j}|$
    b. Create a random solution vector and multiply it by the matrix to create the right hand side vector b. (Use a blas 2 matrix vector multiply routine for this)
    c. Use LAPACK routine sgetrf to factor A into L and U.
    d. Use LAPACK routine sgetrs to solve the linear system using the factorization above.
    e. Time both the factorization and solution parts. Vary the size of the systems as you did in the previous homework matrix matrix multiply experiment.
3. Create a program that solves the same linear systems above by Jacobi's method. This work can be combined with the program of part 2.
    a. Be careful to use the same Coefficient Matrix A and right hand side vector b for this part as for part 2.
    b. Use a vector consisting of all 1s as an initial guess.
    c. Run the iteration until the relative error is less than 10^{-5}. Use the random solution vector created in part 2b above as the exact solution. The relative error is $\frac{\|x-\tilde{x}\|}{\|x\|}$ where $\tilde{x}$ is the approximate solution. This requires an inner product calculation. Again use the blas2 functions for dot product to calculate the error.
    d. Measure the time it takes to converge. (ie for the error to reach the prescribed value)
    e. Compare this time to the time for LAPACK direct solution calculated in part 2 above.

4. Create a program that solves the same linear systems above by the Gauss-Seidel method. This work can be combined with the program of part 2.
    a. Be careful to use the same Coefficient Matrix A and right hand side vector b for this part as for part 2.
    b. Use a vector consisting of all 1s as an initial guess.
    c. Run the iteration until the relative error is less than10^{-5}. Use the random solution vector created in part 2b above as the exact solution. The relative error is $\frac{\|x-\tilde{x}\|}{\|x\|}$ where $\tilde{x}$ is the approximate solution. This requires an inner product calculation. Again use the blas2 functions for dot product to calculate the error.
    d. Measure the time it takes to converge. (ie for the error to reach the prescribed value)
    e. Compare this time to the time for LAPACK direct solution calculated in part 2 above.