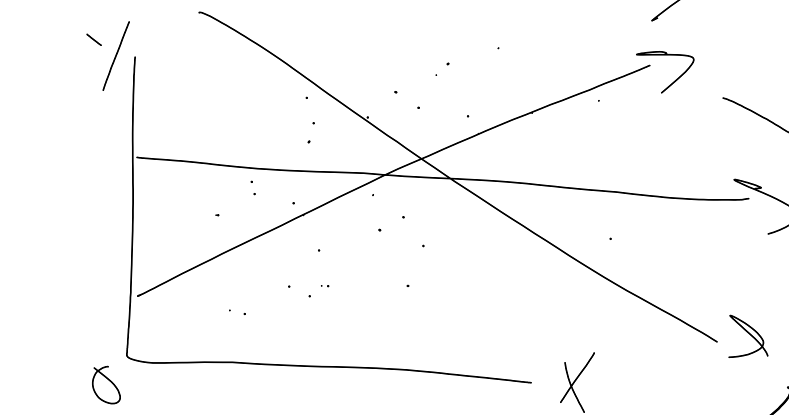
Lecture 2 - Method of Moments Continued

Thursday, January 23, 2020 9:22 AM

Agenda:
TA signups up
Emails vs. discussion board
Stock & Watson book references

SLRM: $Y_i = \beta_0 + \beta_1 X_i + u_i$



the... measure

$E[u] = 0$

Mean ind. sol

$E[u|X] = 0$

Sample analogs

$$\frac{1}{n}\sum_{\forall i} \hat{u}_i = 0$$

$$\frac{1}{n}\sum_{\forall i} X_i \hat{u}_i = 0$$

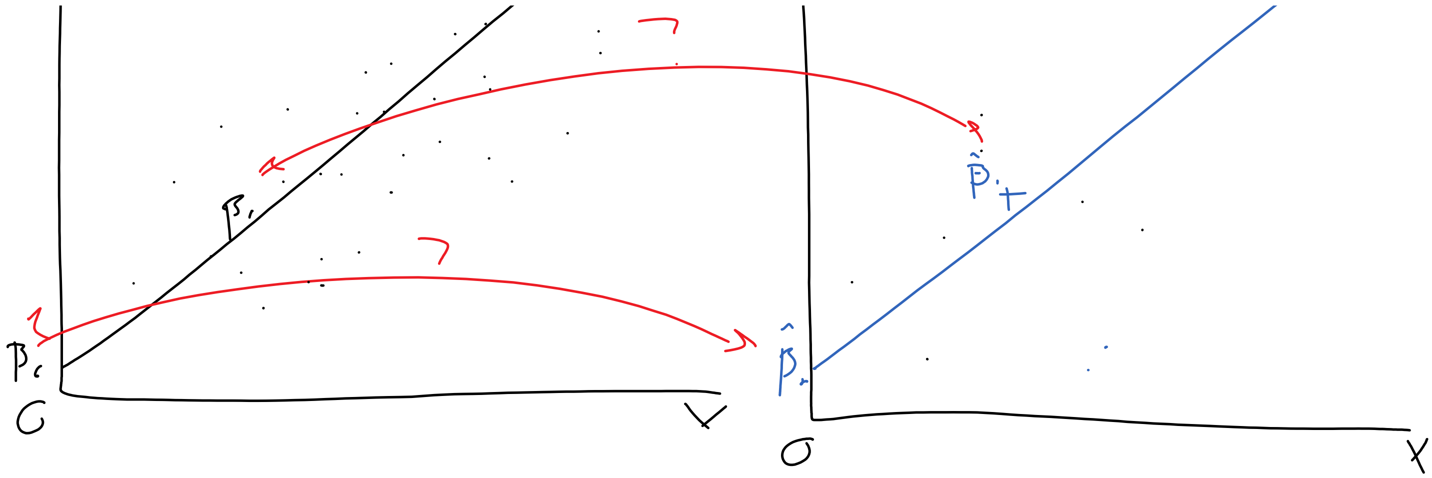
2 eqns
2 unk
$\hat\beta_0, \hat\beta_1$

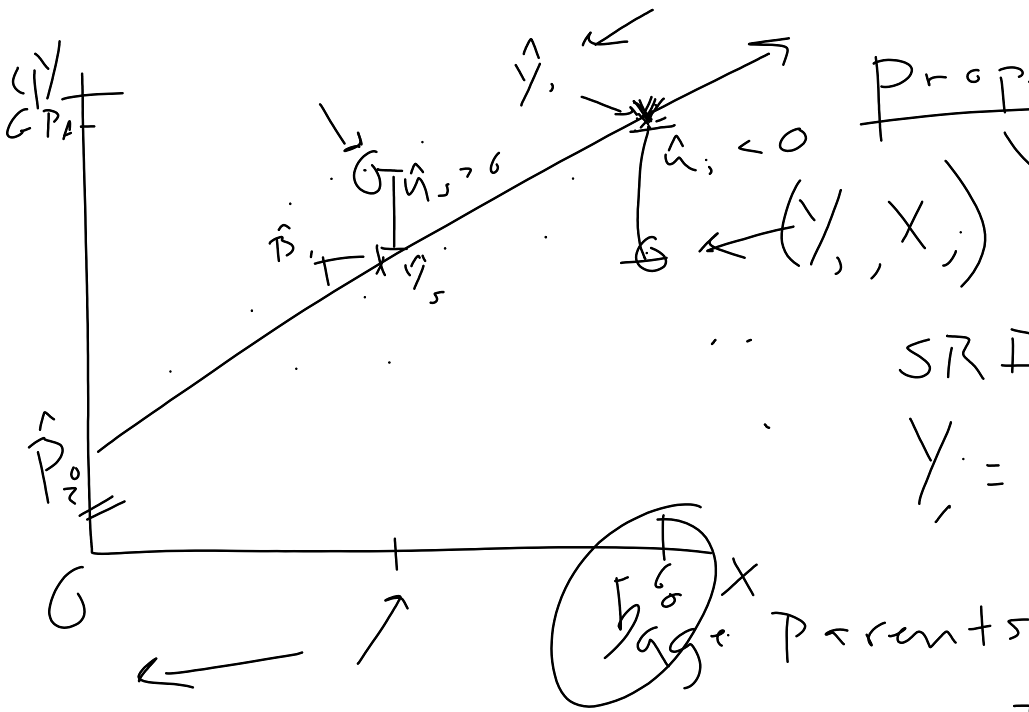
$$\hat\beta_0 = \bar{Y} - \hat\beta_1 \bar{X}$$

$$\hat\beta_1 = \frac{\text{cov}(Y, X)}{\text{var}(X)}$$

P.P

Sample





Properties

pred val

SRF

$$Y_i = \underbrace{\hat{\beta}_0 + \hat{\beta}_1 X_i}_{\hat{Y}_i} + \hat{u}_i$$

$$Var(Y) = \frac{1}{n-1} \sum_{\forall i} (Y_i - \bar{Y})^2$$

$$TSS_Y = \sum_{\forall i} (Y_i - \bar{Y})^2$$

SST_Y

amt of variation in Y

$$ESS = \sum_{\forall i} (\hat{Y}_i - \bar{Y})^2$$

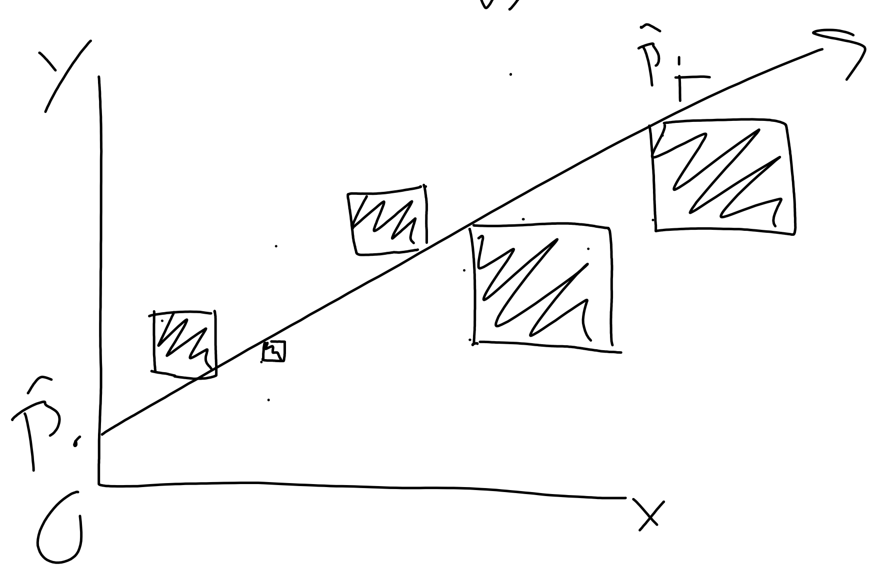
SSE

amt of variation in Y_i explained by X by X

$$RSS = \sum_{\forall i} (Y_i - \hat{Y}_i)^2$$ ← amt var in Y left unexplained by X

SSR

$$= \sum_{\forall i} \hat{u}_i^2$$



$$RSS = \sum_{\forall i} \hat{u}_i^2$$

$$ESS/TSS = R^2 \in [0, 1]$$

"goodness of fit"

→ "share of variation in Y expl by X/model"

$$TSS = ESS + RSS$$

$$(Y) \quad (\hat{Y}) \quad (\hat{u})$$

(Y_i) $(\hat{Y}_i)$ $(\hat{u}_i)$

Would like model that explains the <u>most</u> variation in Y

↳ min. unexplained

$$\min_{\hat\beta_0, \hat\beta_1} \sum_{\forall i} \hat{u}_i^2 \qquad \text{line}/\ Y_i = \hat\beta_0 + \hat\beta_1 X_i + \hat{u}_i$$

$$\hat{u}_i = Y_i - \hat\beta_0 - \hat\beta_1 X_i$$

$$\min_{\hat\beta_0, \hat\beta_1} \sum_{\forall i} (Y_i - \hat\beta_0 - \hat\beta_1 X_i)^2$$

$$\frac{dRSS}{d\beta_0} : \sum_{\forall i} \cancel{2} (Y_i - \hat\beta_0 - \hat\beta_1 X_i) \cancel{-1} = 0$$

$$\sum_{\forall i} Y_i - \sum_{\forall i} \hat\beta_0 - \sum_{\forall i} \hat\beta_1 X_i = 0$$

$$n\bar{Y} - n\hat{\beta}_0 - \hat{\beta}_1 n\bar{X} = \emptyset$$

$$\cancel{n}\bar{Y} - \cancel{n}\hat{\beta}_0 - \hat{\beta}_1 \cancel{n}\bar{X} = \emptyset$$

$$\boxed{\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}}$$

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = \sum_{\forall i} 2(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^{-} X_i = \emptyset$$

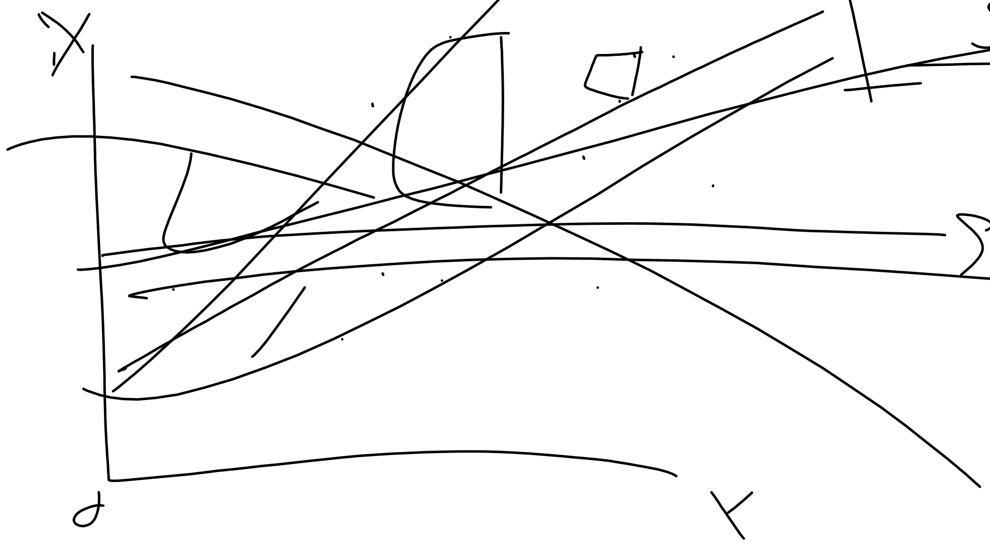
$$\sum_{\forall i} \cancel{2}(Y_i - (\bar{Y} - \hat{\beta}_1 \bar{X}) - \hat{\beta}_1 X_i)\cancel{^{-}} X_i = \emptyset$$

$$\sum_{\forall i} (Y_i - \bar{Y}) X_i + \hat{\beta}_1 (\bar{X} - X_i) X_i = \emptyset$$

$$\sum_{\forall i} (Y_i - \bar{Y}) X_i - \hat{\beta}_1 \sum_{\forall i} (X_i - \bar{X}) X_i = \emptyset$$

$$\hat{\beta}_1 = \frac{\sum_{\forall i} (Y_i - \bar{Y}) X_i}{\sum (X_i - \bar{X}) X_i} = \frac{\operatorname{cov}(Y, X)}{\operatorname{Var}(X)}$$

$$\sum (X_i - \bar{X}) X_i \quad = \mathrm{Var}(X)$$



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