Theory Behind Monte-Carlo Wavefunction Simulation

Jonathan Curtis

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The Monte-Carlo wavefunction approach to problems in quantum optics is outlined well in the paper DOI: 10.1364/JOSAB.10.000524 (Molmer, Castin, Dalibard 1993). The basic idea will be outlined here in the context of a driven/dissipative cavity with a Jaynes-Cummings type non-linearity. For more information on this topic, see for example Carmichael (2015) DOI: 10.1103/PhysRevX.5.031028. We begin with a Hamiltonian of the form

$$H_0 = \omega_0(a^\dagger a + \sigma^\dagger \sigma) + g(\sigma^\dagger a + a^\dagger \sigma)$$

where the

$$\sigma = |0\rangle\langle 1|$$

is the atomic lowering operator and

$$a = \sum_{n=1}^{\infty} \sqrt{n}|n-1\rangle\langle n|$$

is the photonic lowering operator. This Hamiltonian is diagonalized by a change of basis to

$$|n,\pm\rangle = \frac{1}{\sqrt{2}}\left(|n\rangle_{\text{cavity}} \otimes |0\rangle_{\text{atom}} \pm |n-1\rangle_{\text{cavity}} \otimes |1\rangle_{\text{atom}}\right)$$

We consider this system in the presence of a coherent drive field and incoherent dissipation such that the drive Hamiltonian is given by

$$H_D = \mathcal{E}\left(a e^{i\omega_d t} + a^\dagger e^{-i\omega t}\right)$$

where ω_d is the drive frequency and $\mathcal{E}$ is the drive strength. Note that this is not diagonal in terms of the polariton basis. We also have dissipation introduced by the Markovian jump operators

$$L = \sqrt{2\kappa} a$$

where κ is the cavity decay rate. The Monte-Carlo wavefunction approach calls for two ingredients. The first is a non-Hermitian Hamiltonian we can evolve the state by. This is given by

$$H = H_0 + H_D - \frac{i}{2} L^\dagger L$$

which means

$$H = (\delta - i\kappa)a^\dagger a + \delta \sigma^\dagger \sigma + g(a^\dagger \sigma + \sigma^\dagger a)$$

where $\delta = \omega_c - \omega_d$ is the drive detuning from the cavity and we have already passed to the frame co-rotating with the drive field. Let dt be a small time step we will evolve our state over. Then if our wavefunction is given by $|\psi(t)\rangle$ then at time $t+dt$ the wavefunction is given by

$$|\psi(t+dt)\rangle = (\mathbb{1} - iH dt)\,|\psi(t)\rangle$$

This is no longer a normalized state. Instead, this state has norm (to $O(dt)$),

$$\langle \psi(t+dt)|\psi(t+dt)\rangle = 1 - i dt\langle\psi(t)|(H - H^\dagger)|\psi(t)\rangle \equiv 1 - dp$$