$x - z$

$$S = \int_1^2 ds$$

$$ds = \sqrt{dx^2 + dz^2} = \sqrt{1 + z'^2}\, dx$$

$$S = \int_{x_1}^{x_2} \sqrt{1 + z'^2}\, dx$$

$$f(z, z', x) = (1 + y'^2)^{1/2}$$

$$\frac{\partial f}{\partial y} = 0 \quad \& \quad \frac{\partial f}{\partial y'} = \frac{z'}{(1 + z')^{1/2}}$$

$$\frac{d}{dx}\frac{\partial f}{\partial y'} = 0, \quad \frac{\partial f}{\partial y} = \text{const.}$$

$$\frac{z'}{(1+z'^2)^{1/2}} = c$$

$$z'^2 = c^2(1+z'^2)$$

$$z'^2 = c^2 + c^2 z'^2$$

$$z'^2(1-c^2) = c^2$$

$$z'^2 = \frac{c^2}{1-c^2} = \text{constant}$$

$$\int z'_{dx} = \int \sqrt{\text{constant}}\, dx$$

$$\therefore z = \sqrt{\text{const}}\, x + b$$

$$n(z) = n_0(1+\beta z), \quad z = z(x)$$

$$\int_1^2 n(z)\, ds$$

$$ds = \sqrt{1+z'^2}\, dx \qquad z' = \frac{dz}{dx}$$

$$F(z, z', x) = n(z)\sqrt{1+z'^2}$$

$$= n_0(1+\beta z)\sqrt{1+z'^2}$$

$$\frac{\partial F}{\partial z} = n_0 \beta \sqrt{1+z'^2}$$

$$\frac{\partial F}{\partial z'} = n_0(1+\beta z)\frac{z'}{\sqrt{1+z'^2}}$$

$$n_0 \beta \sqrt{1+z'^2} - \frac{d}{dx}\left[n_0 (1+\beta z) \frac{z'}{\sqrt{1+z'^2}} \right]$$

$$n_0 \beta \sqrt{1+z'^2} - \frac{n_0 \beta z'^2}{\sqrt{1+z'^2}} = 0$$