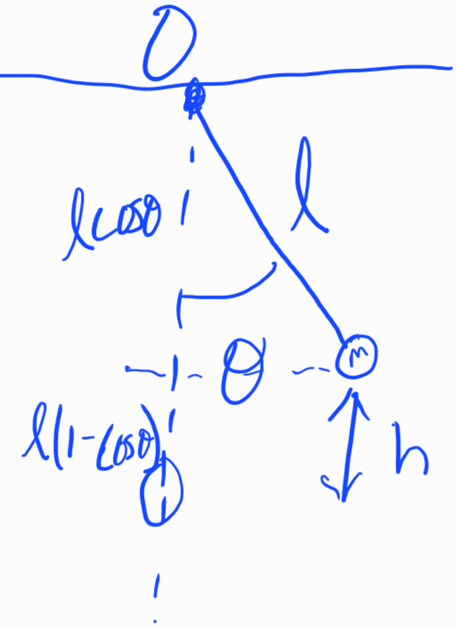
1)



$$\mathcal{L} = \mathcal{L}(\theta, \dot\theta, t)$$

$$T = \frac{1}{2} m l^2 \dot\theta^2$$

$$U = mgh = mgl(1-\cos\theta)$$

$$\mathcal{L}(\theta, \dot\theta) = \frac{1}{2} m l^2 \dot\theta^2 - mgl(1-\cos\theta)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta}$$

$$F = -mgl\sin\theta$$

$$p = ml^2\dot\theta$$

$$-\cancel{m}g\cancel{l}\sin\theta = \frac{d}{dt}\left[ml^2\dot\theta\right] = \cancel{m}l^{\cancel{2}}\ddot\theta$$

$$-\frac{g}{l}\sin\theta = \ddot\theta$$

$$\theta \approx \sin\theta$$

$$\ddot\theta = -\frac{g}{l}\theta = -\omega^2\theta$$

$$\omega = \sqrt{g/l}$$

2)

$$\mathcal{L} = \mathcal{L}(x, \dot{x}, t) = T - U$$

$$T = \frac{1}{2} m \dot{x}^2 \,, \quad U = \frac{1}{2} k x^2$$

$$\mathcal{L}(x, \dot{x}, t) = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}}$$

$$F = \frac{\partial \mathcal{L}}{\partial x} = -kx$$

$$P = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m \dot{x}$$

$$-kx = m \ddot{x} \,, \quad \ddot{x} = -\frac{k}{m} x$$