

1) Find $V(t)$ & $X(t)$

know: $m, F_f = -\alpha V$

$$m \ddot{x} = F_f = -\alpha V$$

$$\frac{dV}{dt} = \underbrace{\frac{-\alpha}{m}}_{\text{const.} \equiv \frac{1}{\tau}} V$$

$$\int \frac{dV}{V} = \int -\frac{1}{\tau} dt$$

$$\int \frac{1}{V} dV = -\frac{1}{\tau} \int dt$$

$$e^{(\ln V)} = e^{(-\frac{1}{\tau} t + c)}$$

$$e^c = V_0 \text{ at } t=0$$

$$V = e^c e^{-t/\tau}$$

$$V(t) = V_0 e^{-t/\tau}$$

$$X = \int V(t) = \int_0^t v_0 e^{-t'/\tau} dt'$$

$$X(t) = v_0 \tau (1 - e^{-t/\tau}) + C$$

$$C = X_0 \text{ @ } t=0$$

$$\therefore X(t) = X_0 + v_0 \tau (1 - e^{-t/\tau})$$

2) Find $v(t)$ & $x(t)$

$$F = F_0 \cos(\omega t + \phi)$$

$$m \frac{dv}{dt} = F_0 \cos(\omega t + \phi)$$

$$\int \frac{dv}{dt} = \int \frac{F_0}{m} \cos(\omega t + \phi)$$

$$= \frac{F_0}{m} \int \cos(\omega t + \phi) =$$

$$= \frac{F_0}{m} \underbrace{\frac{1}{\omega}}_{V_c} [\sin(\omega t + \phi) - \sin \phi] + C$$

$C = V_0$

$$\therefore v(t) = V_0 + V_c [\sin(\omega t + \phi) - \sin \phi]$$

$$X(t) = \int V(t)$$

$$= \int V_c dt + \int V_c [\sin(\omega t + \phi) - \sin \phi]$$

$$= X_0 + V_0 t - V_0 \sin \phi t + \frac{V_c}{\omega} [\cos \phi - \cos(\omega t + \phi)]$$

$$\therefore X(t) = X_0 + \frac{V_c}{\omega} \left[\left\{ \frac{V_0}{V_c} - \sin \phi \right\} \omega t + \cos \phi - \cos(\omega t + \phi) \right]$$