1)

a) $m\ddot{x} = -kx$

b) $E = \frac{1}{2}kA^2$

c) $U = \frac{1}{2}kx^2$

d) $E = T + U$

$$\frac{1}{2}kA^2 = T + \frac{1}{2}kx^2$$

$$T = \frac{1}{2}k(A^2 - x^2)$$

e) $\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$

$$2x^2 = A^2 \rightarrow x = \frac{A}{\sqrt{2}}$$

2.

a) $m\ddot{X} + b\dot{X} + KX = F(t)$

$\ddot{X} + \frac{b}{m}\dot{X} + \frac{K}{m}X = \frac{F(t)}{m}$

$\ddot{X} + 2\beta\dot{X} + \omega_o^2 X = f(t)$

$2\beta = \frac{b}{m}, \quad \omega_o = \sqrt{\frac{K}{m}}, \quad f(t) = \frac{F(t)}{m}$

b) $X(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta)$

c) $\omega_1 = \sqrt{\omega_o^2 - \beta^2}$

d) $\omega_1 = \sqrt{\frac{K}{m} - \left(\frac{b}{2m}\right)^2}$

$= \sqrt{\frac{6}{1} - \left(\frac{2\sqrt{2}}{2}\right)^2} = \sqrt{6 - 4} = \sqrt{2}$