1) $F(x) = -kx + \frac{kx^3}{A^2}$, T_0, $U_0 = 0$

a.) $F(x) = -\nabla U(x)$

$$U(x) = -\int_0^x F(x')\,dx' = \left[-\frac{1}{2}x' + \frac{1}{4}\frac{x'^4}{A^2}\right]_0^x$$

$$U(x) = \frac{1}{2}kx^2 - \frac{1}{4}\frac{kx^4}{A^2}$$

b.) $\Delta T + \Delta U = \Delta E = 0$, $\Delta T = -\Delta U$

$$\Delta T = T - T_0 = -(U - \cancel{U_0}^{\,0})$$

$$T - T_0 = -U = -\frac{1}{2}kx^2 + \frac{1}{4}\frac{kx^4}{A^2}$$

$$T = -\frac{1}{2}kx^2 + \frac{1}{4}\frac{kx^4}{A^2} + T_0$$

c.) $E = T + U$

$$= \cancel{\frac{1}{2}kx^2} - \cancel{\frac{1}{4}\frac{kx^4}{A^2}} \cancel{-\frac{1}{2}kx^2} + \cancel{\frac{1}{4}\frac{kx^4}{A^2}} + T_0$$

$$E = T_0$$

2.) $U(x) = ax^2 - bx^3$

$$F(x) = -\nabla U(x)$$

$$= -\frac{\partial}{\partial x}(ax^2 - bx^3)$$

$$F(x) = -2ax + 3bx^2$$