1)

a) $T = \frac{1}{2} m l^2 \dot{\theta}^2 \qquad U = mgl(1-\cos\theta)$

$$P_\theta = \frac{\partial T}{\partial \dot{\theta}} = ml^2\dot{\theta}$$

b)

$$\dot{\theta} = \frac{P_\theta}{ml^2} \quad , T = \frac{1}{2} \cancel{m} \cancel{l^4} \frac{P_\theta^2}{m^{\cancel{2}} \cancel{l^4}} = \frac{P_\theta^2}{2ml^2}$$

c) $\mathcal{H} = \frac{P_\theta^2}{2m} + mgl(1-\cos\theta)$

$$\dot{\theta} = \frac{\partial \mathcal{H}}{\partial P_\theta} = \frac{P_\theta}{ml^2} \qquad \dot{P}_\theta = \frac{-\partial \mathcal{H}}{\partial \theta} = -mgl\sin\theta$$

d) $\ddot{\theta} = \frac{\dot{P}_\theta}{ml^2} = -\frac{\cancel{m}g\cancel{l}}{\cancel{m}l^{\cancel{2}}}\sin\theta = -\frac{g}{l}\sin\theta$

$$\sin\theta \simeq \theta$$

$$\ddot{\theta} = -\frac{g}{l}\theta = -\omega^2\theta \quad , \omega = \sqrt{\frac{g}{l}}$$

2

a) $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$, $U = \frac{1}{2}k(x^2+y^2) + \lambda xy$.

b)

$P_x = \frac{\partial T}{\partial \dot{x}} = m\dot{x}$ $\quad \dot{x} = \frac{P_x}{m}$

$P_y = \frac{\partial T}{\partial \dot{y}} = m\dot{y}$ $\quad \dot{y} = \frac{P_y}{m}$

$T = \frac{1}{2}m\left(\frac{P_x^2}{m^2} + \frac{P_y^2}{m^2}\right)$

$= \frac{P_x^2 + P_y^2}{2m}$

c) $H = \frac{P_x^2 + P_y^2}{2m} + \frac{1}{2}k(x^2+y^2) + \lambda xy$

d)

$\dot{x} = \frac{\partial H}{\partial P_x} = \frac{P_x}{m}$ $\quad \dot{y} = \frac{\partial H}{\partial P_y} = \frac{P_y}{m}$

$\dot{P}_x = -\frac{\partial H}{\partial x} = -kx - \lambda y$ $\quad \dot{P}_y = -ky - \lambda x$

$\ddot{x} = \frac{\dot{P}_x}{m} = -\frac{k}{m}x - \frac{\lambda}{m}y$ $\quad \ddot{y} = -\frac{k}{m}y - \frac{\lambda}{m}x$