Appendix A

Perturbative QCD photons at low p_T

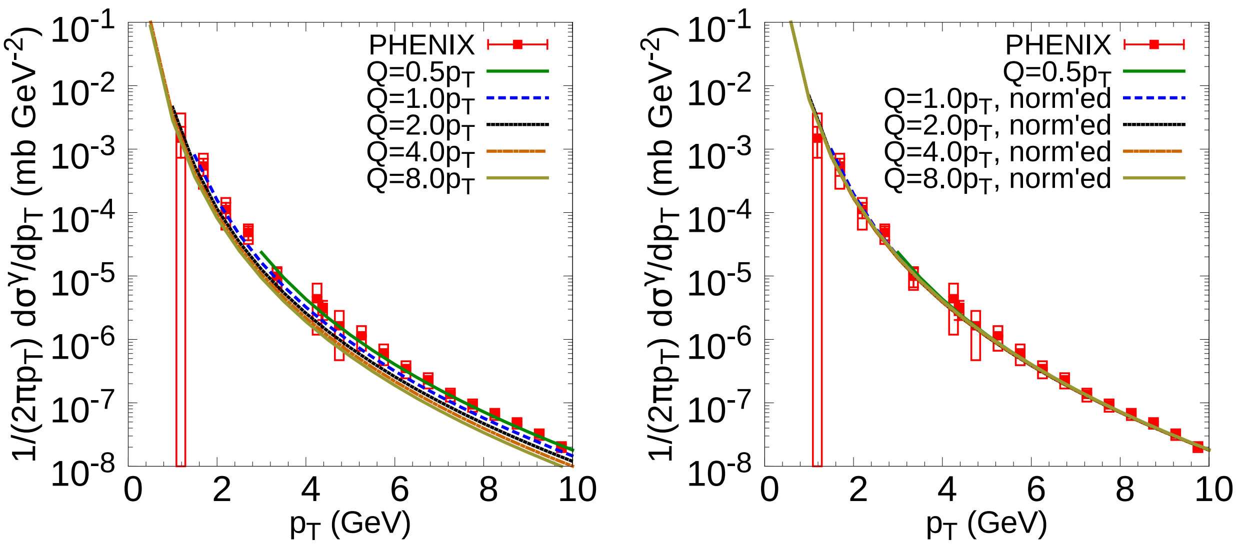
Pertubative QCD is based on the idea that a large momentum exchange occurs in a hadronic collisions, allowing for a part of the cross-section to be computed perturbatively. Photons produced at large transverse momentum can be described by perturbative QCD because the magnitude of their transverse momentum guarantees a large momentum exchange.

It is understood that perturbative QCD eventually breaks down at low transverse momentum, although the exact value of p_T at which this happens is not clear. When global fits of parton distribution functions and fragmentation functions to data are made, this issue has to be considered: data points with scale Q smaller than a chosen minimum scale Q_0 are not used in the fits. The value of Q_0 is typically $1 - 1.5$ GeV. This value provides an estimate for the lowest energy scale at which perturbative QCD should be considered reliable.

For photon production in perturbative QCD, the scales are given by an energy scale of the order of the photon transverse momentum. It is not compulsory to set the scales equal to p_T^γ; there could be a proportionality constant, for example. Even a scale that is related to p_T^γ would do. Such a scale exists for fragmentation photons, where a large transverse momentum parton is produced first, and then produces a photon through fragmentation. In this case, using the energy of the parton as scale would also be justified.

In this thesis, the factorisation, renormalisation and fragmentation scales are all set to $Q = p_T^\gamma/2$. The reason behind this choice can be seen on Figure A.1a, where the perturbative QCD calculation of prompt photons described in Section 5.2.1 is evaluated at different scales $Q = Np_T^\gamma$ and compared to direct photon measurements in proton-proton collisions at $\sqrt{s_{NN}} = 200$ GeV from RHIC [20]. Results are plotted for $Q = Np_T^\gamma$; $N = 1/2$ to $N = 8$, with the calculations going down to $p_T^\gamma = 1.5/N$ GeV due Q_0 being approximately equal to 1.5 GeV in the calculation.

It is apparent from Figure A.1a that a small proportionality constant such as $Q = p_T^\gamma/2$ provides a better description of the available measurements. The choice $Q = p_T^\gamma/2$ in



(a) (b)

Figure A.1: Direct photon spectrum measured in $\sqrt{s_{NN}} = 200$ GeV proton-proton collisions at RHIC compared with (a) perturbative QCD calculations made with different scales Q (b) normalised perturbative QCD calculations (Equation A.1)

Section 5.2.1 was made based on this observation.

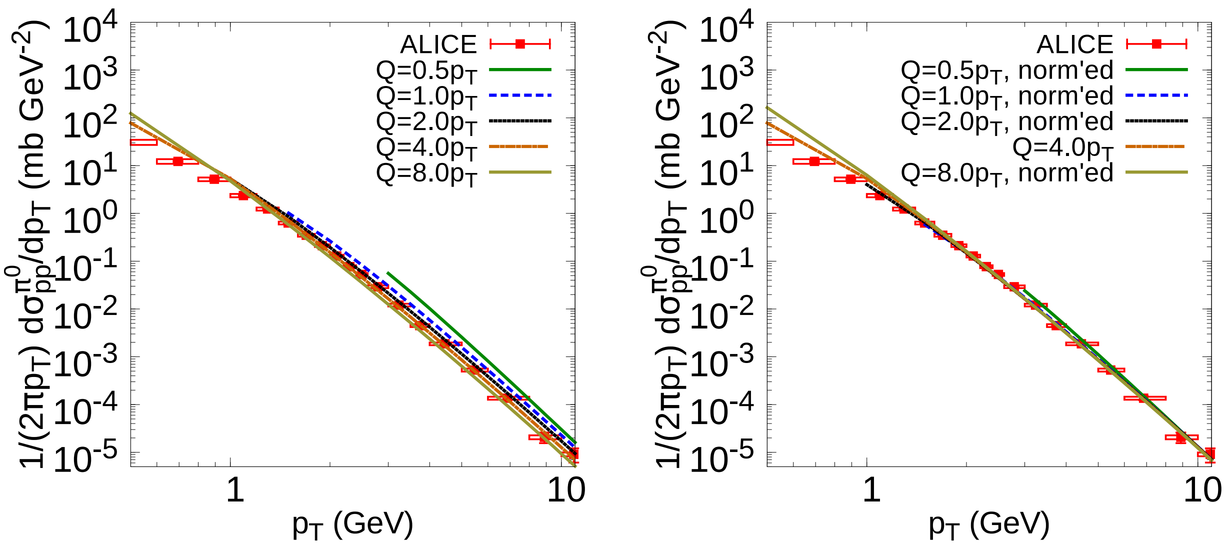
The lowest p_T^γ available for $Q = p_T^\gamma/2$ is $p_T^\gamma = 3$ GeV, assuming $Q_0 = 1.5$ GeV. Computing photons at a lower p_T^γ would require a smaller Q_0, which would imply using the perturbative scale evolution below Q_0. A different approach is used here, based on the scale dependence of the perturbative QCD calculation.

Figure A.1a hints that the main effect of changing the value of N in $Q = Np_T^\gamma$ is a change of normalization of the photon spectrum, although it is difficult to see if the calculations have a different p_T^γ-dependence. This can be verified by rescaling all the calculations by a constant, so that they have the same normalisation. The following formula is used for the normalisation

$$\left.\frac{1}{2\pi p_T}\frac{d\sigma_{pp}^{renorm}}{dp_T}\right|_{Q=Np_T^\gamma} = 1.25N^{0.26}\left[\frac{1}{2\pi p_T}\frac{d\sigma_{pp}}{dp_T}\right] \tag{A.1}$$

so that $Q = p_T^\gamma/2$ is not rescaled, but perturbative QCD calculations made with higher values of Q are normalised up. The result is shown on Figure A.1b. All calculations fall on top of each other, supporting the claim that a change in scale results essentially in a change of normalisation, and not a change in momentum dependence.

The above observation is not trivial. It could very well have been that normalising all the calculations to have the same magnitude at high p_T would have resulted in a different



(a)

(b)

Figure A.2: Same as Figure A.1 for pions measured in $\sqrt{s_{NN}} = 2.76$ TeV proton-proton collisions at the LHC

low p_T-dependence. Instead, all calculations lined up. This finding can be used to compute prompt photons to low p_T by simply using a large proportionality constant between Q and p_T^γ and modifying the overall normalisation so that high p_T data are fitted, which in this case corresponds to the normalisation of $Q \approx p_T^\gamma/2$.

Figure A.1b indicates that the perturbative QCD calculation of prompt photons is in good agreement with direct photon measurements at low p_T^γ, although it overestimates slightly the very lowest point around 1 GeV. This good agreement with measurements at low p_T^γ supports the idea that perturbative QCD calculations can be relied on even at fairly low transverse momentum. To provide further support for this idea, a similar test is made at the LHC.

While there are no low p_T direct photon measurements in proton-proton collisions at the LHC, there are measurements for e.g. neutral pions. It can thus be verified if the same scaling behaviour is observed for pions at the LHC. The perturbative QCD calculation of π^0 for different scales is shown on Figure A.2a, while the normalised results are shown on Figure A.2b. This time, all results were normalised so as to have the same high p_T magnitude as $Q = 4p_T^{\pi^0}$. Note that both axis are logarithmic, unlike the previous figures.

Once again, the normalised calculations line up well, although arguably not as well as for photons. Nevertheless, the normalised perturbative calculations are in good agreement with π^0 measurements down to $p_T^{\pi^0} \sim 1-2$ GeV.

Two conclusions can be drawn from the above. The first one is that any limitation imposed by the presence of an effective lower scale Q_0 in parton distribution functions and fragmentation functions can be sidestepped by using larger proportionality constant between the factorisation/fragmentation scales and the transverse momentum.

The second conclusion is that perturbative QCD appears to provide a good description of the momentum dependence of hard photons and pions down to $p_T \sim 1 - 1.5$ GeV. On the other hand, the present results suggest that below this momentum, the perturbative QCD calculation overestimates the production of particles. With this warning in mind, it can be said that perturbative QCD can provide an appropriate estimate of prompt photons at low p_T.

It is important to note that the value of for Q should not, in theory, be considered a parameter of the model to be adjusted to provide a better description of measurements. Calculations in perturbative QCD are actual predictions, and their scale dependence are uncertainties of the calculations. The higher the order of the calculation in α_s, the smaller the scale uncertainty.

The pragmatic point of view adopted in this thesis is that prompt photons are needed at low p_T to understand the direct photon excess observed in heavy ion collisions, and the best estimate of prompt photons should be used. If a given choice of scale provides a better description of the available direct photon data in proton-proton collisions, than it is reasonable to assume that using this scale will also provide the best description of low p_T prompt photons in heavy ion collisions.