Porto Oscillation Code (POSC)

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Abstract The *Porto Oscillation Code* (POSC) has been developed in 1995 and improved over the years, with the main goal of calculating linear adiabatic oscillations for models of solar-type stars. It has also been used to estimate the frequencies and eigenfunctions of stars from the pre-main sequence up to the sub-giant phase, having a mass between 0.8 and 4 solar masses.

The code solves the linearised perturbation equations of adiabatic pulsations for an equilibrium model using a second order numerical integration method. The possibility of using Richardson extrapolation is implemented. Several options for the surface boundary condition can be used. In this work we briefly review the key ingredients of the calculations, namely the equations, the numerical scheme and the output.

1 Introduction

The *Porto Oscillation Code* (POSC) was initially developed in 1995 to obtain the frequencies of solar models and envelopes. The first description of the code has been given in Monteiro (1996).

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The objective of this paper is to present a summary on how POSC calculates the frequencies of oscillations for stellar models. The paper starts with the basic linear equations describing the oscillations and how these are formulated to be solved numerically. The boundary conditions used and their implementation are also discussed as well as the accuracy of the calculations. We end by listing some of the output values provided by the code and some of the applications where the results of the code have been used.

2 Basic equations for linear perturbations

Our objective here is to review the necessary equations for *non-radial adiabatic oscillations* of spherically symmetric non-rotating stars. By following the work by Unno et al. (1989) it is possible to start from the hydrodynamic equations (continuity, Poisson and conservation of momentum equations), in order to obtain a set of equations describing the radial dependence of the amplitude functions for small perturbations. These perturbations correspond to; P for pressure, Φ for gravitational potential, while $\vec{\xi}$ is the displacement. The solutions are written as

$$P(t,r,\theta,\phi) = P_0(r) + \tilde{P}(r)\ Y_l^m(\theta,\phi)\ e^{i\omega t},$$
$$\Phi(t,r,\theta,\phi) = \Phi_0(r) + \tilde{\Phi}(r)\ Y_l^m(\theta,\phi)\ e^{i\omega t}, \quad (1)$$
$$\vec{\xi}(t,r,\theta,\phi) = \left[\xi_r(r),\ \xi_h(r)\ \frac{\partial Y_l^m}{\partial \theta},\ \frac{\xi_h(r)}{\sin\theta}\ \frac{\partial Y_l^m}{\partial \phi}\right] e^{i\omega t},$$

where the equilibrium configuration of the stars is described by the functions; ρ_0 (for density), P_0 and Φ_0. Here t is time, ω the frequency for the oscilating solutions, (θ, ϕ) the horizontal variables while r is radial distance and $Y_l^m(\theta,\phi)$

the spherical harmonics characterized by the integer numbers l (mode degree) and m (azimuthal order with $m = -l, .., 0, .., l$).

By considering an equation for adiabatic perturbations and after eliminating the dependence on the horizontal coordinates and time, the equations describing the radial amplitude of the small perturbations are obtained in the following form;

$$\left(1 - \frac{S_l^2}{\omega^2}\right)\frac{\tilde{P}}{\rho_0} - \frac{1}{r^2}\left(g_0 - c_0^2\frac{d}{dr}\right)(r^2\xi_r) + \frac{S_l^2}{\omega^2}\tilde{\Phi} = 0,$$

$$\frac{1}{\rho_0}\left(\frac{g_0}{c_0^2} + \frac{d}{dr}\right)\tilde{P} - (\omega^2 - N_0^2)\,\xi_r - \frac{d\tilde{\Phi}}{dr} = 0\,, \quad (2)$$

$$\tilde{P} + \frac{\rho_0 c_0^2 N_0^2}{g_0}\xi_r - \frac{S_l^2}{4\pi G}\tilde{\Phi} + \frac{c_0^2}{4\pi Gr^2}\frac{d}{dr}\left(r^2\frac{d\tilde{\Phi}}{dr}\right) = 0.$$

The equilibrium structure in these equations is also characterized by quantities as gravity g_0 and sound speed c_0;

$$g_0 = -\frac{d\Phi_0}{dr} = -\frac{1}{\rho_0}\frac{dP_0}{dr}\,,$$

$$c_0^2 = \frac{\Gamma_{1,0}\,P_0}{\rho_0} \quad \text{with} \quad \Gamma_1 \equiv \left(\frac{\partial\log P}{\partial\log\rho}\right)_S\,, \quad (3)$$

where the derivative has been calculated at fixed entropy S. There are also two characteristic frequencies; the *Lamb frequency* S_l and the *buoyancy frequency* N_0 (also known as the Brunt-Väissälä frequency), corresponding to

$$S_l^2 = l(l+1)\frac{c_0^2}{r^2}\,,$$

$$N_0^2 = g_0\left(\frac{1}{\Gamma_{1,0}}\frac{d\log P_0}{dr} - \frac{d\log\rho_0}{dr}\right) \quad (4)$$

$$= -g_0\left(\frac{g_0\rho_0}{\Gamma_{1,0}\,P_0} + \frac{d\log\rho_0}{dr}\right).$$

If we consider the following dimensionless variables

$$y_1 = \frac{\xi_r}{r}\,,$$

$$y_2 = \frac{\omega^2}{g}\xi_h = \frac{1}{rg}\left(\frac{\tilde{P}}{\rho} - \tilde{\Phi}\right)\,,$$

$$y_3 = \frac{\tilde{\Phi}}{rg}\,, \quad (5)$$

$$y_4 = \frac{1}{g}\frac{d\tilde{\Phi}}{dr}\,,$$

the equations can be written as

$$r\frac{dy_1}{dr} = \left(\frac{rg_0}{c_0^2} - 3\right)y_1 + \frac{rg_0}{c_0^2}\left(\frac{S_l^2}{\omega^2} - 1\right)y_2 - \frac{rg_0}{c_0^2}y_3,$$

$$r\frac{dy_2}{dr} = -\frac{r}{g_0}(N_0^2 - \omega^2)y_1 - \left(\frac{rg_0}{c_0^2} + \frac{d\log\rho_0}{d\log r}\right.$$

$$\left. + 4\pi G\frac{r\rho_0}{g_0} - 1\right)y_2 - \left(\frac{rg_0}{c_0^2} + \frac{d\log\rho_0}{d\log r}\right)y_3,$$

$$\quad (6)$$

$$r\frac{dy_3}{dr} = \left(1 - \frac{4\pi Gr\rho_0}{g_0}\right)y_3 + y_4,$$

$$r\frac{dy_4}{dr} = -\frac{4\pi Gr^2\rho_0}{c_0^2}\frac{c_0^2 N_0^2}{g_0^2}y_1 - \frac{4\pi Gr^2\rho_0}{c_0^2}y_2$$

$$+ \left[l(l+1) - \frac{4\pi Gr^2\rho_0}{c_0^2}\right]y_3 - \frac{4\pi Gr^2\rho_0}{c_0^2}\frac{c_0^2}{rg_0}y_4.$$

These form the set of equations we need to solve to obtain the radial behaviour of linear adiabatic oscillations of spherically symmetric stars.

3 The equilibrium model

In order to describe the reference/equilibrium model we consider the following dimensionless functions of the equilibrium structure (as defined by Christensen-Dalsgaard 2008b)

$$x \equiv \frac{r}{R},$$

$$a_1 \equiv \frac{m_{r,0}}{r^3}\frac{R^3}{M},$$

$$a_2 \equiv -\frac{1}{\Gamma_{1,0}}\frac{d\log P_0}{d\log r} = \frac{rg_0}{c_0^2},$$

$$a_3 \equiv \Gamma_{1,0}, \quad (7)$$

$$a_4 \equiv \frac{1}{\Gamma_{1,0}}\frac{d\log P_0}{d\log r} - \frac{d\log\rho_0}{d\log r} = \frac{r}{g_0}N_0^2,$$

$$a_5 \equiv \frac{4\pi r^3\rho_0}{m_{r,0}},$$

where M and R are respectively the total mass and radius of the star, while m_r is the mass within a sphere of radius r. These 5 functions are the result of an evolution code, being necessary as the input of the oscillation code.

The four first order differential equations for small amplitudes can now be written simply as

$$x\frac{dy_1}{dx} = (a_2 - 3)y_1 + \left[\frac{l(l+1)}{\sigma^2}a_1 - a_2\right]y_2 + a_2 y_3,$$

$$x\frac{dy_2}{dx} = \left(\frac{\sigma^2}{a_1} - a_4\right)y_1 + (1 + a_4 - a_5)y_2 - a_4 y_3,$$

$$x\frac{dy_3}{dx} = (1 - a_5)y_3 + y_4, \quad (8)$$

$$x\frac{dy_4}{dx} = a_4 a_5 y_1 + a_2 a_5 y_2 + [l(l+1) - a_2 a_5] y_3$$
$$- a_5 y_4,$$

where we have introduced the reduced frequency

$$\sigma^2 = \frac{R^3}{GM} \omega^2. \tag{9}$$

We may write these equations in a vectorial form by defining the matrix, with $L^2 = l(l+1)$,

$$\mathcal{A} = \begin{bmatrix} a_2 - 3 & \frac{L^2}{\sigma^2} a_1 - a_2 & a_2 & 0 \\ \frac{\sigma^2}{a_1} - a_4 & a_4 - a_5 + 1 & -a_4 & 0 \\ 0 & 0 & 1 - a_5 & 1 \\ a_4 a_5 & a_2 a_5 & L^2 - a_2 a_5 & -a_5 \end{bmatrix}. \tag{10}$$

The system of differential equations is then simply written as

$$x\frac{d\vec{y}}{dx} = \mathcal{A} \cdot \vec{y}, \tag{11}$$

where the vector $\vec{y}$ has the components (y_1, y_2, y_3, y_4).

4 Boundary conditions

To complete the required equations it is also necessary to define four boundary conditions. The solution is to be found by integrating the equations between the centre of the star ($r = 0$ or $x = 0$) and the top of the atmosphere ($r \geq R$ or $x \geq 1$). So, in fact we shall be establishing two boundary conditions at $x = 0$ and other two at the surface. The result is an eigenvalue problem with solutions existing for discrete values of σ. These are the eigenvalues associated to the corresponding eigenfunctions, that must satisfy the boundary conditions.

4.1 At the centre

Since we have four dependent variables, the interior boundary conditions correspond to fix the values for two of the dependent variables. The other two are then related to these.

The boundary conditions have to guarantee that the solutions are regular in the singular point, $x = 0$, of the differential equations. So, we start by determining the limiting behaviour of the a_i's when $x \to 0$. For $x \equiv r/R \ll 1$ we can write (the subscript "c" stands for the value at $x = 0$);

$$\rho \sim \rho_c, \quad m_r \sim \frac{4\pi R^3}{3} \rho_c x^3, \quad \text{and} \quad P \sim P_c. \tag{12}$$

These expressions determine the behaviour of the a_i's near the centre as follows from the definitions (7);

$$a_1 \sim \frac{4\pi}{3} \bar{\rho}_c, \quad a_2 \sim 0, \quad a_3 \sim \Gamma_{1c}, \quad a_4 \sim 0, \quad a_5 \sim 3, \tag{13}$$

where $\bar{\rho}_c = R^3 \rho_c / M$.

If we now replace these in the definition (10) of the matrix $\mathcal{A}$, the problem is reduced to a simple system of differential equations with constant coefficients. This is,

$$x\frac{d\vec{y}}{dx} \simeq \mathcal{A}_c \cdot \vec{y}, \tag{14}$$

with

$$\mathcal{A}_c = \begin{bmatrix} -3 & \frac{4\pi \bar{\rho}_c}{3} \frac{L^2}{\sigma^2} & 0 & 0 \\ \frac{3}{4\pi \bar{\rho}_c} \sigma^2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & L^2 & -3 \end{bmatrix}. \tag{15}$$

This has a general solution (non-zero) given by

$$y_j = x^{l-2} \sum_{i=0}^{\infty} \mathcal{Y}_{ij} x^{2i}, \quad j = 1, 2, 3, 4, \tag{16}$$

with

$$\mathcal{Y}_{01} = \frac{4\pi \bar{\rho}_c}{3} \frac{l}{\sigma^2} \mathcal{Y}_{02} \quad \text{and} \quad \mathcal{Y}_{04} = l \mathcal{Y}_{03}. \tag{17}$$

These are the two boundary conditions at the centre: from the values of $\mathcal{Y}_{02}$ and $\mathcal{Y}_{03}$ it is possible to determine $\mathcal{Y}_{01}$ and $\mathcal{Y}_{04}$.

4.2 At the atmosphere

Two more boundary conditions need to be imposed at the top of the atmosphere. In a similar fashion to what has been done for the centre, we now need to established what is the limiting behaviour for the a's (the subscript "S" represents in the following the value at the top of the atmosphere, located at $r_S/R \geq 1$) for $x \to x_S$.

There are different options for imposing a boundary condition at the top of the model (surface). The most commonly used one is to assume an isothermal atmosphere for which we have that,

$$a_1 \sim 1, \quad a_2 \sim a_{2S}, \quad a_3 \sim \Gamma_{1S}, \quad a_4 \sim a_{2S}, \quad a_5 \sim 0. \tag{18}$$

In such an isothermal atmosphere the density decreases exponentially with radius. This behaviour allows one to approximate the actual value of a_{5S} by zero.

The set of equations is now written as,

$$x \frac{d\vec{y}}{dx} \simeq \mathcal{A}_S \cdot \vec{y}, \tag{19}$$

where the matrix $\mathcal{A}$, has been approximate using the asymptotic values of a_i (see 18):

$$\mathcal{A}_S = \begin{bmatrix} a_{2S}-3 & \frac{L^2}{\sigma^2}-a_{2S} & a_{2S} & 0 \\ \sigma^2-a_{4S} & 1+a_{4S} & -a_{4S} & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & L^2 & 0 \end{bmatrix}. \tag{20}$$

By redoing the analysis presented in the previous subsection, and using

$$y_j = x^{-l} \sum_{i=0}^{\infty} \mathcal{Y}_{S,ij} \, x^{2i}, \quad j = 1, 2, 3, 4, \tag{21}$$

it follows that

$$\mathcal{Y}_{S,01} = 2 \frac{(L^2/\sigma^2 - a_{2S})\mathcal{Y}_{S,02} + a_{2S}\mathcal{Y}_{S,03}}{a_{4S}+4-\gamma^{1/2}},$$

$$\gamma = (a_{2S}-a_{4S}-4)^2 + 4(\sigma^2-a_{4S})\left(\frac{L^2}{\sigma^2}-a_{2S}\right), \tag{22}$$

$$\mathcal{Y}_{S,04} = -(l+1)\,\mathcal{Y}_{S,03}.$$

When using the first expression one must be careful since the actual value for $\gamma^{1/2}$ can be imaginary. If it happens the solution will have a propagating component at the boundary, which implies that the wave will be loosing energy at this boundary. This does not correspond to the type of solutions we are looking for (standing waves). Therefore we only consider eigenvalues that are real, corresponding to standing waves, i.e. solutions that are evanescent at the boundaries.

Note that this imposes restrictions on the values the frequency σ can have for possible modes of oscillation. Solutions are only calculated for $\gamma \geq 0$.

Other options for surface boundary conditions are possible (and have been implemented in POSC). The simplest option is to impose full reflection at the top of the model. Such a condition is achieved by setting $\delta P = 0$ at $x = x_S$, giving that

$$\mathcal{Y}_{S,01} = \mathcal{Y}_{S,02} + \mathcal{Y}_{S,03}, \tag{23}$$

instead of the first expression in (22).

5 Calculation of the solutions

In order to calculated the eigenvalues (frequencies of oscillation) of a solar model POSC uses a simple numerical scheme

to solve the set of equations (11) with the boundary conditions (17) and (22) (or one of the other alternatives). In this section we describe briefly how this is done.

The actual expressions implemented in the code are extracted from the basic dimensionless system of 4 differential equations;

$$x \frac{d\vec{y}}{dx} = \mathcal{A} \cdot \vec{y}, \tag{24}$$

where the matrix $\mathcal{A}$ is given in (10).

The functions a_i are as listed in (7) and known on a mesh from the equilibrium model obtained from solving the stellar structure equations. We use the dimensionless frequency σ, related to the actual frequency of oscillation (ω). As discussed above, under the selected boundary conditions, solutions exist only for discrete values of $\omega = \omega_{ln}$. The *mode order n* is associated with the radial structure of the different eigenfunctions that exist for the same mode degree l (the code considers spherical stars, and so the solutions are independent of the azimutal order m).

These values and the corresponding solution $\vec{y}$ are what we are trying to find. The method we use consists in, given a value of the degree l, to determine the values(s) of σ that give a continuous solution at some meeting point (defined below as x_f). This point is where we stop the integration up from the centre, and the integration down from the atmosphere. In other words we find the value(s) of σ that have a global solution satisfying all our four boundary conditions. So what we do in fact is to iterate in σ in order to find the values that give the zeros of a function measuring the fitting of outer and inner solutions at x_f.

5.1 Numerical variables

Due to numerical control of errors and precision of the calculation we redefine the variables for different regions of the model. We do so by estimating which regions of the star are evanescent for a given frequency. The two points used here to define these regions are x_{in} and x_{out}. These depend on the model and the values of ω and degree l, corresponding to the roots of the following equation,

$$\omega^2 (\omega^2 - \omega_c^2) - S_l^2 (\omega^2 - N^2) = 0. \tag{25}$$

The acoustic cutoff frequency ω_c used here is determined by

$$\omega_c \equiv \frac{c_0^2}{4H_\rho^2}\left(1-2\frac{dH_\rho}{dr}\right) \quad \text{where } H_\rho \equiv \left|\frac{d\log\rho}{dr}\right|^{-1}. \tag{26}$$

For the inner (near the centre) evanescent region we redefine the variables according to

$$\vec{y}_{in} = \left(\frac{x}{x_{in}}\right)^{2-l} \vec{y}. \tag{27}$$

Here, x_{in} is the transition point separating this inner region from the zone where the default variables, as given in (24), are used.

For the outer evanescent region (surface layers), above $x = x_{\text{out}}$, we use instead

$$\vec{y}_{\text{out}} = \left(\frac{x}{x_{\text{out}}}\right)^l \vec{y}. \qquad (28)$$

The equations are integrated from $x = 0$ to $x = x_{\text{in}}$ determining $\vec{y}_{\text{in}}$. From there to a fitting point x_f (well within the oscillatory region) we calculate the solution using the equations for $\vec{y}$. Note that the transition from one region to the other is quite natural considering our definitions $\vec{y}_{\text{in}}$ and $\vec{y}_{\text{out}}$ of $\vec{y}$. On the other hand we integrate inward from $x = x_S$ to $x = x_{\text{out}}$ using instead the equations for $\vec{y}_{\text{out}}$. From there, down to x_f we take again the equations for $\vec{y}$.

Resulting from these two integrations we have the two sets of values at $x = x_f$ which are then continuous (after normalization). Since the system of equations is linear, this is so if and only if the value of σ is an eigenvalue. At this point what we actually do is to iterate on σ to find the zeros of the fitting determinant at x_f.

5.2 Method of integration

The method implemented to solve numerically the equations considered above is a shooting method using a second-order differences representation of the equations. It consists in writing the differential equations relating the values at two mesh points, x_n and x_{n+1}, as

$$\vec{y}(n+1) = \vec{y}(n) + \frac{h_n}{2}\left[\frac{d\vec{y}}{dx}(n) + \frac{d\vec{y}}{dx}(n+1)\right] + \mathcal{O}(h_n^3), \quad (29)$$

where $h_n = x_{n+1} - x_n$ and with $\vec{y}(n) \equiv \vec{y}(x_n)$. In order to replace the derivatives we use the different sets of differential equations discussed above for the regions $0 \leq x_{\text{in}} \leq x_f \leq x_{\text{out}} \leq x_s$.

We also have to implement the boundary conditions. It is done by setting the values of y_1 and y_3 at the boundaries (centre and surface) and to calculate the values of y_2 and y_4 (at both Boundaries) from the relations constructed in the previous section. Both linearly independent solutions are found by setting the central/atmospheric values of y_1 equal to one and y_3 alternatively to one and to zero. The actual solution is a linear combination of these two (for the interior solution—up to x_f, as well as for the external solution—down to x_f).

Since we are using a shooting method, from the values of these two solutions ("in" and "out") at x_f, we construct the matching matrix whose determinant has to be zero if σ is an eigenvalue. So the task of finding an eigenvalue is reduced to finding the zero of the determinant for the fitting conditions at x_f.

We maximize the efficiency of the search for the eigenvalues (zeros of the determinant) by using the fact that these values are separated approximately by

$$\Delta\sigma_p^2 \sim \left(\frac{GM}{R^3}\right)^{1/2}\frac{2\pi\sigma}{\int_0^R c_0^{-1}dr} \quad \text{(p-modes)},$$

$$\Delta\sigma_g^2 \sim \left(\frac{GM}{R^3}\right)^{1/2}\frac{2\pi\sigma^3}{\left(l+\frac{1}{2}\right)\int_0^R \frac{N_0^2}{r}dr} \quad \text{(g-modes)}. \qquad (30)$$

5.3 Accuracy of the results

The actual accuracy of the final values of the frequencies are determined by several aspects of the calculation. As it would be expected, the accuracy of the results (frequencies) depends on the accuracy of the equilibrium model being used. Here we do not address this issue, referring the reader to Monteiro et al. (2006) and Lebreton et al. (2008).

But another aspect associated with the equilibrium model, and determining the precision of the calculated eigenvalues, is the mesh on which the equilibrium model is given. To minimise this effect, before calculating the frequencies we produce a re-meshing of the equilibrium model. The actual details of the new mesh depends on the type of model and oscillation modes being calculated. We use a receipt similar to the one discussed by Christensen-Dalsgaard and Berthomieu (1991). This allows us to minimize the errors caused by having too few points where the eigenfunctions are expected to vary more strongly.

Other aspect determining the accuracy of the eigenvalues is of course the numerical method used to integrate the four differential equations discussed above. In our case we have a second order scheme for the integration of the system of differential equations. To this we have also added the use of reduced dependent variables in the regions where the amplitudes of the eigenfunctions would be otherwise very small.

Further to this the code also uses an extrapolation to improve the accuracy of the determination of each frequency. It is known as Richardson Extrapolation. This uses the fact that our second order integration has an error which varies with the inverse of the squared number of mesh points. Using such a fact it can be written that the actual value of the eigenvalue is

$$\sigma^2 = \frac{\alpha}{\alpha-1}\sigma_N^2 - \frac{1}{\alpha-1}\sigma_{N'}^2, \quad \text{with} \quad \alpha = \left(\frac{N}{N'}\right)^2, \qquad (31)$$

where σ_N is the result found for a mesh of N points and $\sigma_{N'}$ for a mesh of N' points. The code uses, by default, $N' \sim N/2$, giving $\alpha \sim 4$. This extrapolation requires extra work but improves significantly the accuracy of the numerical frequencies (see Moya et al. 2008).

The behaviour of the frequencies with increasing number of mesh points is an internal checkpoint that allows one to
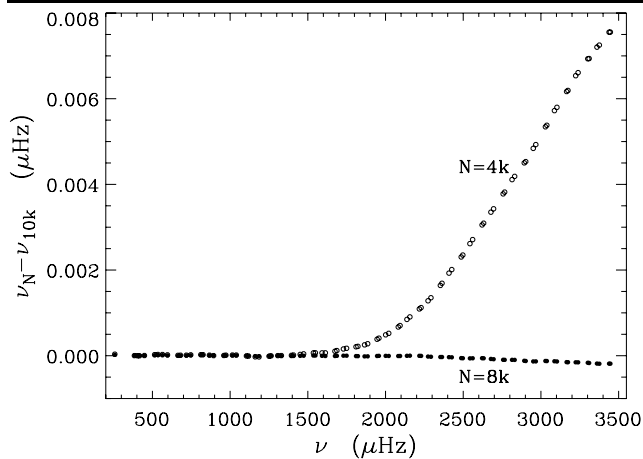
Fig. 1 Comparison of the frequencies obtained using an equilibrium model provided with a different number of mesh points. The reference is for a 10 k mesh. Only differences for frequencies with $l = 0, 1, 2, 3$ and $100\,\mu\text{Hz} \leq \nu \leq 3500\,\mu\text{Hz}$ are shown

identify where the actual frequencies are no longer affected by the precision of the integration scheme. Such a comparison is shown in Fig. 1 where the frequencies obtained for a solar model in a mesh of 10 k points is compared with the frequencies obtained using the same model with 4 k and 8 k mesh points. We have also performed a detailed comparison with the results from ADIPLS (Christensen-Dalsgaard 2008a) to have an external check on the computation. Further comparisons of Posc with other codes has been performed recently by Moya et al. (2008).

In general (when using a model in a mesh of ∼6 k points) Posc frequencies of oscillation for solar-type stars have an estimated numerical uncertainty below 0.001 µHz. This is below the current observational errors of solar frequencies. Similar values are obtained for oscillation modes of stars of different masses and ages if the mesh is adequately adapted (in terms of number of points and its distribution) to the eigenfunctions being calculated (g or p modes).

6 Output

The main output of the code are the values of the frequencies of linear adiabatic oscillations of an equilibrium model of a star. The calculation to be done is defined by an interval in frequency ($\omega_a \leq \omega \leq \omega_b$) and in mode degree ($l_a \leq l \leq l_b$).

6.1 Mode classification

The mode order of each eigenvalue is obtained using a method similar to the phase diagram as described by Unno et al. (1989). It consists in counting the number of times the solution crosses the line $y_1 \equiv 0$ in the plane (y_1, y_2). If the cross is clockwise it counts as (-1) otherwise as ($+1$).

When $l = 0$ (radial modes) an additional ($+1$) cross is considered.

To a total negative counting of the crosses corresponds a g-mode while p-modes have positive counting results, with the number corresponding to the mode order. The solutions corresponding to f-mode eigenvalues have a total of zero counts.

6.2 Mode inertia and eigenfunctions

The eigenfunctions are provided in different formats (several normalisations and/or combinations) depending on what is required. These are obtained from $\vec{y}$ and correspond to combinations of the functions,

$$\frac{\xi_r(r)}{\xi_r(R)}, \quad \frac{\xi_h(r)}{\xi_h(R)}, \quad \frac{\tilde{P}(r)}{\tilde{P}(R)}, \quad \frac{\tilde{\Phi}(r)}{\tilde{\Phi}(R)}. \tag{32}$$

The equilibrium structure is also used to calculate different normalizations of the eigenfunctions.

The code provides, in addition to the mode parameters and frequencies, the mode inertia as given by,

$$E_{ln} \equiv \frac{4\pi}{M} \frac{\int_0^R [\xi_r^2(r) + L\,\xi_h^2(r)] r^2 \rho\,\mathrm{d}r}{\xi_r^2(R) + L\,\xi_h^2(R)}. \tag{33}$$

7 Conclusion

This work provides a brief description of Posc—the Porto Oscillation Code. This code has been developed mainly for calculating linear adiabatic oscillations of stellar models for stars similar to the Sun (in mass). The code is written in Fortran 77 and is modular. It is prepared to accept input models in the AMDL[1] format. Tools are also available to convert almost any available stellar model output to the required format to be used by Posc.

The code has been applied to several cases, namely the Sun (Monteiro 1996; Monteiro et al. 1996; Monteiro and Thompson 2005) and other stars (Cunha et al. 2003; Fernandes and Monteiro 2003), including pre-main sequence models (Ruoppo et al. 2007). It has also been used to produce the frequencies of reference grids of stellar evolution models for asteroseismology (Marques et al. 2008).

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[1]See the description of some file formats for stellar evolution models at http://www.astro.up.pt/corot/ntools/

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