SEISMIC PROBING OF OUTER REGIONS OF THE SUN

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(Received 28 February, 1990; in revised form 25 October, 1990)

Abstract. The inverse scattering problem for reconstruction of the structure of reflecting potential from the observed frequency dependence of the phase shift of reflected acoustic waves is considered. The linearized formulation of the ill-posed inverse problem is used, which is solved using a perturbation technique. The potential perturbation of the standard model as a combination of five B-splines leads to a constructive solution of the discrepancy problem between the observational and theoretical frequencies of the 5-min oscillations. The discrepancy is reduced by an order of magnitude. The corresponding change of the shape of the reflecting potential is interpreted as a requirement of a general increase of convection efficiency in the standard solar model. In this way, the agreement of the oscillation frequencies of high degree is also improved.

1. Introduction

The asymptotic inversion technique for high-frequency acoustic waves is an effective means of investigation of the solar interior structure using the observed 5-min oscillation frequencies (see, e.g., review by Vorontsov and Zharkov, 1989, and references therein). When constructing the appropriate asymptotic equation for eigenfrequencies, the fact is taken into account that an asymptotic description is invalid in the thin layer near the solar surface, where the physical parameters vary on a scale smaller than the oscillation wavelength. This difficulty is eliminated by matching the asymptotic solutions in the interior with exact non-asymptotic solutions near the surface, leading to the eigenfrequency equation which contains an additional frequency-dependent phase shift (Vorontsov and Zharkov, 1989; Brodsky and Vorontsov, 1988). This phase shift is determined by the structure of the outermost solar layers where the reflection of internal acoustic waves occurs. For a given solar envelope model, the phase shift can be computed from the profile of corresponding acoustic potential. If the frequency dependence of the phase shift is determined somehow from the observational oscillation frequencies, it can be used, at least in principle, to infer the profile of the acoustic potential and thus to study the structure of the outermost solar layers.

The splitting of the experimental information, contained in the oscillation frequencies, yields the frequency dependence of the phase shift in the form of the function (Brodsky and Vorontsov, 1988)

$$\bar{\beta}(\omega) = -\omega^2 \frac{\mathrm{d}}{\mathrm{d}\omega} (\bar{\alpha}/\omega), \tag{1}$$

where ω is angular frequency and $\alpha(\omega)$ is the frequency-dependence of the phase shift. This dependence is determined by the profile of the reflecting potential $V^2(\tau_\alpha)$ in the

Solar Physics **133:** 149–154, 1991.

model and the value of $\bar{\alpha}(\omega)$ can be calculated for each ω as the asymptote of the solution of the initial value problem for the phase equation (Vorontsov and Zharkov, 1989)

$$\frac{\mathrm{d}}{\mathrm{d}\tau_\alpha}(\pi\alpha) = \frac{V^2}{\omega}\cos^2[\omega\tau_\alpha - \pi/4 - \pi\alpha(\tau_\alpha)] \tag{2}$$

for sufficiently high values of τ_α with the corresponding boundary condition at the surface $r = R_\odot$, where the 'acoustic depth' $\tau_\alpha(r) = \int_r^{R_\odot} \mathrm{d}r/c$, c is the adiabatic sound speed.

The function $\bar{\beta}(\omega)$ is the asymptote of the corresponding initial value problem for the equation

$$\frac{\mathrm{d}}{\mathrm{d}\tau_\alpha}(\pi\beta) = \frac{V^2}{\omega}\{1 + \cos\vartheta(\tau_\alpha) + [0.5\vartheta(\tau_\alpha) + \pi/4 + \pi\beta(\tau_\alpha)]\sin\vartheta(\tau_\alpha)\}\,,$$
$$\vartheta(\tau_\alpha) = 2[\omega\tau_\alpha - \pi/4 - \pi\alpha(\tau_\alpha)]\,, \tag{3}$$

which should be solved simultaneously with Equation (2).

The difference between the observational and theoretical (for the standard model 1 of Christensen-Dalsgaard, 1982) frequency dependencies of the phase shift is very large and its character is systematic (see Figure 1). It determines the main source of the discrepancy between the observational and theoretical frequencies of the 5-min oscillations (Vorontsov and Zharkov, 1989) and is related with the structure of the outer reflecting solar layers. This discrepancy can be studied in the framework of the direct problem, which is the construction and verification of the different physical models of the solar envelope. Such an approach, however, may be long and not simple 'try and see' method. The study of the inverse scattering problem for the reconstruction of the structure of realistic reflecting potential $V^2(\tau_\alpha)$ from the observed phase shift of the reflected acoustic waves $\beta(\omega)$ is thus of special interest. The change in the shape of the resultant reflecting potential in comparison with the theoretical one must indicate in

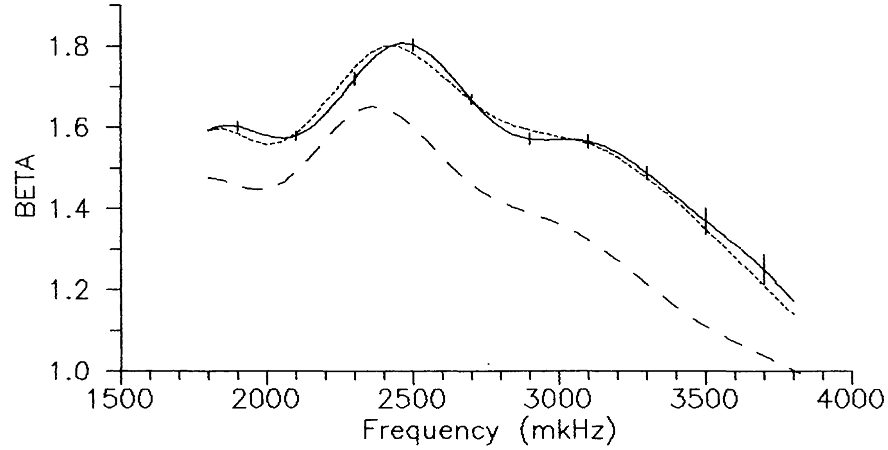

Fig. 1. Frequency dependence of the phase shift. Solid curve with error bars is inferred from observational data (Brodsky and Vorontsov, 1988); dashed line corresponds to the standard model 1; dotted curve was computed for the perturbed acoustic potential, inverted from the observational $\beta(\omega)$.

what way one should modify the model for the elimination of the discrepancy between the frequency dependencies of the phase shift.

2. Perturbation Theory for the Solution of the Inverse Problem

Let us assume, that the complete perturbed reflecting potential $V^2(\tau_\alpha)$ is a sum of the reference potential V_0^2 and some additional potential V_1^2, and the last one may be considered as a small correction compared to V_0^2:

$$\begin{aligned} &V^2(\tau_\alpha) \approx V_0^2(\tau_\alpha) + V_1^2(\tau_\alpha)\,, \\ &V_1^2(\tau_\alpha) \ll V_0^2(\tau_\alpha)\,. \end{aligned} \tag{4}$$

Similarly, let us represent the values of the total phase shift

$$\begin{aligned} &\alpha(\omega, \tau_\alpha) \approx \alpha_0(\omega, \tau_\alpha) + \alpha_1(\omega, \tau_\alpha)\,, \\ &\alpha_1(\omega, \tau_\alpha) \ll \alpha_0(\omega, \tau_\alpha)\,, \end{aligned} \tag{5}$$

and also

$$\begin{aligned} &\beta(\omega, \tau_\alpha) \approx \beta_0(\omega, \tau_\alpha) + \beta_1(\omega, \tau_\alpha)\,, \\ &\beta_1(\omega, \tau_\alpha) \ll \beta_0(\omega, \tau_\alpha)\,. \end{aligned} \tag{6}$$

If we substitute Equations (4), (5), (6) into (2), (3) and neglect higher-order terms, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\tau_\alpha}(\pi\alpha_1) = \frac{V_0^2}{\omega}\,\pi\alpha_1 \sin\vartheta_0 + \frac{V_1^2}{\omega}\cos^2 0.5\vartheta_0\,, \tag{7}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\tau_\alpha}(\pi\beta_1) = {}& \frac{V_0^2}{\omega}\{\pi(\beta_1 + \alpha_1)\sin\vartheta_0 - 2\pi\alpha_1(\omega\tau_\alpha - \pi\alpha_0 + \pi\beta_0)\cos\vartheta_0\} + \\ & + \frac{V_1^2}{\omega}\{1 + \cos\vartheta_0 + (\omega\tau_\alpha - \pi\alpha_0 + \pi\beta_0)\sin\vartheta_0\}\,, \end{aligned} \tag{8}$$

where $\vartheta_0 \equiv 2[\omega\tau_\alpha - \pi/4 - \pi\alpha_0(\tau_\alpha)]$.

For the unperturbed potential $V_0^2(\tau_\alpha)$ we choose the reflecting potential (Brodsky and Vorontsov, 1988) of the standard solar model 1 of Christensen-Dalsgaard (1982), which is shown in Figure 2. Assuming that the complete perturbed potential $V^2(\tau_\alpha)$ does not differ from the unperturbed potential in the vicinity of the temperature minimum (i.e., in the region of the plateau of $V_0^2(\tau_\alpha)$ near the surface, $\tau_\alpha = 0$), we establish boundary conditions for Equations (7) and (8):

$$\begin{aligned} &\alpha_1(\tau_\alpha = 0) = 0\,, \\ &\beta_1(\tau_\alpha = 0) = 0\,. \end{aligned} \tag{9}$$

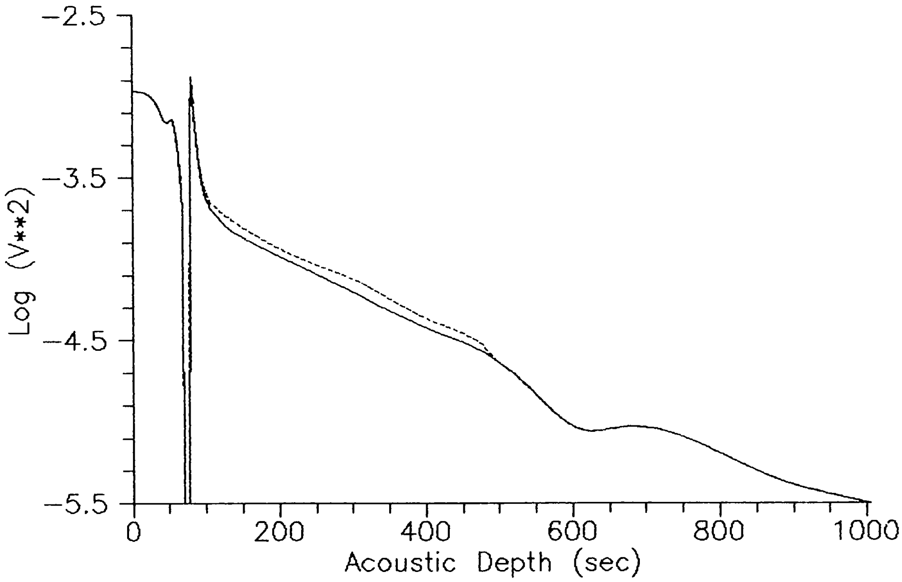

Fig. 2. Profile of the reflecting acoustic potential versus acoustic depth τ_α. Solid curve corresponds to the standard solar model 1; dotted line is the result of the inversion of observational data (i.e., reconstructed in the process of the solution of inverse problem).

Equations (7) and (8) with boundary conditions (9) determine the corresponding initial value problem for the perturbation of phase shift, if the perturbation of the potential $V_1^2(\tau_\alpha)$ is known. Note that this problem should be solved simultaneously with problems (2) and (3) for the unperturbed quantities and with corresponding boundary conditions (see paper by Brodsky and Vorontsov, 1988). Integration of (8) with respect to τ_α from 0 to $\tau_{\max}$, taking into account (9), gives the integral equation of Fredholm's type:

$$\pi\beta_1(\omega, \tau_{\max}) = \int_0^{\tau_{\max}} \Bigg[\frac{V_0^2}{\omega} \{\pi(\alpha_1 + \beta_1)\sin\vartheta_0 - 2\pi\alpha_1(\omega\tau_\alpha - \pi\alpha_0 + \pi\beta_0)\cos\vartheta_0\} + \frac{V_1^2}{\omega}\{1 + \cos\vartheta_0 + (\omega\tau_\alpha - \pi\alpha_0 + \pi\beta_0)\sin\vartheta_0\} \Bigg] \mathrm{d}\tau_\alpha , \tag{10}$$

$\omega_1 \leq \omega \leq \omega_2$, ω_1 and ω_2 determine the interval of the observed frequencies. When solving Equation (10), we suppose that

$$\beta_1(\omega, \tau_{\max}) = \bar{\beta}_{\text{obs}}(\omega) - \bar{\beta}_{\text{theor.}}(\omega) . \tag{11}$$

Let the r.m.s. error of $\bar{\beta}_{\text{obs}}$ equal δ. The problem (10) is an example of the ill-posed linear inverse problem.

We look for the solution of (10) in the form

$$V_1^2(\tau_\alpha) = \sum_{m=1}^{N} C_m B_m(\tau_\alpha) ; \tag{12}$$

$\tau_1 \le \tau_\alpha \le \tau_2$, where $B_m(\tau_\alpha)$ are cubic B-splines with the distribution of the break points equidistant in τ_α for some interval (τ_1, τ_2), C_m are unknown constants. So, the complete perturbed potential will be

$$V^2(\tau_\alpha) = V_0^2(\tau_\alpha) + V_{1\delta}^{2*}(\tau_\alpha), \tag{13}$$

where the asterisk symbolizes the choice in (12) of the definite constants C_i^* $(i = 1, \ldots, N)$ determined using least-squares technique, and subscript 'δ' indicates that only the approximate solution is sought, with the precision defined by the r.m.s. error of the observational data. We use the function $\beta_1^*(\omega, \tau_{\max}) \equiv \bar{\beta}_1^*(\omega)$, obtained from (7)–(10) with $V_1^2(\tau_\alpha) = V_{1\delta}^{2*}(\tau_\alpha)$, for the determination of the phase shift which corresponds to the 'corrected' solar model:

$$\bar{\beta}_{\text{model}}(\omega) = \bar{\beta}_0(\omega) + \bar{\beta}_1^*(\omega). \tag{14}$$

Then we compare the 'corrected' function (14) with the observed $\bar{\beta}(\omega)$ by verifying the condition

$$\left\{ \int_{\omega_1}^{\omega_2} [\bar{\beta}_{\text{model}}(\omega) - \bar{\beta}_{\text{obs}}(\omega)]^2 \, d\omega \right\}^{1/2} \le \delta. \tag{15}$$

If condition (15) is satisfied, then the problem (10) may be considered to be solved: $V_{1\delta}^{2*}(\tau_\alpha)$ is the approximate solution of the problem. In the case when the condition (15) is not satisfied, the next iteration should be performed similarly. If, in principle, the solution for $V_{1\delta}^2(\tau_\alpha)$ may be found in the form (12), then the process is repeated cyclically (reiterate) until condition (15) is satisfied. If it is not possible to select some linear combination (12) for $V_1^2(\tau_\alpha)$ to satisfy condition (15), the problem is not solvable in the given class of functions.

3. Results and Discussion

For numerical computations of the potential perturbation $V_1^2(\tau_\alpha)$ an interval of τ_α was chosen, which extends from the region of maximum of the potential $V_0^2(\tau_\alpha)$ when $\tau_1 = 80$ s to the acoustic depth $\tau_2 = 500$ s (the second helium ionization zone). The reasons for such a choice are: (1) Below the second helium ionization zone the potential decreases rapidly, practically does not depend on the model parameters and is determined mainly by the value of gravity at the surface. (2) The structure of the regions above this interval has little influence on the phase shift of low-frequency waves, because the low-frequency waves are practically completely reflected by the lower layers. At the same time the discrepancies are similar for high and low frequencies. In the indicated interval the system of five cubic B-splines with zero's boundary conditions was chosen for the computations. The result of six iterations is shown in Figure 1. Condition (15) is satisfied. In this figure one can see a good coincidence of the functions $\beta(\omega)$ between the perturbed model and the observed one; the discrepancy is reduced by an order of magnitude. The corresponding change in the shape of the complete potential is shown in Figure 2.

What kind of modifications in the physical model of the envelope can produce the resultant correction of the reflecting potential? In the investigated region, the stratification of the matter in the convective zone is very close to the adiabatic one, so we may assume the square of Brunt–Väisälä frequency $N^2 = 0$. Taking into consideration that the pressure scale height $H \ll R_\odot$ and gravity acceleration $g \approx$ const., the expression for the acoustic potential (Vorontsov and Zharkov, 1989) can be reduced to the form:

$$V^2 \approx \frac{g^2}{16c^2}\left[(1 + \Gamma_1)(3 - \Gamma_1) + 2(1 + \Gamma_1)\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\ln p} - \left(\frac{\mathrm{d}\Gamma_1}{\mathrm{d}\ln p}\right)^2 + \right.$$
$$\left. + 4\Gamma_1 \frac{\mathrm{d}^2\Gamma_1}{\mathrm{d}(\ln p)^2}\right], \tag{16}$$

where $c^2 = \Gamma_1 p/\rho$ is the sound speed squared, p is the pressure, ρ is the density.

The investigated depth range occupies the helium and hydrogen ionization zones, where the adiabatic exponent $\Gamma_1 = (\mathrm{d}\ln p/\mathrm{d}\ln\rho)_s$ varies substantially. Nevertheless, the quantitative study of expression (16) shows that no reasonable changes in the profile of $\Gamma_1(p)$ can produce this large deviation of V^2 (about 20%), which is obtained as the solution of the inverse problem. In this case the necessary increase of V^2 can be obtained only by a general decrease of c^2. Note that the decrease of the specific entropy in the adiabatic part of the convective zone leads to exactly the same effect (Ulrich and Rhodes, 1977)! The decrease of specific entropy corresponds to a general increase of the convective efficiency.

It is worth noting that the increase of convective efficiency leads also to better agreement for the oscillation frequencies of high degree modes (see, e.g., review by Vorontsov and Zharkov, 1989, and references therein) and leads to some increase of the convection zone depth. But the convection zone depth is well defined by helioseismological data (e.g., Vorontsov, 1988), and the compensation of this effect will probably allow us to put some constraint on the opacity at the base of convection zone.

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