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Search for solar cycle changes in the signature of rapid variation in BiSON data

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ABSTRACT

The second helium ionization zone and the base of the convective envelope are regions of rapid variation of solar structure which introduce characteristic signatures into the frequencies of p-mode oscillations. These signals provide a direct seismological method to probe the acoustic properties of these regions. In this work we isolate these signatures in over 9 yr of low-degree BiSON data and extract information on the acoustic depth and local properties from each signal. Any temporal variations are investigated by fitting the signals extracted from 432, 864 and 1728-d spectra. The extracted parameters are found to be in agreement over the different length spectra and within one formal standard deviation of the values obtained for model 'S'. There is no evidence found for any systematic variation in the acoustic depth, width or magnitude of the second helium ionization zone, which suggests any activity-dependent disturbance to the near surface layers does not propagate down to this layer. The convection zone signal does show some temporal variation that may be correlated with solar activity, although further analysis with current data is required. The isolation of these signatures in low-degree data confirms that this method can be used to provide structural information on Sun-like stars once similar asteroseismic data become available.

Key words: Sun: interior – Sun: oscillations.

1 INTRODUCTION

Regions of rapid variation of solar structure create a characteristic periodic signature in the frequencies of p-mode oscillations (Gough 1990; Basu, Antia & Narasimha 1993; Monteiro, Christensen-Dalsgaard & Thompson 1993; Roxburgh & Vorontsov 1994). This signature can be analysed to extract the location of the region (from the period) and physical parameters describing the localized disturbance (from the amplitude). The two main such regions are the second helium ionization zone ($\sim$0.98 $R_\odot$), where there exists a distinct bump in the adiabatic exponent, and the base of the convective envelope ($\sim$0.71 $R_\odot$), caused by discontinuities in sound speed derivatives. These two regions introduce oscillatory perturbations to the frequencies of solar p-modes relative to a fictitious Sun without such features.

The first studies into the signature of rapid variation used second, fourth and higher-order differences in the oscillation frequencies of modes of degree $5 \leqslant \ell \leqslant 20$ to extract information on, in particular, the signal from the base of the convective envelope. Later work introduced other methods to remove the smooth component of the oscillation frequencies and isolate the signature of rapid structural change. This signal has since been used to put limits on the extent of overshoot from the convection zone into the radiative interior (Basu, Antia & Narasimha 1994; Monteiro, Christensen-Dalsgaard & Thompson 1994; Christensen-Dalsgaard, Monteiro & Thompson 1995; Basu 1997).

By applying a variational principle for non-radial adiabatic oscillations, equations have been derived for the signals from both the base of the convective envelope and the second helium ionization zone (Monteiro et al. 1994; Monteiro & Thompson 1998). Although these signals are conceptually similar, they have different amplitude and period behaviour reflecting the acoustic depth and properties of each region. Monteiro et al. (2001) reported that the signal from the convection zone base, isolated in 4 yr of SOI-MDI data (using only those modes that penetrate into the radiative interior), indicates a possible temporal variation in acoustic depth over that period.

It is well established that solar p-mode frequencies change over the solar activity cycle (Woodard & Noyes 1985; Elsworth et al. 1990, 1994; Chaplin et al. 1998, 2001, 2002) and that the main influence for the frequency variations is a perturbation to the near surface layers. Helioseismic inversions to calculate variations in sound speed in the outer layers become unreliable above 0.98 $R_\odot$ (e.g. Antia et al. 2001). It is therefore useful to use the He II ionization signature as a diagnostic tool for the physical parameters in that region.

In this work, low-degree ($\ell \leqslant 3$) p-mode eigenfrequencies covering the declining phase of activity cycle 22 and most of the

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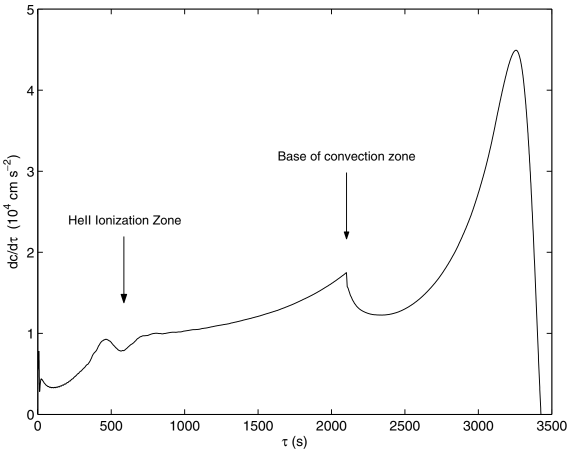

Figure 1. First derivative of sound speed with respect to acoustic depth for model 'S'.

rising phase of cycle 23 are used to study any temporal or activity-dependent variations in the properties of the second helium ionization zone and the base of the convective envelope. The frequencies used are obtained from the ground-based Birmingham Solar Oscillations Network (BiSON),[1] a network of six instruments distributed in longitude making unresolved 'sun as a star' observations. A smooth component to the oscillation frequencies is removed and a two-component fit is applied to the remaining signal. The precision of the extracted parameters from the base of the convective envelope is limited by the uncertainties in low-degree data, which are of the order of the amplitude of the signal. However, with careful extraction techniques the convection zone base signature can be isolated in BiSON p-mode frequencies. The amplitude of the He II ionization signature is greater by an order of magnitude and is easily isolated in unresolved data. These results have interesting implications for uncovering these properties in Sun-like stars once similar asteroseismology data becomes available (see, for example, Miglio et al. 2003).

2 THE CONVECTION ZONE BASE SIGNATURE

The frequencies of p-mode oscillations are dependent on the adiabatic sound speed of the medium through which they propagate. Any rapid variation in the sound speed introduces a periodic signature in the frequencies with a 'period' determined by the round-trip travel time of an acoustic wave from the surface to the region of rapid variation. This is twice the *acoustic depth* of the localized disturbance, with the acoustic depth defined as

$$\tau(r) = \int_r^R \frac{\mathrm{d}r}{c(r)}, \tag{1}$$

where $c(r)$ is the sound speed at radius r and R is the total solar radius.

At the base of the convective envelope the temperature gradient changes from being radiative to adiabatic, introducing an abrupt change into the first derivative and, therefore, a discontinuity into the second derivative of the sound speed at that location. This is illustrated in Fig. 1 for the sound speed calculated for model 'S' of Christensen-Dalsgaard (see Christensen-Dalsgaard et al. 1996). Using a variational principle for non-radial adiabatic oscillations it can be shown that this abrupt transition introduces a periodic component into the frequency of p-modes which pass this region. For low-degree data ($\ell \leqslant 4$) this is given by (Monteiro et al. 1993, 1994)

$$\delta\omega_{cz} \simeq \left[a_1^2\left(\frac{\tilde{\omega}}{\omega}\right)^4 + a_2^2\left(\frac{\tilde{\omega}}{\omega}\right)^2\right]^{1/2} \cos\left[2\left(\omega\bar{\tau}_{cz} + \phi_1\right)\right], \tag{2}$$

where a_1 and a_2 are constants, ϕ_1 is a constant associated with the phase of the eigenfunctions at the surface and $\tilde{\omega}$ is a fiducial frequency introduced for convenience (where $\tilde{\omega}/2\pi = 2500\ \mu$Hz). Also, $\bar{\tau}_{cz} \equiv \tau_{cz} + a_\phi$ depends on the acoustic depth τ_{cz} of the discontinuity plus an offset $a_\phi \equiv -\pi\, \mathrm{d}\alpha/\mathrm{d}\omega$ due to the frequency dependence of the surface phase function $\alpha(\omega, \ell)$. By fitting for a number of solar models, Monteiro et al. (1994) determined $a_\phi \sim 200$ s.

The amplitude of the signal at the fiducial frequency (known as $A_{2.5}$) is found to discriminate well between solar models with different degrees of overshoot (Christensen-Dalsgaard et al. 1995). Any significant temporal variation in this parameter may be an indication of structural changes at the base of the solar convective envelope.

3 THE He II IONIZATION SIGNATURE

The thermodynamics of the solar interior are modified in regions of partial ionization. Of crucial importance is the value of the first adiabatic exponent, Γ_1, defined as the logarithmic derivative of pressure with respect to density taken at constant entropy, i.e.

$$\Gamma_1 = \left(\frac{\mathrm{d}\ln P}{\mathrm{d}\ln\rho}\right)_s. \tag{3}$$

This in turn is related to the adiabatic sound speed by

$$c^2 = \frac{\Gamma_1 P}{\rho}, \tag{4}$$

which is assumed to vary only with depth. The region of the second ionization of helium can be seen as a clear bump in Γ_1 and has a corresponding effect on the sound speed. It is this localized perturbation to the sound speed that gives rise to an oscillating signature in the p-mode frequencies. Fig. 2 shows the bump in Γ_1 calculated for model 'S'.

Any significant variation in the sound speed or other physical parameters at (or near to) the second helium ionization zone will

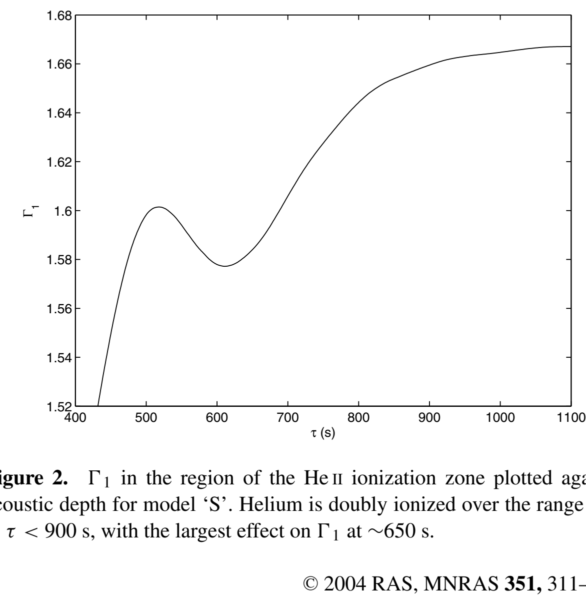

Figure 2. Γ_1 in the region of the He II ionization zone plotted against acoustic depth for model 'S'. Helium is doubly ionized over the range $500 < \tau < 900$ s, with the largest effect on Γ_1 at ~650 s.

[1] http://bison.ph.bham.ac.uk

result in a shift in either the position, width or relative magnitude of this bump. Such a variation may not indicate a solely localized disruption and could be caused by structural changes outside the He II ionization zone affecting the properties in this region. There is considerable dispute over which layers are affected by the solar activity cycle. Studies have shown that thermal disturbances in the near-surface superadiabatic layers reproduce the observed frequency shifts but do not account for variations in luminosity and suggest that in addition a fibril magnetic field could be responsible (Balmforth, Gough & Merryfield 1996). Dziembowski et al. (2000) found evidence of activity driven structural changes below the He II ionization zone, peaking at $\sim$0.935 $R_{\odot}$, owing to magnetic and/or thermal perturbations. If such changes are thermally driven they could have an effect on the location of the He II ionization zone despite being localized elsewhere.

Using an asymptotic theory and applying a variational principle it can be shown that for low-degree data ($\ell \leqslant 5$) the second helium ionization zone gives rise to a perturbation of the p-mode oscillation frequencies of (Monteiro 1996; Monteiro & Thompson 1998):

$$\delta\omega_{he} \simeq -\frac{3}{4\tau_t}\left(\frac{\delta\Gamma_1}{\Gamma_1}\right)_{\tau_{he}} \frac{\sin^2(\omega\beta)}{\omega\beta} \cos\left[2\left(\omega\tau_{he} + \phi_2\right)\right], \tag{5}$$

where τ_t is the total acoustic radius, $(\delta\Gamma_1/\Gamma_1)_{\tau_{he}}$ is the relative magnitude of the bump in Γ_1 (which is referred to as $\delta_{\rm obs}$), β measures approximately the acoustic halfwidth of the bump, τ_{he} is the acoustic depth below the surface where the bump is located and ϕ_2 is a constant associated with the phase of the oscillation eigenfunctions at the solar surface.

4 EXTRACTING THE SIGNAL

Any region in which there is a rapid variation of the background state imposes a signature on the oscillation frequencies of all modes passing through that region. All of the low-degree modes used have lower turning points below the base of the convection zone and upper turning points above the He II ionization zone.

The extraction procedure involves removing a sufficiently smooth contribution to the frequencies $\nu = \omega/2\pi$, for each degree ℓ as a function of radial order n. The remaining combination of He II ionization zone and convection zone base signatures are then fitted by a two-component fit,

$$\delta\omega = \delta\omega_{cz} + \delta\omega_{he}, \tag{6}$$

using a weighted non-linear least squares method. The choice for the smooth contribution to the frequencies must be sufficient to remove the general trend but not over-fit the data and lose the He II ionization signal. To facilitate this a second derivative smoothing spline was chosen as the smooth contribution.

For each degree (in each spectrum) a smooth contribution s was constructed for smoothing parameter q so that it minimizes

$$q\sum_i\left[\nu_i - s(n_i)\right]^2 + (1-q)\int\left(\frac{\mathrm{d}^2 s}{\mathrm{d}n^2}\right)^2 \mathrm{d}n. \tag{7}$$

The resultant smooth function is a fourth-order spline with simple knots at the non-end point integer values of the mode order n_i. A weighted smoothing spline (described in de Boor 1978) would also include the variance in each frequency within the summation in equation (7). However, it was found that the sharp increase in linewidth (and hence in observational uncertainty) at high frequency results in a weighted approach over-favouring low-radial-order modes and reducing the amplitude of the residual signal at low frequency. As the extracted signal discards the three lowest and three highest-radial-order modes to prevent using the near end points of the smooth contribution, the signal extracted comprises modes with very similar uncertainties and so a non-weighted approach was used.

The smoothing parameter q was varied between 0.1 and 0.9 and each time the residuals were fitted to equation (6). For each q the adjusted R-square and reduced χ^2 goodness of fit parameters for the two-component fit were calculated, parameters closer to unity indicating a better fit. This was then used to tune the smoothing parameter and extract both signatures simultaneously.

The non-linear least squares fitting to the extracted signal involves eight variables and was found to be very sensitive to the initial value chosen for the convection zone base depth parameter, $\bar{\tau}_{cz}$, in equation (2). Fits were performed with a range of initial values for $\bar{\tau}_{cz}$ and the solutions indicated by a stable plateau of fitted parameters.

Fig. 3 shows that a value of $q \sim 0.42$ extracts residuals that best fit equation (6) in the case of one particular 864-d spectrum. The two-component fit for this degree of smoothness is shown in Fig. 4. A small variation was found in the optimum choice of smoothing parameter between each data set but a single value must be chosen

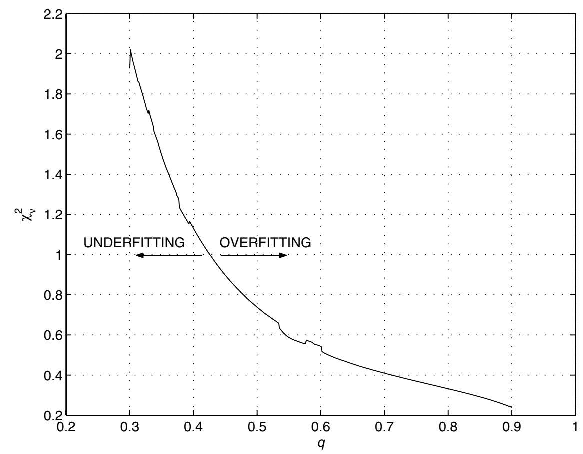

Figure 3. Smoothing parameter tuning using the reduced χ^2 for a single 864-d BiSON data set (starting on 1993 May 13).

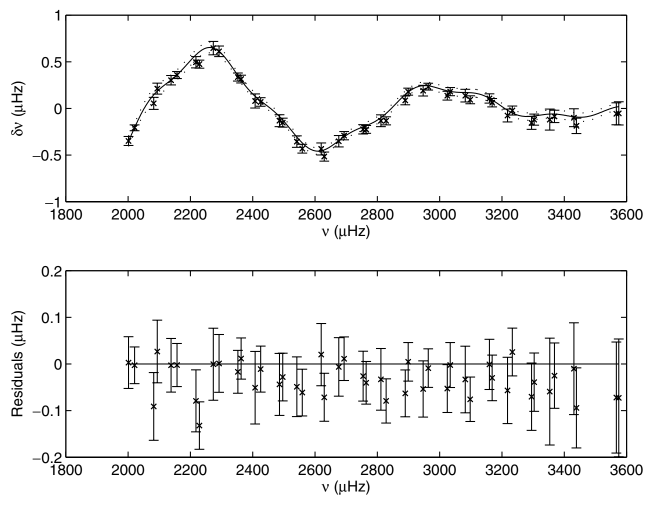

Figure 4. Two-component fit to the extracted signal for a single 864-d BiSON data set ($\delta\nu = \delta\omega/2\pi$), with 95 per cent confidence bounds (dotted) and residuals.

for all sets to prevent the introduction of a bias in the extracted signal amplitude and frequency. The mean optimal smoothing parameter for all spectra was determined to be 0.425. A small variation in q was accommodated in the Monte Carlo simulations performed to obtain estimates of the uncertainty in each of the fitted parameters.

For each set of frequencies, 100 simulations were performed with normally distributed random scatter on each mode frequency in accordance with the observational uncertainty. The value of the extraction parameter, q, for each simulation was chosen from a random normal distribution centred at 0.425 with standard deviation 0.05 in accordance with the distribution of optimum q values. The simulations indicated that where a frequency and a phase were fitted simultaneously, as was the case for the acoustic depth parameters, a strong anti-correlation existed between the two extracted values.

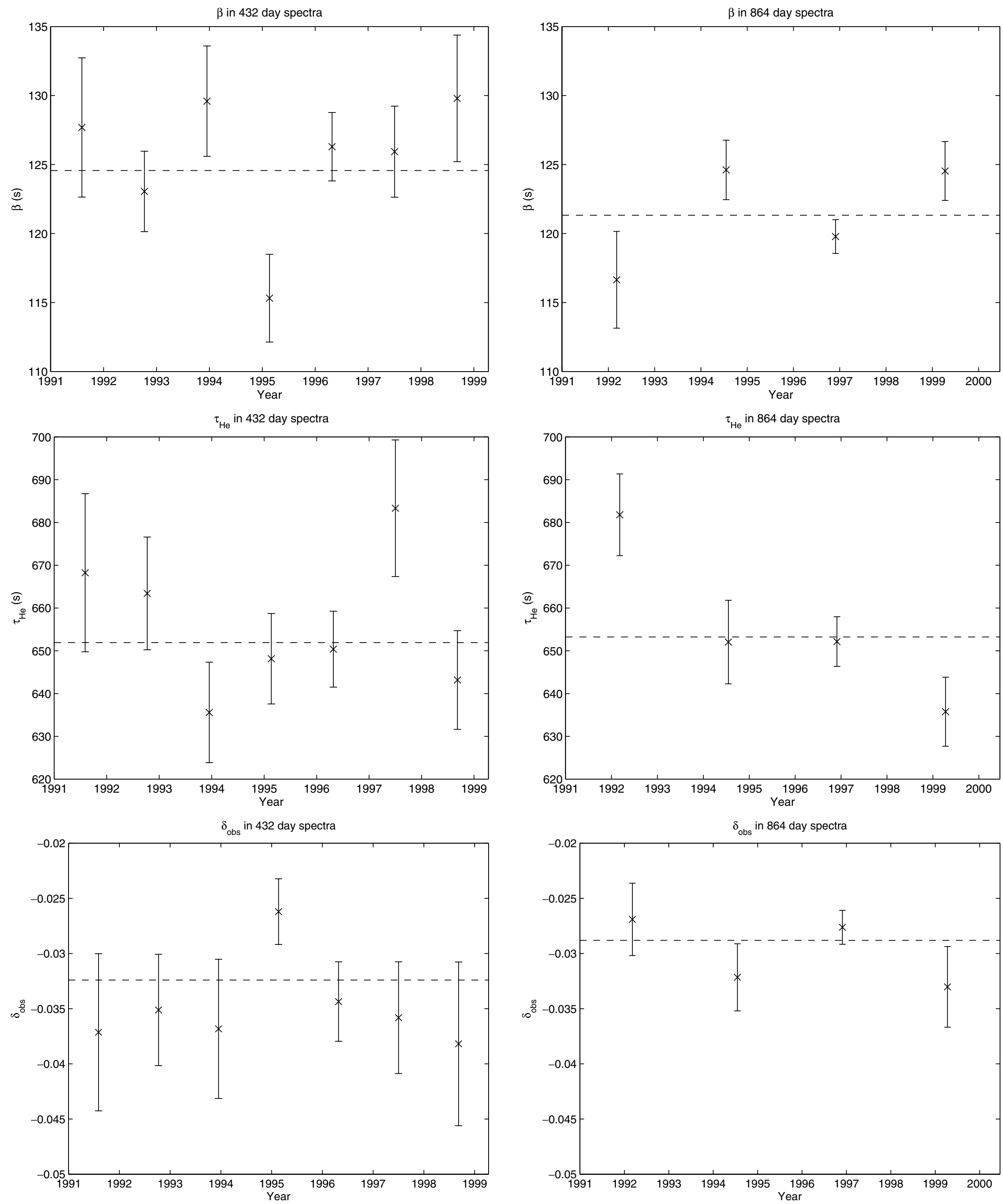

Figure 5. Fitted parameters for the He II ionization signature isolated in 432 and 864-d BiSON spectra, with uncertainty estimates from simulations with normally distributed scatter. Top panels show the fitted values for β, the approximate acoustic halfwidth of the bump in Γ_1. Centre panels show the results for the acoustic depth of the ionization zone. Bottom panels show any variation in $\delta_{\rm obs}$, the relative magnitude of the bump in Γ_1. Dotted lines indicate the weighted mean value for each set of results.

This was caused by the fitting routine compensating for a slight change in frequency by shifting the phase. The distributions of extracted acoustic depth parameters from the simulations were found to be centred on the values extracted from the fits to the real data, indicating that no bias was introduced as a result of this artefact. However, as the standard deviations of these distributions were used as an estimate of the uncertainty in each extracted parameter, the anti-correlation may have caused an increase in error estimates for the acoustic depth parameters.

5 RESULTS

The analysis outlined in the previous section has been performed on the frequencies from a number of different length BiSON spectra covering a nine-and-a-half year period from the beginning of 1991 until mid 2000. This includes seven 432, four 864 and two 1728-d spectra for modes of degree $\ell \leqslant 3$ generated from observations taken by the BiSON instruments. Extraction and fitting was also performed on the frequencies of model 'S' for the same modes observed in 864-d BiSON spectra. Uncertainties were imposed on the model 'S' frequencies equal to the mean observational errors for the corresponding modes in the 864-d data. Figs 5 and 6 show the key parameters extracted for the He II ionization signature and the convection zone base signature for each fitted set of spectra. The weighted mean parameters and fitted parameters from model 'S' are shown in Table 1.

The early BiSON spectra (pre mid-1992) were obtained from a network of only four (or occasionally three) stations (Elsworth et al. 1995; Chaplin et al. 1996), reducing the duty cycle and increasing observational uncertainties. The combined signatures from these early spectra prove problematic to fit, particularly the extraction of parameters governing the convection zone base signal. Later spectra are of higher quality and give more reliable estimates of these parameters.

These later BiSON results are in good agreement over different spectra lengths and the mean results are within one formal standard deviation of the parameters obtained from the frequencies of model 'S'.

6 DISCUSSION

This work has succeeded in extracting the two periodic components of the signature of rapid variation from low-degree BiSON data in order to measure the properties of the He II ionization zone

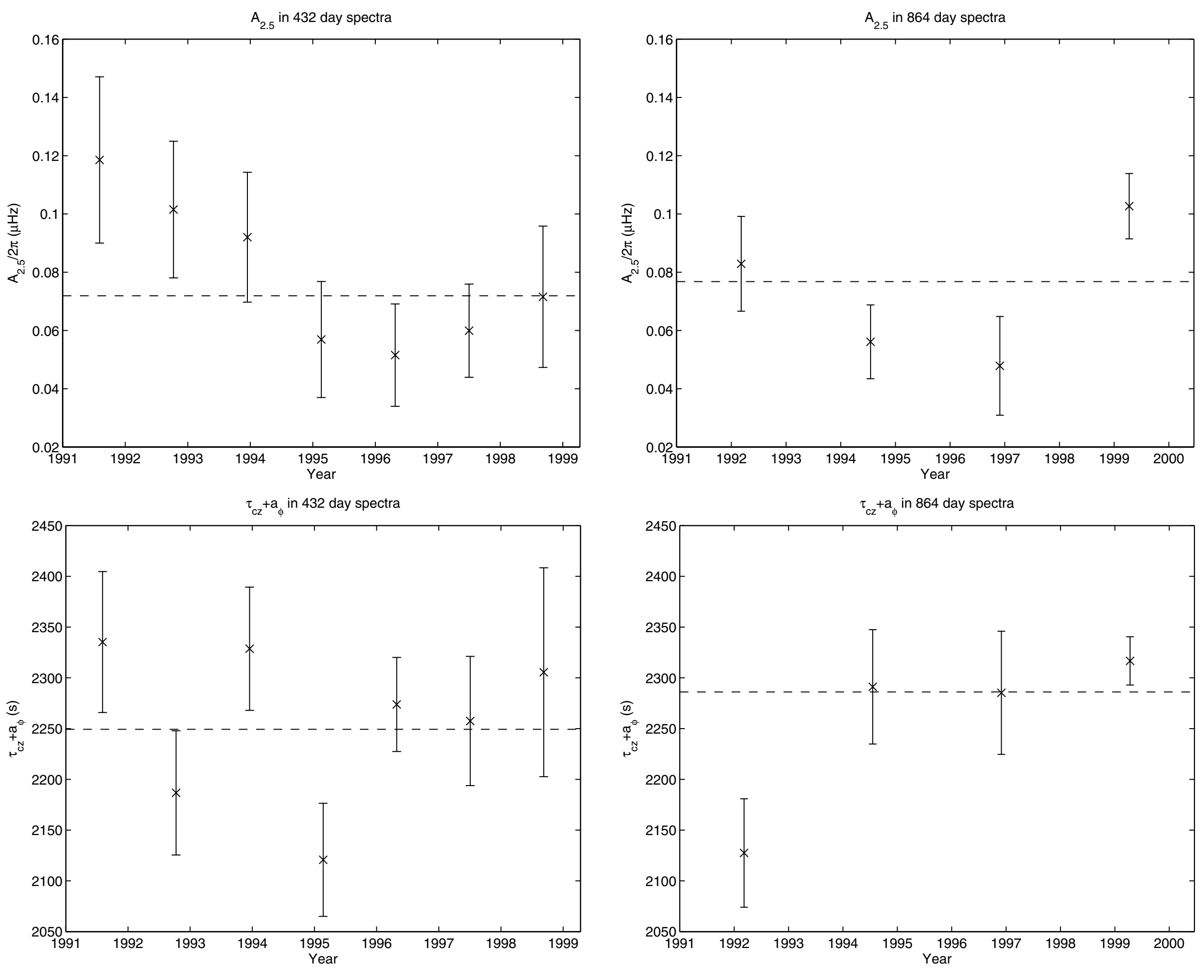

Figure 6. Fitted parameters for the convection zone base signature in 432 and 864-d BiSON spectra, with uncertainty estimates from simulations with normally distributed scatter. Top panels show the variation in $A_{2.5}$, the amplitude of the convection zone signal at 2500 μHz. Bottom panels show fitted values for the acoustic depth of the base of the convection zone plus the a_ϕ systematic offset. Dotted lines indicate the weighted mean value for each set of results.

Table 1. Weighted mean values (with weighted uncertainties) for key parameters in 432, 864 and 1728-d BiSON spectra; and model 'S' parameters from a single fit with uncertainties from Monte Carlo simulations with normally distributed scatter indicative of BiSON 864-d observational errors.

Data set	β (s)	τ_{he} (s)	$\delta_{\rm obs}$	$A_{2.5}/2\pi$ (μHz)	$\bar{\tau}_{cz}$ (s)
432-d BiSON	124.6 ± 1.3	651.9 ± 4.5	-0.032 ± 0.002	0.072 ± 0.008	2249 ± 23
864-d BiSON	121.3 ± 0.9	653.2 ± 3.9	-0.029 ± 0.002	0.077 ± 0.007	2286 ± 19
1728-d BiSON	122.4 ± 0.9	651.4 ± 7.0	-0.030 ± 0.002	0.058 ± 0.007	2232 ± 21
Model 'S'	122.5 ± 2.4	642.0 ± 9.8	-0.026 ± 0.003	0.084 ± 0.020	2300 ± 64

and the convection zone base. Although the results are of lower precision than from previous work using higher-degree data, these results show that the signatures can be extracted from unresolved measurements.

The results for the He II ionization zone show no significant variation or activity dependence in the magnitude or location of this region over the time-scale of the observations. At the level of precision of the data, there is no evidence that solar cycle dependent changes in the surface layers propagate down to more than 500-s acoustic depth ($\sim$0.986 $R_\odot$).

There is some variation in the amplitude and acoustic depth parameters in the convection zone base signature. To within the precision and temporal resolution of the extracted parameters for the convection zone base it is difficult to observe the same trend in the acoustic depth that was reported in Monteiro et al. (2001). Earlier work has used higher-degree data to obtain the convection zone signal, which has introduced a degree-dependent term into equation (2). This term can be neglected for sufficiently low-degree data in which the extracted signature displays no ℓ dependence. This is certainly true for $\ell \leqslant 3$, however the reduction in the number of modes leads to greater uncertainties in the extracted parameters.

The amplitude of the convection zone signal does show some variation that could be correlated with solar activity; however, the precision of the results limits the significance of such a claim. This will be further investigated with the analysis of 12 yr of BiSON observations with improvements to the time series generation and multi-station data combination techniques. The increase in the precision of the p-mode frequencies and the availability of data after the maximum of cycle 23 may confirm any cyclic behaviour at the base of the convective envelope.

The constant surface offset a_ϕ for model 'S' is found to be 199 $\pm$ 64, in good agreement with the values determined for the solar models in Monteiro et al. (1994). This implies a mean true acoustic depth for the base of the convection zone of 2059 $\pm$ 65, which agrees with the currently accepted value of $\sim$2100. The extracted parameters for the helium ionization zone are also consistent with expected values.

With current and forthcoming stellar seismological programs providing low-degree p-mode frequencies, these results confirm that it should be possible to extract information directly on the regions of rapid structural variation in Sun-like stars.

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