

in the effect. That the eleven-year period is not more conspicuous in weather records is perhaps surprising. If a sub-multiple of the period were consistently conspicuous, it would, in my opinion, be more surprising.

THE PERIOD OF SIMPLE VERTICAL OSCILLATIONS IN THE ATMOSPHERE.

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If the atmosphere is in stable equilibrium, any small element which is displaced in the vertical direction, upward or downward, is acted upon by forces of restitution which are proportional to the distance through which the element is displaced. The subsequent motion of the particle should therefore be a simple harmonic motion about its original position. The derivation of the period of this oscillation is as follows.

Let the height above the earth's surface be denoted by h , and let the absolute temperature at height h be denoted by T , so that T is a function of h . For simplicity, we shall assume that a unit mass of air, originally at height h and temperature T is displaced to a height $h + dh$ where the surrounding air at the same level has a temperature $T + dT$. The displaced air takes up a temperature $T + dT'$ at its new level. The motion being supposed adiabatic, $dT' = -\beta dh$ where β is equal to the adiabatic lapse rate.

In its displaced position the air is acted upon by the following forces:—

- (1) Its own weight,
- and (2) the force of buoyancy equal to the weight of air displaced by it at its new level.

The first of these is equal to g and acts downward. The second is equal to $g \frac{T + dT'}{T + dT}$ acting upward.

The resultant force acting upon the displaced mass is therefore $-g + g \frac{T + dT'}{T + dT}$ acting vertically upward. This may be written

$$g \left\{ -1 + 1 + \frac{dT' - dT}{T} \right\}$$

neglecting small quantities of the second order, or

$$\frac{g}{T} \left\{ \frac{dT'}{dh} - \frac{dT}{dh} \right\} dh,$$

$$\text{or } -\frac{g}{T} \left\{ \beta + \frac{dT}{dh} \right\} dh.$$

The condition that this resultant force shall tend to restore the displaced mass towards its original position is that $\beta + \frac{dT}{dh}$ shall be positive, or that $\beta > -\frac{dT}{dh}$. In other words, the lapse rate must be less than the adiabatic.

The resultant force on unit mass is equal to the acceleration produced, and may therefore be written in the form $\frac{d^2}{dt^2}(dh)$. We then have

$$\frac{d^2}{dt^2}(dh) + \frac{g}{T} \left(\beta + \frac{dT}{dh} \right) dh = 0.$$

With the proviso that $\beta + \frac{dT}{dh}$ is positive, this equation represents a simple harmonic motion about the original position, with a period of oscillation

$$2\pi \left\{ \frac{g}{T} \left(\beta + \frac{dT}{dh} \right) \right\}^{\frac{1}{2}} \text{ seconds.}$$

In an isothermal atmosphere, $\frac{dT}{dh} = 0$ and the period of oscillation is

$$\frac{2\pi}{\sqrt{g\beta/T}}. \text{ Taking } T \text{ as } 300^\circ, \text{ we find a period of oscillation of}$$

350 seconds, or rather less than 6 minutes. For inversions, the period of oscillation is less than this, and as the lapse rate increases from zero up to the adiabatic value, the period increases indefinitely from 6 minutes upward. It can be readily seen that if the lapse rate is $(1 - 1/n)$ times the adiabatic the period is $350\sqrt{n}$ seconds. Thus if the lapse rate is one half the adiabatic, the period is $350\sqrt{2}$ seconds, or just over 8 minutes. When the lapse rate is two thirds the adiabatic, the period is $350\sqrt{3}$ seconds, or almost exactly 10 minutes. When the lapse rate becomes equal to the adiabatic there is no oscillation and any displaced particle will stay where put. For lapse rates greater than the adiabatic, instability occurs, and the displacement increases exponentially.

If the air is saturated, β must be interpreted as the adiabatic lapse rate for saturated air, or roughly half its original value. The period of oscillation in an isothermal saturated atmosphere is then 8 minutes, it being assumed that raindrops do not fall out. This assumption is probably justified in the case of oscillations of such short period.

Thus the periods of oscillation which correspond to the most frequently observed lapse rates lie between 6 and 10 minutes. Periods of this duration are frequently observed on the microbarograph traces, and it is in the hope that the simple result given above may be of assistance to those investigating microbarograph traces, that the present note has been written.

Note added November 15, 1926.

The above discussion formed a portion of one of a series of lectures on Dynamical Meteorology delivered at the School of Meteorology during the years 1921-1924. Since this paper was sent to the Society my attention has been drawn by Dr. L. F. Richardson to a paper by Väisälä, *Soc. Scient. Fennica, Comm. Phys. Maths.*, 2, 19, 1925, p. 38, in which the same formula for the length of the period is derived.

DISCUSSION.

Mr. F. J. W. WHIPPLE pointed out that the author of the paper had not demonstrated that an element of the atmosphere could perform simple oscillations of the type that was suggested. One element could not move without displacing the surrounding air. The investigation of the motion presented a very difficult problem in hydrodynamics. The real virtue of the paper was that it indicated the order of magnitude of the period of possible oscillations. If the problem of a localised vertical oscillation could be solved, the difference between the actual lapse rate and the adiabatic lapse rate would appear in the solution. The period of oscillation would be some multiple of the period given in the paper and vary inversely as the square root of the difference in question. Mr. Whipple thought this was as far as it was legitimate to go.

In reply to Mr. Whipple, Mr. BRUNT writes:—I did not fail to realise that while the real problem of oscillations in the atmosphere is a hydrodynamical problem, the paper discusses an artificial problem in particle dynamics. I find it difficult, however, to believe that a solution by hydrodynamical methods can alter the form or magnitude of the solution very considerably, and this is borne out by the fact that my formula gives the right order of magnitude for the microbarograph periods. The paper is put forward as a useful basis for the investigation of microbarograph records, as it gives a provisional relationship between the period of oscillation and the lapse rate, and it is nowhere suggested that the simple oscillation which gives the period derived is itself the physical mechanism behind the periodic oscillations shown by the microbarograph.

The Soaring Flight of Birds.

A note published in a recent number of *Mitteilungen des Aero-nautischen Observatoriums Lindenberg* contains an account of an interesting observation of the soaring flight of a stork made during a kite-balloon ascent on May 2, 1926. The bird was kept under constant observation through a range-finder for no less than 45 minutes, during which time not the slightest motion of the wings could be detected. The readings of the range-finder show that during an interval of twenty minutes the height only varied between 502 m. and 518 m., the distance from the point of observation increasing from 1625 m. to 2000 m.

The kite ascent furnishes particulars of the conditions in the free atmosphere. In the lowest 120 metres the lapse rate was adiabatic and the wind velocity increased from 9 m/s. at the surface to 15 m/s. Above this surface layer there was an inversion about 300 m. thick, in which the temperature increased by 1°·8 C. and the wind velocity increased further to 22 m/s. Above that level the velocity fell off rapidly, and at 1750 metres it had the comparatively low value of 4 m/s. Instinctively the bird had selected the most favourable level for its flight, in this case the level of maximum wind velocity at the top of the inversion. Turbulence at the surface of separation suggests itself as the source of support.

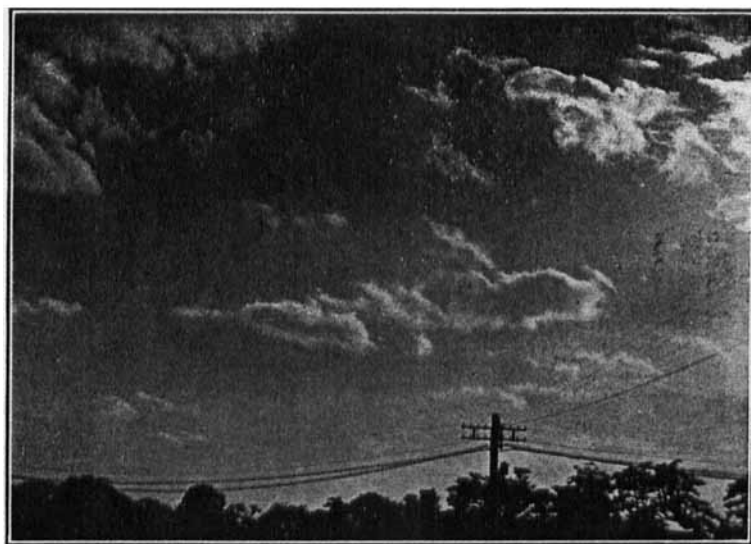
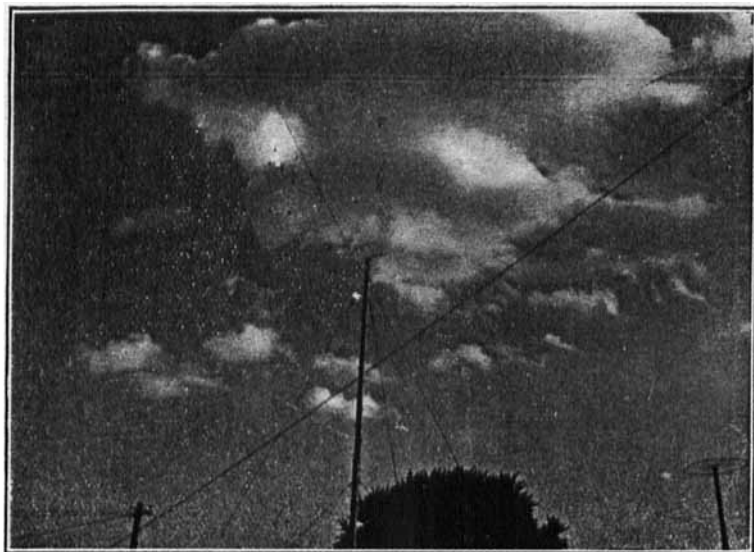


FIG. 1.—Clouds resembling animals. Above, crabs crawling; below, boats or rats.



FIG. 2.—Butterfly cloud (Tyō-Tyō-Gumo).

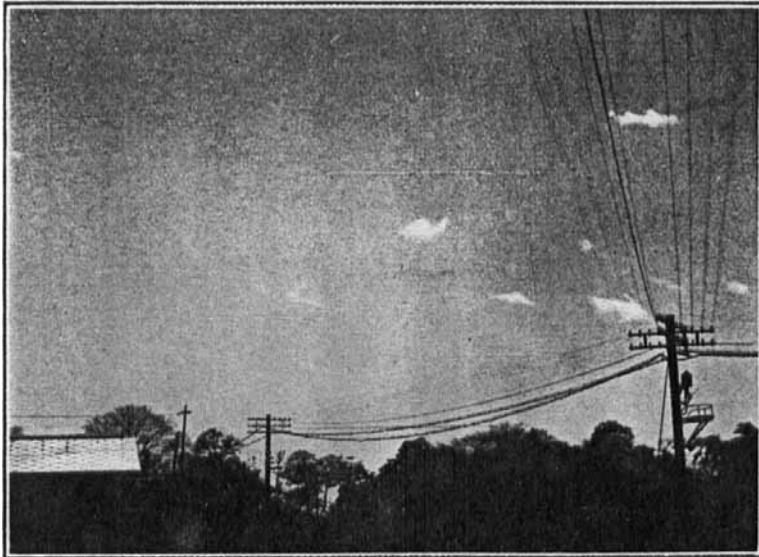


FIG 3.—Forerunner of a line squall. Drifting towards E., photographed facing S.

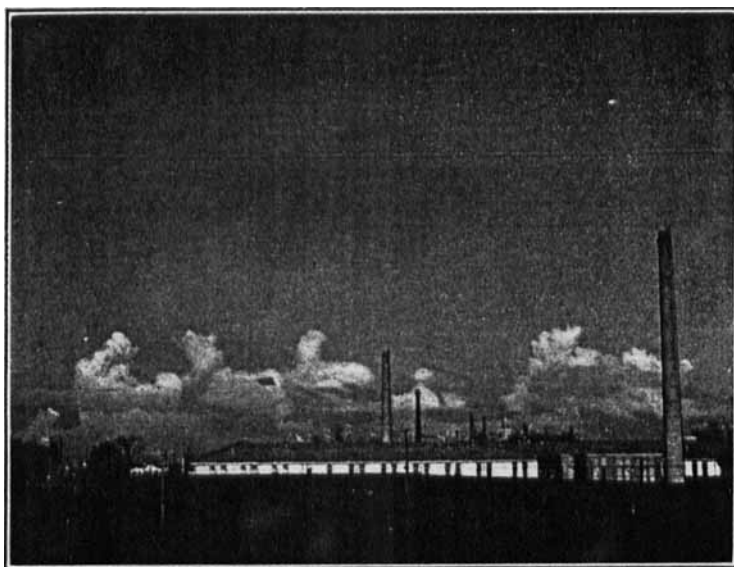


FIG. 4a.

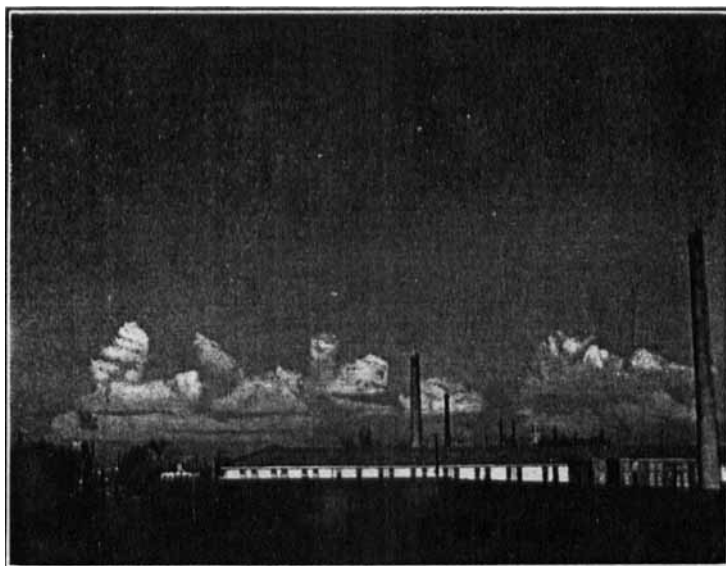


FIG. 4b.

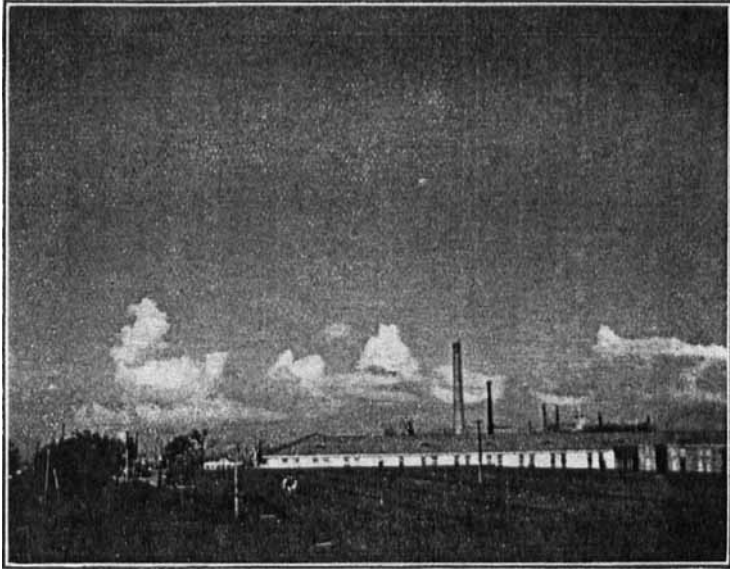


FIG. 4c.



FIG. 4d.

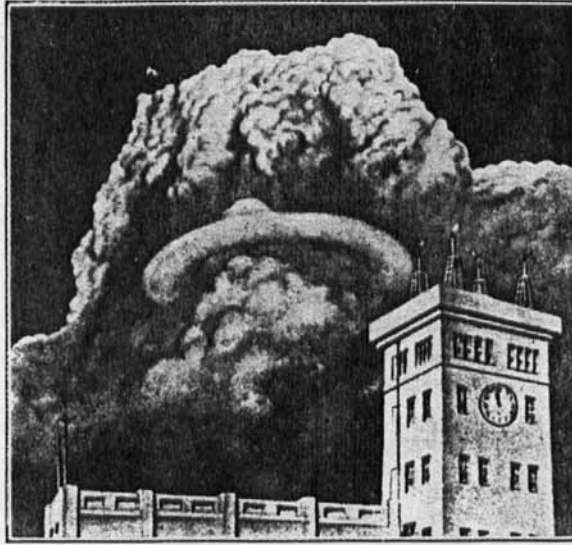
FIG. 4.—Screwing up of towering cumulus.



FIG. 5.—Screwing structure of cirrus.



FIG. 6.—Cumulo-Nimbus belonging to a "front" formation.



FIGS. 7 & 8.—Gigantic cumulus over Tokyo, September, 1923.