THE ASTROPHYSICAL JOURNAL, 306:L37–L40, 1986 July 1


# DETERMINATION OF STELLAR AGES FROM ASTEROSEISMOLOGY

ROGER K. ULRICH
Department of Astronomy, University of California, Los Angeles
*Received 1986 February 18; accepted 1986 March 27*

## ABSTRACT

This *Letter* shows that measurements of the stellar analog of the solar 5 minute oscillations can permit the determination of the radius and age of isolated stars. The key frequencies of oscillation correspond to pairs of modes differing by two in the degree of the spherical harmonic describing the angular dependence of the motion and by one in the overtone order of the modes. The frequency pairs are very nearly degenerate, and adequate frequency resolution will require a nearly unbroken time sequence extending over 15 days.

*Subject headings:* stars: evolution — stars: pulsation

## I. INTRODUCTION

Asteroseismology involves the study of stellar oscillations as a diagnostic tool. Christensen-Dalsgaard (1984) has used the number of simultaneously excited modes to distinguish between the study of pulsating stars like the Cepheids and the study of asteroseismology. The classical pulsating stars involve only a few modes—typically the fundamental radial mode and one or two radial overtones—whereas the Sun and presumably solar-type stars involve oscillations with a large number of modes excited simultaneously. The case of the Sun has provided the best opportunity for the seismic analysis of an astrophysical body partly because the Sun is very bright and partly because the surface is well resolved and the spatial structure of the oscillations can be determined. The oscillation modes in the Sun which are most prominent are those with periods near 5 minutes. These oscillations have been measured in integrated sunlight by several groups—Claverie *et al.* (1979, 1981), Grec, Fossat, and Pomerantz (1980), and Woodard and Hudson (1983*a*, *b*).

The quality and variety of information obtained from the solar data raises the question of whether similar data could be obtained for other stars. Indeed there are two reports of experiments currently in the literature wherein possible detections are discussed (Noyes *et al.* 1984; Gelly, Grec, and Fossat 1986). The measurement is exceedingly difficult because of the small amplitude of the motions—on the order of 50 cm $s^{-1}$. Christensen-Dalsgaard and Frandsen (1983) have estimated the amplitude of the oscillations based on the assumption that the motion is excited by convection. They estimate that the amplitudes should be up to about 10 times larger for stars of 1.5 $M_\odot$ than for the Sun. However, they also apply their method to the Sun and obtain an estimate which is below what is observed. Consequently, their assumed excitation mechanism may not be the only cause for the solar and stellar oscillations. In any case, if such oscillations are to be detected in more than a handful of stars, a major experimental effort will be required.

One of the most striking of the qualities of seismic analysis of the Sun or stars is the high precision possible in the determination of the frequencies. Christensen-Dalsgaard (1984) has pointed out that the frequencies are most sensitive to the radius of the star—a quantity which is usually not well known. A principal result from asteroseismic observations will then be in setting a very precise constraint on the radius of stars. Using this approach, Demarque, Guenther, and van Altena (1985) have shown that the preliminary frequency spacing for $\alpha$ Cen from Gelly, Grec, and Fossat (1986) is in serious disagreement with the observed stellar radius. In another case Guenther and Demarque (1985) have shown that the data for $\varepsilon$ Eri from Noyes *et al.* (1984) are in reasonably good agreement with the expected stellar parameters of mass, radius, and age. Inferences may be possible in areas other than radius such as the age of a star. Christensen-Dalsgaard (1984) has noted that some frequency differences depend on stellar age due to the conversion of hydrogen to helium in the stellar core and the attendant reduction in the speed of sound.

Each of the modes of oscillation observed in the integrated sunlight can be classified according to the degree $l$ of the spherical harmonic $Y_l^m(\theta, \phi)$ which describes the pattern of the motion over the stellar surface and the radial order $n$ which gives the number of nodes in the radial direction. I denote the frequencies by $\nu_{l,n}$. The frequencies pairs $(\nu_{0,n}, \nu_{2,n-1})$ and $(\nu_{1,n}, \nu_{3,n-1})$ are nearly degenerate. The frequencies differ for members of these pairs because the lower $l$ modes penetrate closer to the stellar center than do the higher $l$ modes and because the mode structures are very nearly identical outside the center. As long as the common effects of the outer layers can be calibrated or eliminated, the frequency differences between members of these pairs can provide a measure of the structure of the stellar center including the abundance gradient—an age indicator. One key property of the outer parts of the star is the sound travel time between center and surface. One of the most striking characteristics of the seismic spectrum—its regularly repeating pattern—is directly related to the travel time. I denote the frequency interval of this repeat pattern by:

$$\Delta\nu = (\nu_{l,n+1} - \nu_{l,n}).$$

In this *Letter* I show that the combinations of mode frequencies involving $(\nu_{0,n} - \nu_{2,n-1})/\Delta\nu$ and $(\nu_{1,n} - \nu_{3,n-1})/\Delta\nu$ can be used to measure stellar ages independent of such outer layer properties as the stellar radius. The principal observational requirement beyond the need for ultra-low noise velocity measurements is the need to make the measurements as nearly continuously as possible over a time base of 10–20 days.

## II. STELLAR MODEL CALCULATION

The calculations reported here are an extension of the methods used in previous studies. Two independent codes are used—an interior code which calculates the nuclear evolution of the model (Bahcall *et al.* 1982) and an "atmosphere" code which calculates the detailed structure of the model (Henyey, Vardya, and Bodenheimer 1965; Ulrich 1970; Ulrich and Rhodes 1977, 1983). The interior code follows the evolution of the nuclear abundances but uses a very large outer zone to step over the superadiabatic regions and other atmospheric complications. The "atmosphere" code has been extended to calculate the entire stellar model and treats the outer layers in detail but uses the abundances for each evolved model provided by the interior code. Both codes share the key physics subroutines which calculate or look up the nuclear energy generation rate and opacities. The equation of state for both codes includes the effects of the Coulomb corrections and scattering state collective effects; however, the interior program assumes complete ionization, while the atmosphere code treats the partial ionization of the principal elements up to oxygen explicitly. Both codes use a shooting integration method, but the interior code uses fourth-order Runge-Kutta and about 110 mesh points, whereas the atmosphere code uses a trapezoidal rule and about 400 mesh points. The atmosphere code integrates inward from a point in the corona to a radius of about 0.01 $R$, while the interior code integrates both ways to a matching point typically at 0.2 in the mass. The inner 0.01 of the radius in the atmosphere code is treated analytically.

The physical input of nuclear physics has been discussed by Bahcall *et al.* (1982). The present calculations have been improved relative to the procedures reported by Bahcall *et al.* (1982) through the use of a substantially finer grid of temperature and density in the opacity tables. The Los Alamos Opacity Library discussed by Huebner *et al.* (1977) was used to generate these tables. The density of the opacity grid is high enough that an interpolation scheme is adequate and I have used a simple linear logarithmic interpolation. The abundance for the table is that given by Ross and Aller (1976). Opacities for temperatures lower than $10^6$ K are taken from a subroutine written by Vardya (1964) which has the electron pressure and temperature as input parameters. The dominance of $H^-$ at low temperatures where radiative transport is important makes the electron pressure the appropriate input parameter. The gravitational and thermal energy is now included in the atmosphere code. Because these energy sources are very small on the main sequence, this improvement has almost no effect on the models or their derived frequencies.

Two types of model can be computed with the atmospheric code—solar models in which the values of $Y$ and $l/H$ are adjusted until the model matches the observed solar radius and luminosity and nonsolar models in which the radius and luminosity are adjusted until the central boundary conditions are met. In the models reported in this letter, the values of $Y$, $Z$, and $l/H$ have been held fixed at the values derived from the match to the Sun which are 0.2572, 0.0173, and 1.5669, respectively.

Three evolutionary sequences were computed for 0.8 $M_\odot$, 1.0 $M_\odot$, and 1.2 $M_\odot$ using the interior code. At three to six points along the evolutionary track the detailed interior structure was calculated using the atmosphere code. The integration for the models begins in the corona at about 5% above the stellar surface. The coronal structure is scaled from the solar temperature assuming that the temperature at each height above the photosphere is proportional to the effective temperature. In addition to these evolutionary sequences, three models near the Sun were computed with altered values of $Y$, $Z$, and $l/H$ in order to determine the sensitivity of the derived results to uncertainties in these parameters. Some characteristics of the models used for the frequency calculations are given in Table 1.

TABLE 1

CHARACTERISTICS OF THE STELLAR MODELS

| $t$ ($10^9$ yr) | $R/R_\odot$ | $L/L_\odot$ | $T_e$ (K) | $r_{ceb}/R$ | $T_c$ ($10^6$ K) | $X_c$ |
|---|---|---|---|---|---|---|
| | | | $M = 0.8\ M_\odot$ | | | |
| 0.5...... | 0.705 | 0.247 | 4854 | 0.689 | 11.3 | 0.708 |
| 10.0...... | 0.749 | 0.319 | 5021 | 0.690 | 12.5 | 0.402 |
| 20.0...... | 0.845 | 0.489 | 5244 | 0.670 | 15.3 | 0.072 |
| | | | $M = 1.0\ M_\odot$ | | | |
| 0.1...... | 0.886 | 0.722 | 5659 | 0.741 | 13.3 | 0.715 |
| 0.4...... | 0.892 | 0.736 | 5669 | 0.740 | 13.4 | 0.693 |
| 1.6...... | 0.916 | 0.798 | 5709 | 0.741 | 13.8 | 0.610 |
| 4.7...... | 1.002 | 1.012 | 5790 | 0.744 | 15.5 | 0.354 |
| 8.5...... | 1.199 | 1.453 | 5796 | 0.729 | 18.7 | 0.000 |
| | | | $M = 1.2\ M_\odot$ | | | |
| 0.05 .... | 1.140 | 1.770 | 6244 | 0.853 | 15.5 | 0.711 |
| 1.2...... | 1.224 | 2.088 | 6281 | 0.863 | 16.7 | 0.527 |
| 2.0...... | 1.295 | 2.338 | 6280 | 0.864 | 17.7 | 0.387 |
| 2.4...... | 1.352 | 2.477 | 6236 | 0.859 | 18.8 | 0.312 |
| 3.0...... | 1.446 | 2.697 | 6161 | 0.845 | 19.4 | 0.215 |

## III. MODEL FREQUENCIES

The frequencies were computed for each of the detailed stellar models using the procedures described by Ulrich and Rhodes (1983). The atmospheric boundary condition was applied at the top of the lower chromosphere just below the temperature rise to the 10,000 K region. For each model frequencies for modes with $0 < l < 3$ and $3 < n < 30$ were computed. Because individual modes can have their frequencies perturbed by a resonance in the chromosphere and because the analysis of observational data will undoubtedly involve the combination of frequencies in a summary form, I have fitted the frequencies for each value of $l$ to a polynomial of the form

$$\nu = b_0 + b_1(n - n_0) + b_2(n - n_0)^2 + b_3(n - n_0)^3,$$

where $n_0$ is an arbitrary zero point that I have taken as 18 for $l = 0$ and 1 and as 17 for $l = 2$ and 3. The range of $n$ was restricted to $10 < n < 25$ in order to permit the above formula to represent the frequencies accurately, and the resulting least-squares fit has an rms deviation of only 0.6 $\mu$Hz or less for all cases. This rms deviation is mostly a result of using a third-order polynomial to fit the frequencies and is not random. The pattern of deviation of the frequencies from the fitting line is similar for all the models so that the error in frequency splitting is much less than this rms deviation. Only the coefficients $b_0$ and $b_1$ behave in a regular enough manner to be easily used. A second-order fit provides a slightly poorer representation of the data and has $b_2 = 0.12$–$0.20$ $\mu$Hz. When the third-order fit is used the division of the representation of the curvature of the frequency pattern between $b_2$ and $b_3$ depends roughly on mass with the $b_3$ term being most important for the $M = 0.8\ M_\odot$ case and the $b_2$ term being most important for the $M = 1.2\ M_\odot$ case. There is also a slight trend for the $b_3$ term to become more important in the less evolved models. These higher terms will need further study to determine which if any can be used as diagnostics of interior structure.

Both $b_0$ and $b_1$ behave in ways that can be easily applied to diagnosing the interior state of a star. Since $b_0$ represents an averaged frequency at $n = n_0$, I will refer to this term as $\nu_l$. The choice of $n_0$ for $l = 0$–$3$ made in the previous paragraph selects those pairs of frequencies which are nearly degenerate. The frequency splitting of the nearly degenerate pairs contains information about the structure of the stellar core including the abundance gradient. Information about the sound travel time is contained in $b_1$ which now plays the role of $\Delta\nu$. The fundamental period of pulsating stars like the Cepheids is also a measure of the sound travel time so that we can expect the inverse of the frequency spacing to behave like the pulsation period. The dimensionless quantity $Q = \text{period} * (\bar{\rho}/\bar{\rho}_\odot)^{1/2}$, where $\bar{\rho}$ and $\bar{\rho}_\odot$ are the average stellar and solar densities, has a value ranging between 0.04 and 0.06 for the pulsating variables. The analogous quantity $(\bar{\rho}/\bar{\rho}_\odot)^{1/2}/\Delta\nu$ has a value close to 0.08 for the models studied in this *Letter*. In order to preserve ready correspondence with the observations, I plot in the top part of Figure 1 the quantity $\Delta\nu * (\bar{\rho}_\odot/\bar{\rho})^{1/2}$. This quantity is sufficiently slowly varying that $\bar{\rho}$ can be determined to better than 1% with a very crude guess for the stellar mass and age (the value of $\Delta\nu$ will have to be known to better than 0.1% before the degeneracies can be split). The age indicators $(\nu_0 - \nu_2)$ and $(\nu_1 - \nu_2)$ scale with the average stellar density in the same way as $\Delta\nu$ so that the density dependence can be removed by studying $\delta_{02} = (\nu_0 - \nu_2)/\Delta\nu$ and $\delta_{13} = (\nu_1 - \nu_3)/\Delta\nu$. These ratios are given in the lower parts of Figure 1. Also shown in Figure 1 as $\odot$ symbols are the observed values of the frequency spacings for the Sun plotted at the correct solar age. If the frequency splitting can be measured with an accuracy of 0.5 $\mu$Hz it should be possible to determine the age of a star with an accuracy of about $10^9$ yr$*(\tau/\tau_\odot)$.

The frequency splitting for solar models has been given in several previous studies—Christensen-Dalsgaard and Gough (1980), Shibahashi, Noels, and Gabriel (1983), and Ulrich and Rhodes (1983). The values for $\nu_0 - \nu_2$ from these three studies are, respectively, 10.1, 10.7 and 9.2 $\mu$Hz for a value of $Z$

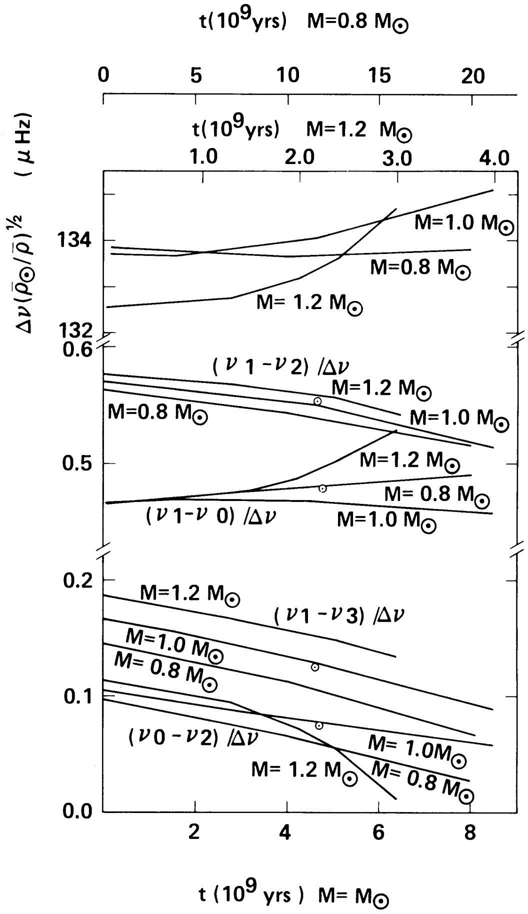


FIG. 1.—Frequencies and frequency splittings as functions of the time scaled according to $\tau/\tau_\odot$. The actual time scales are shown along the top and bottom of the figure for the models as labeled. The top part of the figure gives the repeat frequency interval $\Delta\nu$ for the three evolutionary sequences. These results show that the analog to the pulsating star constant $Q$ is also constant for the nonradial oscillations. The age sensitive splittings $(\nu_0 - \nu_2)/\Delta\nu$ and $(\nu_1 - \nu_3)/\Delta\nu$ are given in the bottom part of the figure. The middle section gives two other splittings which might be helpful in determining the mass of the star. The 1.2 $M_\odot$ model develops a convective core at $2 * 10^9$ yr.

near 0.02. This frequency difference for the calculations reported on in this *Letter* is 10.3 $\mu$Hz. The earlier calculations by Ulrich and Rhodes (1983) gave a smaller value because the central gradient of hydrogen was poorly represented and was too steep. The differences between calculated values of $\nu_0 - \nu_2$ from these separate investigations are small enough that if they represent the theoretical uncertainty in the calculations, then these uncertainties will not influence the determination of stellar ages.

The models are defined by a number of input parameters —$Y$, $\alpha = l/H$, $Z$, $\log M/M_\odot$ and $t * (\tau/\tau_\odot)$, where $\tau/\tau_\odot$ is the ratio of the nuclear time scale for the star to the nuclear time scale for the Sun. I use the time to reduce the value of $X_c$ to one-half the initial value as the measure of the nuclear time scale. The values of $\tau/\tau_\odot$ for the 0.8 $M_\odot$ and 1.2 $M_\odot$ models are 2.5 and 0.47, respectively. In order to represent the derivatives on a uniform basis, I multiply $Y$ by 100 and $Z$ by

TABLE 2

SENSITIVITIES OF DERIVED PROPERTIES—$\partial y/\partial x$

| $x$ | $y$ | | | | |
|---|---|---|---|---|---|
| | $\log T_e$ | $\log L/L_\odot$ | $\delta_{02}$ | $\delta_{13}$ |
| 100 $Y$ ......... | 0.0037 | 0.027 | 0.00095 | 0.0016 |
| 1000 $Z$ ......... | 0.0030 | −0.014 | −0.00093 | −0.0014 |
| $l/H$ ........... | 0.028 | 0.0073 | −0.0049 | −0.0097 |
| $\log M/M_\odot$ .... | 0.54 | 4.8 | 0.095 | 0.19 |
| $t*(\tau/\tau_\odot)$....... | 0.0020 | 0.034 | −0.0042 | −0.0076 |

1000 so that the likely changes in both these parameters are of order unity. I denote the input parameters by $x$. We wish to derive as output the quantities $\log T_e, \log L, \delta_{02} = (\nu_0 - \nu_2)/\Delta\nu$, and $\delta_{13} = (\nu_1 - \nu_3)/\Delta\nu$. I denote the output parameters by $y$. We will in general wish to derive the input parameters by combining observations of the output parameters. Table 2 gives the matrix of $\partial y/\partial x$ for $x$ and $y$. The quantities are all dimensionless except for $t$ which is in units of $10^9$ yr. The output quantities $y$ are given along the top of the table, and the input parameters are given in the left-hand column of the table.

Observations adequate to permit the measurement of $\nu_0 - \nu_2$ will also permit a very accurate measurement of $\Delta\nu$. As noted above $\bar{\rho}$ can then be inferred with good accuracy. Consequently, the only changes in $\alpha$ and $Y$ permitted will be those subject to the constraint that $\bar{\rho}$ remains constant. Thus a key quantity is the sensitivity of $\delta_{02}$ to $\alpha$ when $Y$ is adjusted so that $\bar{\rho}$ is held constant. This partial derivative is $(\partial\delta_{02}/\partial\alpha)_{\bar{\rho}} = 0.003$. Since changes in $\delta_{02}$ resulting from evolution are in the range 0.02, uncertainties in $l/H$ should not compromise our ability to infer stellar ages.

## IV. DISCUSSION

The calculations reported on in this *Letter* show that it may be possible to determine the nuclear ages of stars from combinations of frequency splittings that should be measurable if stellar oscillations like the solar 5 minute oscillations can be detected. If such measurements can be made on relatively faint stars then several new types of study will become possible:

1. The sequence of star formation during the early stages of the formation of our Galaxy could be studied by correlating stellar kinematic properties with age.
2. Measurements on Population II stars could set an interesting lower bound on the age of the universe.
3. Tests of stellar models based on the comparison to cluster color-magnitude diagrams can be tightened by eliminating age as a free parameter.
4. The temperature scale for stellar spectrophotometry can be improved through the determination of precise stellar radii.
5. The helium abundance can be estimated for stars of known distance and surface temperature.

These ideas and applications are meant to be only representative of the possible applications of asteroseismology. The primary issue at present is that of the limiting magnitude of the experimental method. The present detection methods use only a very restricted spectral range and have a limiting magnitude of about 2. No system using a broad spectral range is currently in use, and stability of any broad-band system will be a key issue. The conclusions that could be drawn from a successful series of measurements would appear to justify a significant effort to develop appropriate instrumentation.

This research was supported in part by NASA grant NAGW 472 at UCLA.

## REFERENCES

Bahcall, J. N., Huebner, W. F., Lubow, S. H., Parker, P. D., and Ulrich, R. K. 1982, *Rev. Mod. Phys.*, **54**, 762.
Christensen-Dalsgaard, J. 1984, in *Proc. of the Workshop on Space Research Prospects in Stellar Activity and Variability*, ed. A. Mangeney and F. Praderie (Meudon: Observatory de Paris-Meudon), p. 11.
Christensen-Dalsgaard, J., and Frandsen, S. 1983, *Solar Phys.*, **82**, 469.
Christensen-Dalsgaard, J., and Gough, D. 1980, *Nature*, **288**, 544.
Claverie, A., Isaak, G. R., McLeod, C. P., van der Raay, H. B., and Roca Cortes, T. 1979, *Nature*, **282**, 591.
Claverie, A., Isaak, G. R., McLeod, C. P., and van der Raay, H. B. 1981, *Nature*, **293**, 443.
Demarque, P., Guenther, D. B., and van Altena, W. F. 1985, in *Proc. of the Workshop on the Seismology of the Sun and Other Stars*, ed. D. Gough (Cambridge: Cambridge University Press), in press.
Gelly, B., Grec, G., and Fossat, E. 1986, *Astr. Ap.*, in press.
Grec, G., Fossat, E., and Pomerantz, M. 1980, *Nature*, **288**, 541.
Guenther, D. B., and Demarque, P. 1985, in *Proc. of the Workshop on the Seismology of the Sun and Other Stars*, ed. D. Gough (Cambridge: Cambridge University Press), in press.
Henyey, L., Vardya, M. S., and Bodenheimer, P. 1965, *Ap. J.*, **142**, 841.
Huebner, W. F., Merts, A. L., Magee, N. H., and Argo, M. 1977, Los Alamos Scientific Laboratory Report LA-6760-M.
Noyes, R. W., Baliunas, S. L., Belserene, E., Duncan, D. K., Horne, J., and Widrow, L. 1984, *Ap. J. (Letters)*, **285**, L23.
Ross, J. E., and Aller, L. H. 1976, *Science*, **191**, 1223.
Shibahashi, H., Noels, A., and Gabriel, M. 1983, *Astr. Ap.*, **123**, 283.
Ulrich, R. K. 1970, *Ap. Space Sci.*, **7**, 183.
Ulrich, R. K., and Rhodes, E. J., Jr. 1977, *Ap. J.*, **218**, 521.
______. 1983, *Ap. J.*, **265**, 551.
Vardya, M. S. 1964, *Ap. J. Suppl.*, **8**, 277.
Woodard, M., and Hudson, H. S. 1983*a*, *Solar Phys.*, **82**, 67.
______. 1983*b*, *Nature*, **305**, 589.

ROGER K. ULRICH: Department of Astronomy, University of California, Los Angeles, CA 90024