Stellar model fits and inversions

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The recent asteroseismic data from the CoRoT and *Kepler* missions have provided an entirely new basis for investigating stellar properties. This has led to a rapid development in techniques for analysing such data, although it is probably fair to say that we are still far from having the tools required for the full use of the potential of the observations. Here I provide a brief overview of some of the issues related to the interpretation of asteroseismic data.

1 Introduction

The goals of the asteroseismic analysis are evidently to improve our knowledge about stellar properties and our understanding of stellar structure and evolution. More specifically, simple model fits, or even direct use of scaling relations, provide estimates of the masses and radii of stars, while more detailed analyses, including also other observations of the stars, yield more precise values of mass and radius as well as an estimate of the stellar age. Such determinations clearly have substantial broad interest, not least for stars known to be hosts of planetary systems. However, from the point of view of stellar astrophysics we clearly wish more information, and indeed we can go much further from the data now becoming available, not least from the CoRoT and *Kepler* missions (Michel et al. 2008; Baglin et al. 2009; Borucki et al. 2010; Gilliland et al. 2010; Koch et al. 2010). Recent or upcoming highlights are the inferences on the core structure and rotation of red-giant stars (e.g., Bedding et al. 2011; Beck et al. 2012; Mosser et al. 2012a,b) and the detection of effects in solar-like oscillations of acoustic glitches in main-sequence stars (Mazumdar et al. 2012a,b). Fortunately, both missions have now been extended until 2016.

Much of the data fitting or other form of analysis is done within the framework of stellar models characterized by a specific set of parameters. A crucial goal of asteroseismology is to test whether such models adequately represent real stars and, if not, how the model physics should be improved. Inadequate modelling should be reflected in a failure of the models to fit the observed stellar properties, particularly the frequencies, to within their observational errors, for any choice of parameters characterizing the model. At this point one then has to determine how the models should be further

improved, perhaps through inverse analyses to try to localize the cause of the discrepancy. It is evident that the identification of such significant departures requires great care in the statistical analysis of the data, and it is probably fair to say that we have so far not quite reached this point.

In this brief review I concentrate on the determination of stellar structure based on solar-like oscillations, with main emphasis on stars on or near the main sequence. However, it should be noted in passing that striking results have also been obtained based on CoRoT and *Kepler* data for more massive stars showing heat-engine driven pulsations (Degroote et al. 2010) and for the subdwarf B stars (e.g., Van Grootel et al. 2010). Also, extensive ground-based campaigns have yielded fundamental results for white dwarfs (e.g., Metcalfe et al. 2004). Thus asteroseismic investigations are covering a very broad range of stellar properties and stellar evolutionary stages.

2 Properties of stellar oscillations

As a background for the discussion, I present a brief overview of the relevant properties of stellar oscillations. Much more detailed descriptions were provided, e.g., by Christensen-Dalsgaard (2004) and Aerts et al. (2010).

2.1 Basic properties

We consider only oscillations that are solar-like, in the sense that they are intrinsically damped and excited by the acoustic noise generated by near-surface convection, which reaches near-sonic speeds (e.g., Goldreich & Keeley 1977; Houdek et al. 1999). This mechanism depends on the near-surface properties of the oscillations and the time-dependence of the most vigorous convection (Kjeldsen & Bedding 2011), making it most efficient at frequencies approaching the acoustic cut-off frequency in the

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stellar atmosphere, approximately given, in terms of cyclic frequency, by

$$\nu_{ac} \simeq \frac{c_{phot}}{4\pi H_p} \propto \frac{GM}{R^2 T_{eff}^{1/2}} \,, \tag{1}$$

where c_{phot} and H_p are the sound speed and pressure scale height at the stellar photosphere. The second expression assumes the ideal gas law and that the atmospheric temperature scales like the effective temperature T_{eff}; here G is the gravitational constant, and M and R are the mass and radius of the star.

In relatively unevolved stars, on the main sequence, the modes in the frequency range of efficient driving are acoustic modes, with cyclic frequencies that are asymptotically given by

$$\nu_{nl} \simeq \Delta\nu(n + l/2 + \epsilon) - \delta_{0l} \tag{2}$$

(Vandakurov et al. 1967; Tassoul 1980; Gough 1986), where n and l are the radial order and spherical harmonic degree of the mode. Here $\Delta\nu$ is approximately the inverse acoustic diameter of the star,

$$\Delta\nu = \left(2 \int_0^R \frac{dr}{c} \right)^{-1} \,, \tag{3}$$

where c is the adiabatic sound speed and r is the distance to the centre of the star. As expected from homology arguments this scales approximately as the square root of the mean density of the star,

$$\Delta\nu \propto \left(\frac{M}{R^3} \right)^{1/2} \,. \tag{4}$$

Also, ϵ is a phase depending on the near-surface properties of the mode and δ_{0l} is a small correction that in main-sequence stars is sensitive to the central regions of the star.

In terms of the observed frequencies the asymptotic properties in Eq. (2) are often utilized by considering the large frequency separation

$$\Delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l} \,, \tag{5}$$

and the small frequency separations

$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l+2} \,. \tag{6}$$

Here $\Delta\nu_{nl}$ approximates $\Delta\nu$, although with some variations with frequency that largely arise from the near-surface acoustic glitches (see Section 3.3); $\delta\nu_{nl}$, as δ_{0l}, provides a measure of conditions in the core of the star and hence, for main-sequence stars, the stellar age.

The amplitudes of the modes are determined by a balance between the energy input from the stochastic excitation and the damping. This leads to a bell-shaped distribution of amplitudes with frequency (Goldreich et al. 1994) with a typically relatively well-defined maximum at a frequency ν_{max}. There is observational evidence that ν_{max} scales as ν_{ac}, i.e.,

$$\nu_{max} \propto \frac{GM}{R^2 T_{eff}^{1/2}} \tag{7}$$

(e.g., Brown et al. 1991; Bedding & Kjeldsen 2003; Stello et al. 2008), with some although perhaps not full theoretical justification in terms of the near-surface properties of the modes (Belkacem et al. 2011).

The large and small frequency separations and ν_{max} form the basis for ensemble studies of large samples of stars and hence are typically determined by automated pipeline analysis methods. A careful comparison of such techniques was provided by Verner et al. (2011a).

As the star evolves beyond the main sequence, the core contracts strongly and hence the internal gravitational acceleration becomes very large. Furthermore, hydrogen burning leaves behind strong gradients in the chemical composition. These effects lead to a marked increase in the so-called buoyancy frequency N, which for an ideal gas can be approximated by

$$N^2 \simeq \frac{g^2\rho}{p}(\nabla_{ad} - \nabla + \nabla_\mu) \,, \tag{8}$$

where p is pressure, ρ density, g is the local gravitational acceleration, $\nabla = d\ln T / d\ln p$, T being temperature, ∇_{ad} is the adiabatic value of ∇ and $\nabla_\mu = d\ln\mu/d\ln p$, where μ is the mean molecular weight. N is the characteristic frequency for internal gravity waves, and when N is large the star has *mixed modes*, which behave like acoustic modes in the outer parts of the star and as gravity modes in the deep interior. For sufficiently high N, found in sub-giant and red-giant stars, such mixed modes have frequencies as high as, or exceeding, ν_{ac} and hence are excited efficiently by the stochastic near-surface processes (e.g., Dupret et al. 2009). Given the dependence of N on the structure of the deep interior of the star, including the composition profile, these modes are powerful diagnostics of the stellar interior.

High-order asymptotic g modes are uniformly spaced in period Π_{nl}, which asymptotically satisfies

$$\Pi_{nl} \simeq \frac{\Pi_0}{\sqrt{l(l+1)}}(n + \epsilon_g) \,, \tag{9}$$

where

$$\Pi_0 = 2\pi^2 \left(\int N\frac{dr}{r} \right)^{-1} \,, \tag{10}$$

the integral being over the region where the buoyancy frequency exceeds the oscillation frequency, and ϵ_g is a phase. For red giants this leads to a very dense spectrum of high-order g modes interacting with the acoustic modes in the region of stochastic excitation. For modes with $l = 1$ several such mixed modes are typically observed in the vicinity of each acoustic mode. This allows a determination of the corresponding period spacing $\Delta\Pi_1 = \Pi_0/\sqrt{2}$ which, according to Eq. (10), is a diagnostic of the deep stellar interior where N is large (Beck et al. 2011; Bedding et al. 2011; Mosser et al. 2011).

In a spherically symmetric star the frequencies do not depend on the azimuthal order m of the spherical harmonic. Rotation lifts this degeneracy, giving rise to a rotational splitting which depends on the internal rotation of the star,

weighted by the eigenfunction of the mode. Observations of the rotational splitting in the Sun have provided detailed information about the solar internal rotation (for a review, see Thompson et al. 2003), and *Kepler* has provided striking observations of the splitting of solar-like oscillations in evolved stars (e.g., Beck et al. 2012; Deheuvels et al. 2012; Mosser et al. 2012b). However, here I ignore rotation and concentrate on the determination of stellar properties based on the average multiplet frequencies ν_{nl}.

2.2 Near-surface problems

Modelling of near-surface layers of a solar-like star is complicated by the presence of convection. The temperature structure of the model is determined by the treatment of convective energy transport, often using the mixing-length formulation (Böhm-Vitense 1958), and the models normally ignore the dynamical effects of convection, in the so-called turbulent pressure. Also, the frequency calculations are most often carried out in the adiabatic approximation, even though the oscillations are strongly nonadiabatic in the superficial layers. However, the treatment of the nonadiabatic effects needs a description of the interaction between convection and pulsations, through the perturbation to the convective flux; also, the perturbation to the turbulent pressure may affect both the frequencies and the stability of the modes. An overview of these complications was given by Houdek (2010). It should be noticed that the uncertain aspects of the structure and oscillation modelling are all concentrated very near the surface. Thus they have little effect on modes of low frequency which are evanescent in this region, with a very low amplitude compared with the amplitude in the bulk of the star.

These near-surface problems dominate the differences between observed solar oscillation frequencies and frequencies of solar models (Christensen-Dalsgaard et al. 1996). In this case it is possible to separate the near-surface effects from the differences between the Sun and the model in the deeper parts of the star. A convenient way to do so is to use the differential form of the Duvall asymptotic expression for the frequencies (Christensen-Dalsgaard et al. 1989), according to which the frequency differences $\delta\nu$ between the Sun and a model satisfy

$$S_{nl} \frac{\delta\nu_{nl}}{\nu_{nl}} \simeq \mathcal{H}_1(\nu_{nl}/L) + \mathcal{H}_2(\nu_{nl}) , \qquad (11)$$

where $L = \sqrt{l(l+1)}$ and S_{nl} is a scale factor which can be calculated from the model, and which may be chosen to be close to 1 for low-degree modes. The term in $\mathcal{H}_1$ depends on the location of the inner turning point of the mode, determined by ν/L, and reflects the difference in sound speed between the Sun and the model throughout the star, whereas $\mathcal{H}_2$ contains the contribution from the near-surface region. In accordance with the above comment the arbitrary additive constant in the definition of $\mathcal{H}_2$ may be chosen such that $\mathcal{H}_2$ is zero at low frequency. In Fig. 1 the solid curve shows $\nu\mathcal{H}_2(\nu)$ determined in this manner, corresponding to

the frequency shift $\delta\nu$ caused by the surface effects in the Sun.

In principle, these effects must be taken into account in any analysis of frequencies of solar-like oscillations. They affect not only the individual frequencies but also frequency combinations such as the large and small frequency separations (cf. Eqs 5 and 6). Roxburgh & Vorontsov (2003a) pointed out that separation ratios, such as

$$r_{02} = \frac{\nu_{n0} - \nu_{n-1\,2}}{\nu_{n1} - \nu_{n-1\,1}} , \qquad (12)$$

can be constructed which are essentially independent of the near-surface problems (see also Otí Floranes et al. 2005; Roxburgh 2005). These combinations therefore provide clean diagnostics of the core properties of the stars.

To fit individual frequencies it is necessary to correct for the near-surface effects. For distant stars, where only low-degree modes are observed, an unconstrained determination of the frequency corrections cannot be carried out. Kjeldsen et al. (2008) suggested to base the correction on the solar results, fitting the observed frequencies to

$$\nu_{nl} = \nu_{nl}^{(\text{best})} + a \left(\frac{\nu_{nl}}{\nu_0} \right)^b , \qquad (13)$$

where $\nu_{nl}^{(\text{best})} = r\nu_{nl}^{(\text{ref})}$ are the frequencies of the best-fitting model, taking into account the near-surface correction, and $\nu_{nl}^{(\text{ref})}$ are frequencies of a computed reference model, assumed to be close to the best model. The scaling with the constant factor r assumes that the frequencies of the models are related by homology. In Eq. (13) the exponent b is obtained from fits to solar frequencies, and ν_0 is typically taken to be an average of the observed frequencies, which is generally close to ν_{max}. Kjeldsen et al. provide a procedure for determining the amplitude a of the correction and the frequency scale factor, by fitting the mean large frequency separation and average frequency for radial modes.

Mathur et al. (2012) carried out detailed fits to a number of stars observed by *Kepler*, including the Kjeldsen et al. (2008) correction. They found that the amplitudes a roughly scaled as $\Delta\nu$. According to Eq. (2) this corresponds to a roughly constant average shift in the phase ϵ. A similar shift was found by White et al. (2011a) in a comparison of fitted values of ϵ from observed and model frequencies. This suggests that a more detailed analysis of the phase, including how it is affected by the near-surface layers, may provide a deeper insight into the properties of the frequency correction.

Although the power-law correction assumed in Eq. (13) has some theoretical support in an analysis of simple near-surface modifications (Christensen-Dalsgaard & Gough 1980) it is clearly only an approximation to the frequency differences obtained for the Sun (see Fig. 1). An alternative is to base the correction more closely on the functional form obtained in the solar case. It is perhaps not unreasonable to assume that the relevant frequency scale is the acoustic cut-off frequency ν_{ac} and hence to replace Eq. (13) by

$$\nu_{nl} = \nu_{nl}^{(\text{best})} + \tilde{a}\mathcal{G}_\odot(\nu_{nl}/\nu_{\text{ac}}) , \qquad (14)$$
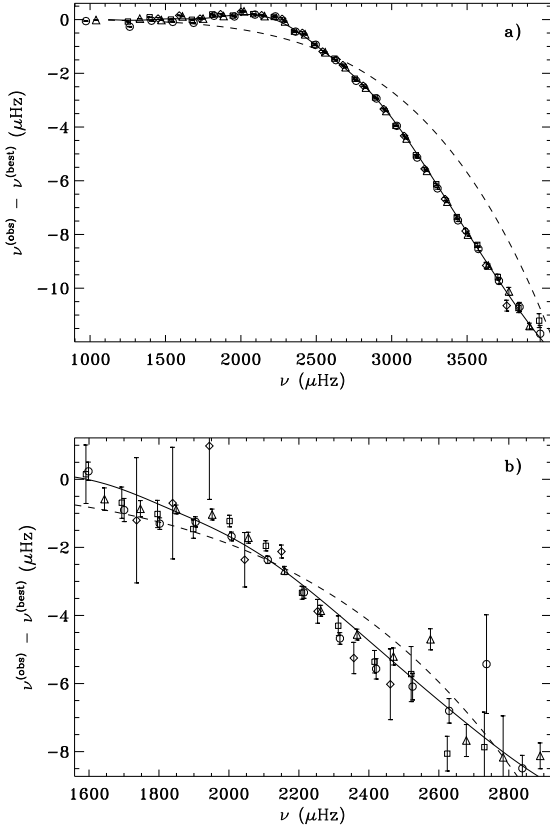
Fig. 1 In panel a) the solid line shows the surface contribution to the differences between the solar and model frequencies, inferred from analysis of modes over a broad range in degree. The symbols show the result of fitting this function to BiSON frequencies (see Chaplin et al. 2002) of degree $l = 0-3$. For comparison, the dashed line is the corresponding fit of a power law (cf. Eq. 13) with exponent $b = 4.9$ In panel b) the symbols show the the differences between observed frequencies of 16 Cyg A (Metcalfe et al. 2012) and frequencies of a fitted model, without the surface correction that was used in the fit. That surface correction is shown by the dashed line, while the solid line shows the scaled solar surface contribution from panel a), assumed to be a function of the frequency in units of the acoustic cut-off frequency.

where the function $\mathcal{G}_\odot$ is determined from $\nu\mathcal{H}_2(\nu)$, illustrated in Fig. 1, as obtained from fitting solar frequencies over a broad range of degrees. The scale factor $\tilde{a}$ and the frequency scale r can be determined from a least-squares fit to the differences between the observed and the reference-model frequencies. In Fig. 1 the symbols illustrate the result of applying this procedure to observed solar frequencies of degree $l = 0-3$, corresponding to what can be observed in distant stars, and using the same reference model as was used to determine $\mathcal{G}_\odot$. Here the scale factor was extremely close to 1. For comparison, the dashed curve shows the result of applying the Kjeldsen et al. (2008) procedure to the

same set of frequencies, with the same reference model. It is evident that, while the power law reproduces the general rapid increase of the effect with frequency, the detailed fit to the extensive and very accurate solar frequencies is somewhat questionable.

As a more realistic illustration of the fit to the surface effect Fig. 1b shows results for 16 Cyg A, based on the *Kepler* observations presented by Metcalfe et al. (2012). I made a fit of the observed frequencies to stellar models, using the Kjeldsen et al. (2008) surface-term procedure, and subsequently determined the best-fitting surface correction according to Eq. (14). The dashed and solid lines show the power-law surface term and the term corresponding to Eq. (14), respectively, while the symbols show the differences between the stellar and (uncorrected) model frequencies. Here the advantage of using the solar fit is less dramatic, but certainly still significant. Given that 16 Cyg A is quite similar to the Sun (with $M = 1.1 M_\odot$ and $T_{\mathrm{eff}} = 5825\,\mathrm{K}$) that is probably not surprising. However, it is interesting that data are now becoming available of sufficient quality to test more detailed properties of the surface effects in stars other than the Sun. Also, the results suggest that there might be an advantage in using the solar surface functional form in fits to observed frequencies, rather than the power law, at least for stars in the vicinity of the Sun. In any case a comparison of the results of using these two representations might provide some measure of the systematic error introduced in the fit by the use of the surface correction.

It is obvious that the assumption underlying these near-surface corrections, namely that they resemble the solar correction, is in general questionable, increasingly so for stars with properties further from those of the Sun. In fact Doğan et al. (2010) were unable to find a solar-like correction for the F5 star Procyon A. Thus some care is required in applying the correction, and further tests are required to investigate whether it might lead to systematic errors in the outcome of the fits to the observed frequencies. It is evident that the broad range of stars observed with *Kepler* provides an excellent basis for such tests. In the longer run the goal must obviously be to improve the modelling of the near-surface layers. In the solar case some improvement has resulted from the use of detailed hydrodynamical models of the structure of the outermost layers (e.g., Rosenthal et al. 1999), while the success of using time-dependent convection modelling in the frequency calculations has so far been somewhat questionable (Houdek 2010; Grigahcène et al. 2012).

3 Fitting stellar models

3.1 Using scaling relations

The simplest analysis of asteroseismic data is based on the overall properties of the oscillations, typically the large separation $\Delta\nu$ and the frequency ν_{max} of maximum power. Us-

ing the scaling relations, Eqs (4) and (7), normalizing to solar values $\Delta\nu_\odot$ and $\nu_{\max,\odot}$, we immediately obtain

$$\frac{R}{R_\odot} = \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right)\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-2}\left(\frac{T_{\text{eff}}}{T_{\text{eff},\odot}}\right)^{1/2} \quad (15)$$

$$\frac{M}{M_\odot} = \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right)^3\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-4}\left(\frac{T_{\text{eff}}}{T_{\text{eff},\odot}}\right)^{3/2} \quad (16)$$

(Kallinger et al. 2010a; Mosser et al. 2010). Assuming that T_{eff} has been determined, this provides values of R and M, independently of any models. However, it should be noticed that $\Delta\nu$ and $\nu_{\max}$ appear to quite high powers, particularly in the expression for the mass, and hence the results are sensitive to errors in the observed quantities.

It is obvious that such a determination of the stellar mass and radius depends on the validity of the assumed scalings. Since the homologous relation between the relevant stars is not exact, we may expect departures from the scaling of $\Delta\nu$. This was investigated by White et al. (2011b), for stars on the main sequence and the ascending red-giant branch. They found a variation of a few per cent which was mainly a function of effective temperature; since a significant departure from homology is caused by the varying depth of the convective envelope, which is largely determined by the effective temperature, such a dependence is not unexpected. Miglio et al. (2012) found a difference of about 3 per cent between $\Delta\nu$ in models of ascending red-giant stars and helium burning clump stars at given mean density, related to the differences in their internal structure. It should also be kept in mind that the near-surface problems discussed in Section 2.2 have an effect on $\Delta\nu$, given their strong frequency dependence. This may cause systematic errors in the results of the fit.

The scaling of $\nu_{\max}$, Eq. (7), has no fully secure theoretical basis, since it is not yet possible to make reliable predictions of the amplitudes of stochastically excited modes or their dependence on frequency. However, as mentioned above, the relation has substantial observational support. A crucial test of this so-called *direct technique* for analysing the asteroseismic data, or other similar asteroseismic techniques to determine the stellar radius, is evidently comparison to independent determinations from the stellar angular diameter and distance. Such tests appear to confirm the validity of the scaling relations (Huber et al. 2012; Silva Aguirre et al. 2012). A similar test of mass determinations for individual stars has apparently not been possible. However, application to red-giant stars in open clusters provides a test that the resulting masses span the expected small interval (Basu et al. 2011; Miglio et al. 2012).

A more strongly constrained, and hence potentially more precise, determination of stellar parameters can be obtained by fitting the observed asteroseismic quantities, together with the effective temperature and composition, to grids of stellar models. The determination of $\Delta\nu$ and $\nu_{\max}$ for the models in the grid can be based either on the scaling relations or, in the case of $\Delta\nu$, on computed frequencies for the models (e.g., Stello et al. 2009; Basu et al. 2010).

The accuracy of the results is clearly sensitive to the stellar models, and hence it is important to test the effect on the fits of using grids computed with different evolution codes (Gai et al. 2011; Basu et al. 2012). The grid-based fitting was extended to include also the small frequency separations by Quirion et al. (2010) in the so-called SEEK algorithm, which also included a statistical analysis of the results.

These techniques provide a fast and potentially automated way to obtain basic stellar properties from the asteroseismic data, and hence they are well suited to the analysis of large samples of stars, in what has been called *ensemble asteroseismology*[1]. This has been extremely valuable for stellar population studies for main-sequence stars (e.g., Chaplin et al. 2011; Verner et al. 2011b) and red giants (Kallinger et al. 2010b; Mosser et al. 2010; Hekker et al. 2011a,b; Miglio et al. 2012).

3.2 Fits to individual frequencies

The basic asteroseismic quantities $\Delta\nu$ and $\nu_{\max}$ can be determined from data with even rather poor signal-to-noise levels and, as discussed above, may provide a determination of stellar global properties. However, with the long timeseries obtained from CoRoT and *Kepler* it is often possible to determine individual frequencies ν_{nl} (e.g., Appourchaux et al. 2008, 2012; Barban et al. 2009; Campante et al. 2011; Mathur et al. 2011). This obviously provides far more information about the stellar properties.

The analysis is typically carried out by fitting the frequencies to a set of models, characterized by parameters $\{\mathcal{P}_k\}$ which typically include the mass, age and initial chemical composition of the star as well as, possibly, the mixing length and a parameter characterizing convective-core overshoot. In its simplest form the analysis aims at minimizing

$$\chi^2_\nu = \frac{1}{N-1}\sum_{nl}\left(\frac{\nu_{nl}^{(\text{obs})} - \nu_{nl}^{(\text{mod})}}{\sigma(\nu_{nl})}\right)^2 , \quad (17)$$

as a function of the parameters $\{\mathcal{P}_k\}$. Here N is the number of observed frequencies $\nu_{nl}^{(\text{obs})}$, $\nu_{nl}^{(\text{mod})}$ are the model frequencies, possibly corrected for near-surface effects according to Eq. (13) and $\sigma(\nu_{nl})$ are the standard errors in the observed frequencies. To this may be added a similar term from other observed properties of the star, such as effective temperature, surface gravity, [Fe/H], and, when available, radius and luminosity, to obtain the χ^2 which is minimized.

As is common in multi-parameter fits, χ^2 often shows multiple local minima, and a blind search risks landing in what is not the globally smallest value. Genetic algorithms provide a relatively efficient technique for localizing the global minimum (e.g., Metcalfe & Charbonneau 2003). An implementation for the analysis of solar-like

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[1] although, according to Douglas Gough, *sinasteroseismology* might be a more appropriate term

oscillations was presented by Metcalfe et al. (2009); this also included a local analysis based on singular-value decomposition (SVD) to locate more precisely the true minimum, based on the typically relatively coarsely sampled genetic fit, and determine the statistical properties of the fit (see also Brown et al. 1994). This has been implemented in the AMP package[2] for analysis of solar-like oscillation frequencies, which has been made available to the asteroseismic community (Woitaszek et al. 2010). This is based on the Aarhus stellar evolution code (ASTEC; Christensen-Dalsgaard 2008a) and pulsation code (ADIPLS; Christensen-Dalsgaard 2008b).

Although the genetic algorithm is effective in localizing the absolute minimum in χ^2, there may be parameters which provide a fit that differs insignificantly from this best fit. In some cases this may correspond to parameters rather different from those of the best fit (e.g., Metcalfe et al. 2010). Such non-uniqueness, which is a clear indication of the intrinsic nonlinearity of the dependence of the frequencies on the parameters, requires further study. It is likely that it may be broken by increased precision in the observations, an increased range of the observed frequencies or possibly the inclusion of additional observables in the fit.

An unconstrained fit may yield optimal fits that can be judged to unphysical from independent information. An example is the fit by Mathur et al. (2012), using AMP, to 22 *Kepler* stars; here the initial helium abundance Y_0 resulting from the fit in several cases was substantially below the cosmologically determined primordial helium abundance $Y_P = 0.248$ (Steigman 2007). In some cases a value was obtained at the lower limit of the range in Y_0 allowed in the fit. Although exceptionally interesting, if correct, the low value of Y_0 is more likely caused by problems in the fitting or the underlying models. A related, if less evident, example could be an inferred Y_0 and initial heavy-element abundance Z_0 which depart strongly from the normally assumed relation from Galactic chemical evolution. Also, for an otherwise sun-like star a value of the inferred mixing length very far from the solar calibration might be suspect.

Potential problems of this nature can be avoided by including prior information in the fit, although one must obviously be careful not to let the result be too strongly influenced by perhaps unwarranted prejudice. Such prior information can be incorporated through a Bayesian approach, as was also done in the SEEK algorithm (Quirion et al. 2010). An interesting example, applied to fits of individual frequencies to grids of stellar models, was provided by Gruberbauer et al. (2012). They demonstrated that this procedure can also be used to suppress the effect of the surface problems. A statistical characterization of inferred stellar parameters, using also Bayesian priors, was obtained by Bazot et al. (2012) using a Markov Chain Monte Carlo algorithm in a fit to observations of α Cen A, although at very considerable computational expense.

In evolved stars the observation of mixed modes provides much more stringent constraints on the stellar parameters. The buoyancy frequency (cf. Eq. 8) changes rapidly with age and so, therefore, do the frequencies of modes with a strong g-mode component. This provides a very precise determination of the stellar age (e.g., Metcalfe et al. 2010).[3] A powerful technique for the analysis of data with mixed modes was presented by Deheuvels & Michel (2011) and applied to CoRoT data (for an application to *Kepler* data, see Deheuvels et al. 2012). Also, Benomar et al. (2012) showed how additional information could be obtained from the detailed properties of the frequencies of such mixed modes.

A major breakthrough in the seismic analysis of red giants has been the detection of mixed dipolar modes with a strong g-mode component (Beck et al. 2011; Bedding et al. 2011; Mosser et al. 2011). These appear as a 'dipolar forest', a group of peaks around the position of the dipolar acoustic resonances where the modes have their strongest p-mode character. The period spacings between the modes in such groups reflect the uniform period spacing for pure g modes (cf. Eq. 9), although with departures caused by the interaction with the acoustic cavity. It is possible to extrapolate the observed period spacings to determine the true g-mode spacing $\Delta\Pi_1$ and hence constrain the properties of the buoyancy frequency in the core of the star (e.g., Stello 2012). It was found by Bedding et al. (2011) that $\Delta\Pi_1$ provides a clear distinction between stars on the ascending red-giant branch, where the energy comes solely from the hydrogen-burning shell around an inert helium core, and the 'clump stars', where in addition there is helium fusion near the centre (see also Mosser et al. 2011). This distinction comes in part from the fact that the helium-burning stars have a small convective core, where consequently the buoyancy frequency is essentially zero (Christensen-Dalsgaard 2012a). An asymptotic expression, characterizing the frequencies in terms of the period spacing and the coupling between the gravity-wave and acoustic regions, was derived by Goupil (in preparation), based on a general asymptotic analysis (Shibahashi 1979; Unno et al. 1979) (see also Christensen-Dalsgaard 2012b). This was used by Mosser et al. (2012a) for an ensemble analysis of red giants observed by *Kepler*. Procedures for detailed fits to the observed frequencies of mixed dipolar modes are under development, and will probably benefit from an application of this asymptotic expression.

3.3 Analysis of acoustic glitches

In the analysis of stellar oscillation frequencies information about specific features of the star may be obtained through suitable combinations of frequencies, often based on asymptotic analyses of the properties of the oscillations. An important example is the effect of sharp features,

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[2] Asteroseismic Modeling Portal; see http://amp.ucar.edu/

[3] It should perhaps be emphasized that the *accuracy* of such determinations still depends sensitively on uncertainties in the stellar modelling.
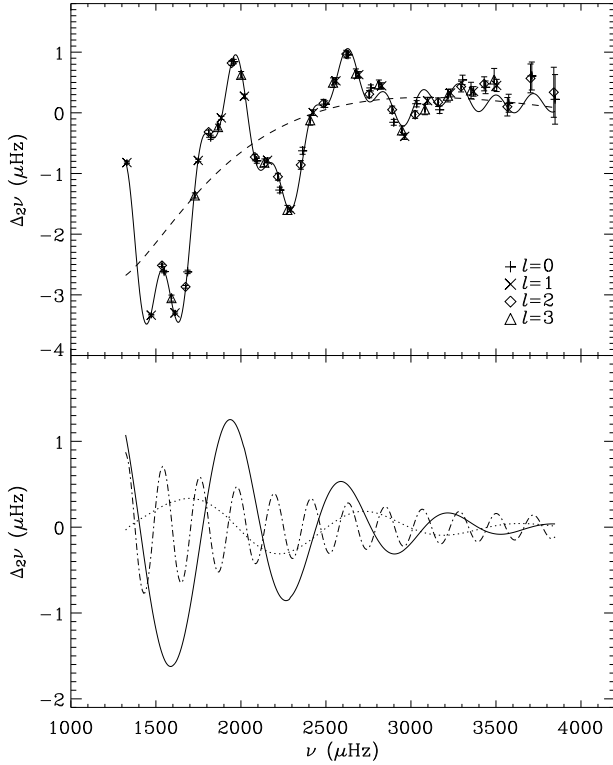
Fig. 2 Effects of acoustic glitches on observed solar frequencies. In the top panel the symbols show second differences obtained from BiSON observations (see Chaplin et al. 2002). The solid curve shows the fit consisting of the effect of the glitches from the ionization zones and the base of the convective envelope, as well as a slowly varying near-surface contribution (shown by the dashed line). The lower panel shows the individual glitch contributions: the dotted and solid lines show the contributions from the first and second helium ionization zones, respectively, and the dot-dashed line shows the contribution from the base of the convective envelope. From Houdek & Gough (2007).

i.e., properties in the star that vary on a scale substantially smaller than the local wavelength of the oscillations (Gough & Thompson 1988; Vorontsov 1988; Gough 1990). This causes an oscillatory variation in the frequency, depending on the phase of the eigenfunction at the location of the feature; the period of the variation depends on the location of the feature, while its amplitude, as a function of frequency, depends on the detailed properties of the feature. In the case of solar-like oscillations these effects are related to *acoustic glitches*, with rapid variations in sound speed. An important example arises in the second helium ionization zone, as a result of the variation in the adiabatic exponent $\Gamma_1 = (\partial \ln p / \partial \ln \rho)_{\mathrm{ad}}$ (the derivative being at constant specific entropy), and hence in the sound speed, given by $c^2 = \Gamma_1 p / \rho$. Glitches also occur at boundaries of convective regions where the temperature gradient varies rapidly, the sound-speed variation possibly enhanced by variations in composition.

To isolate the effects of the acoustic glitches slower variations in the frequencies must be eliminated. An effective way of doing that is to consider the second differences, as a function of mode order:

$$\Delta_2 \nu_{nl} = \nu_{n-1\,l} - 2\nu_{nl} + \nu_{n+1\,l} \qquad (18)$$

(e.g., Gough 1990) or possibly higher differences; obviously, it is important also to consider the error properties of the resulting diagnostics. Houdek & Gough (2007) made a careful analysis of the second difference of solar low-degree frequencies, taking into account the hydrogen and helium ionization zones and the base of the convective envelope; their fit, clearly separating these contributions, is shown in Fig. 2. Alternatively, one can consider suitable filtering of the observations (Pérez Hernández & Christensen-Dalsgaard 1994a) or subtraction of a fitted smooth function from the frequencies (Monteiro et al. 1994; Monteiro & Thompson 2005).

The effects of the second helium ionization obviously depend on the helium abundance in the stellar convection zone. This has been used to determine the solar envelope helium abundance from helioseismic data (e.g., Vorontsov et al. 1991; Antia & Basu 1994; Pérez Hernández & Christensen-Dalsgaard 1994b). It was shown by Pérez Hernández & Christensen-Dalsgaard (1998) and Basu et al. (2004) that a similar analysis is in principle possible for other main-sequence stars, the effect increasing with increasing stellar mass. The properties of the base of the solar convective envelope have been analysed in considerable detail using the resulting acoustic glitch, with emphasis on determining the nature of possible convective overshoot (e.g., Basu & Antia 1994; Monteiro et al. 1994; Roxburgh & Vorontsov 1994; Christensen-Dalsgaard et al. 2011). A similar analysis is, at least in principle, possible on the basis of stellar observations of low-degree modes (Monteiro et al. 2000; Mazumdar & Antia 2001). Ballot et al. (2004) made a detailed analysis of the potential of determining the depth of stellar convection zones from the frequency variations caused by the resulting glitch. Also, Mazumdar (2005) analysed the diagnostic potential of the signatures of acoustic glitches.

The effects on the frequencies of acoustic glitches are quite subtle and hence it is only with the extensive data from CoRoT and *Kepler* that it has been possible to identify them for solar-like oscillations in stars other than the Sun. Based on analysis of CoRoT observations by Carrier et al. (2010), Miglio et al. (2010) found the signature of the glitch caused by the second helium ionization zone in a red giant. They used the inferred depth of the ionization zone, together with other asteroseismic parameters, to obtain a purely seismic determination of the mass and radius of the star; however, the data were not yet of sufficient quality to allow a determination of the envelope helium abundance. Mazumdar et al. (2012a) analysed CoRoT observations of a main-sequence star slightly more massive than the Sun and identified a very clear signature of the second helium ionization zone

and a weak and somewhat ambiguous signature of the base of the convective envelope. Very encouraging results were obtained by Mazumdar et al. (2012b) for several stars observed with *Kepler*.

There may be circumstances where the effects of acoustic glitches might hide or significantly affect other asteroseismic diagnostics. Thus Houdek & Gough (2011) 'corrected' solar data for the effect of the glitches associated with the second helium ionization zone and the base of the convective envelope before using the resulting glitch-free frequencies to infer the seismic age and heavy-element abundance of the Sun, from an asymptotic analysis of the effect of the stellar core. Of course, the by-products obtained in such a glitch analysis are also of substantial diagnostic value!

Sharp features in the sound speed and other seismically relevant quantities are produced by the composition structure resulting from the evolution of stars with convective cores. This is particularly dramatic in moderate-mass stars where the mass contained in the convective core grows with age, resulting in a composition (and hence sound-speed) discontinuity if diffusion is neglected. Cunha & Metcalfe (2007) derived a diagnostic applicable to this case and showed that it provided a measure of stellar age. This and similar diagnostic quantities were further analysed by Cunha & Brandão (2011). A similar analysis was carried out by Silva Aguirre et al. (2011), demonstrating the potential of such diagnostics to constrain the size of the convective core, including the effects of semiconvection which remain a serious uncertainty in the modelling of stellar evolution.

With the continuing operations of both CoRoT and *Kepler* and the resulting longer timeseries there is clearly a substantial potential for this type of analysis. The depth of convective envelopes is an important parameter in the modelling of stellar dynamos assumed responsible for stellar magnetic cycles, while independent determinations of the helium abundance for a range of stars would obviously be very valuable for studies of the Galactic chemical evolution and as a constraint on Big-Bang nucleosynthesis.

4 Inversions

The analysis in the preceding section generally characterizes the stellar properties in terms of a limited set of parameters. In particular, in fits such as the minimization of χ^2 (cf. Eq. 17) the model is characterized by the parameters $\{\mathcal{P}_k\}$. Such fits implicitly assume that the physics of the modelling is correct, which is obviously doubtful. Indeed, perhaps the most interesting aspect of the asteroseismic analysis is to falsify this assumption and, ideally, determine how the modelling should be improved.

In the simple minimization of χ_ν^2 an inconsistency in the modelling is implied if the minimum of χ_ν^2 is substantially bigger than unity. However, this is clearly only significant if in fact the errors in the observations have been estimated correctly, which in itself is not a trivial task. A single mode with an erroneously low estimated error (or a single misidentified frequency) could lead to an unrealistically large value of χ_ν^2. Thus care, and perhaps some conservatism, is required in the interpretation of the results of the fits.

If indeed there is convincing evidence that the large value of χ^2 is significant, the task is obviously to determine the cause of the discrepancy, in terms of the stellar modelling. This would benefit greatly from the ability to locate the cause of the frequency differences between the star and the best model. Independently of these issues of model fitting, it is clearly of interest to obtain information about the stellar structure that is not tied to specific models, as exemplified by the glitch analysis discussed in Section 3.3. Here I provide a brief discussion of *inverse analyses* with this aim.

Such techniques have been extremely successful in helioseismic analysis of solar oscillation data (see, for example Gough et al. 1996; Christensen-Dalsgaard 2002, for reviews). Here the mode set is sufficiently rich that it is possible to infer the sound speed and density in a very large fraction of the solar interior; although this is typically based on determining corrections to a reference model, the results are generally insensitive to the precise choice of model (Basu et al. 2000). The analysis is based on the assumption that the model is sufficiently close the true solar structure that the frequency differences $\delta\nu_{nl}$ between the Sun and the model can be obtained from an expression linearized in the differences in structure:[4]

$$\frac{\delta\nu_{nl}}{\nu_{nl}} = \int_0^R \left(K_{c^2,\rho}^{nl} \frac{\delta_r c^2}{c^2} + K_{\rho,c^2}^{nl} \frac{\delta_r \rho}{\rho} \right) \mathrm{d}r + Q_{nl}^{-1} \mathcal{G}(\nu_{nl}) .$$ (19)

Here $\delta_r c^2$ and $\delta_r \rho$ are the differences, at fixed r, between the Sun and the reference model in squared sound speed and density, and $\mathcal{G}$ is a function that accounts for the near-surface problems of the model, with a scale factor Q_{nl}^{-1} that depends on the inertia of the modes. The kernels $K_{c^2,\rho}^{nl}$ and K_{ρ,c^2}^{nl} are computed from the eigenfunctions of the modes of the reference model.

The inverse analyses, based on techniques originally developed in geophysics (e.g., Backus & Gilbert 1968), consist of combining relations of the form given in Eq. (19) in such a way as to obtain an estimate of, for example, $\delta_r c^2/c^2$ that is localized near point, $r = r_0$, in the Sun. If this can be done throughout the star an estimate is obtained of the sound-speed difference between the Sun and the model. Given the extremely accurate solar oscillation frequencies spanning degrees from 0 to at least 200 this can in fact be done with high precision between $0.07R$ and $0.95R$ (e.g., Basu et al. 1997). The inferences are obtained as linear combinations of the frequency differences and can be expressed as averages of the true sound-speed difference, weighted by averaging kernels $\mathcal{K}_{c^2,\rho}(r_0, r)$. The success of

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[4] Formally, an expression of this form follows from the fact that adiabatic oscillations satisfy a variational principle.

the inversion is characterized by the properties of $\mathcal{K}$, particularly its width, and the error in the inferred differences. The properties of the inversion are typically determined by parameters that, for example, control the balance between localizing $\mathcal{K}$ and minimizing the errors. The choice of these *trade-off parameters*, and other aspects of the helioseismic inversions, was discussed by Rabello-Soares et al. (1999).

In the stellar case we are typically, for the foreseeable future, restricted to modes of degree $l = 0-3$ and hence the potential for inversion is much more restricted. Brief reviews of the related issues were provided by Thompson & Christensen-Dalsgaard (2002) and Basu (2003), concentrating on solar-like stars. Given the limited information it is advantageous to choose a description of the stellar interior that, as far as possible, depends on a single function of position in the star, while still recognizing that the modes observed are predominantly of acoustic nature. Using that $c^2 = \Gamma_1 p/\rho$ a reasonable choice is the squared isothermal sound speed $u = p/\rho$. In fact, in much of solar-like stars $\Gamma_1 \simeq 5/3$ and hence the acoustic properties are fully characterized by u. Assuming furthermore that the equation of state of the stellar matter is known with sufficient accuracy, Γ_1 can be found as a function $\Gamma_1(p, \rho, \{X_i\})$, where $\{X_i\}$ is the composition. Characterizing the dependence on composition simply by the helium abundance Y by mass[5] we can rewrite Eq. (19) as

$$\frac{\delta\nu_{nl}}{\nu_{nl}} = \int_0^R \left(K_{u,Y}^{nl} \frac{\delta_r u}{u} + K_{Y,u}^{nl} \delta_r Y \right) \mathrm{d}r + Q_{nl}^{-1} \mathcal{G}(\nu_{nl}) , \tag{20}$$

where in addition we used the equations of hydrostatic equilibrium and mass to express the dependence on $\delta_r p$ and $\delta_r \rho$ in terms of $\delta_r u$.

A second important difference between helioseismic and asteroseismic inverse analysis is that for stars, unlike the case for the Sun, we do not have accurate independent determinations of the mass and radius. To separate the inverse analysis from the determination of M and R we can rewrite Eq. (20) in terms of $x = r/R$, the dimensionless function $\hat{u} = (R/GM)u$ and the dimensionless frequencies $\sigma_{nl} = 2\pi(R^3/GM)^{1/2}\nu_{nl}$:

$$\frac{\delta\sigma_{nl}}{\sigma_{nl}} = \int_0^1 \left(K_{u,Y}^{nl} \frac{\delta_x \hat{u}}{\hat{u}} + K_{Y,u}^{nl} \delta_x Y \right) \mathrm{d}x + Q_{nl}^{-1} \hat{\mathcal{G}}(\sigma_{nl}) . \tag{21}$$

Here I somewhat crudely assumed that the differences in R and M between the star and the reference model are absorbed in $\hat{\mathcal{G}}$.

A final difference between the helioseismic and asteroseismic case concerns the linear approximation underlying Eq. (19). It is perhaps reasonable, not least from the tight constraints on mass, radius, age and luminosity, to assume that solar models are so close to the solar structure that the linearized representation of the differences is sufficiently

accurate. This is not as obviously the case for other stars. In fact, analyses by Christensen-Dalsgaard & Thompson (in preparation) indicate substantial departures from linearity for frequencies of models differing in mass only by a few percent from the reference model. These issues need further investigation before reliable inversions using this technique can be carried out.

There is relatively limited experience with this type of inversion as applied to solar-like stars; in particular it is only with the recent space-based data that sufficiently extensive data are available to make it realistic to consider such analyses. Early tests of inversion of artificial data for low-degree modes were made by Gough & Kosovichev (1993a,b). Basu et al. (2002) tested inversions for model frequencies for reasonably realistic sets of stellar artificial data, with both models having solar mass and radius. Based on Eq. (21) they found that relatively localized averaging kernels could be constructed in the stellar core, and that the inferred $\delta_r u$ showed clear evidence for the core mixing imposed in the test model. An interesting variant of the linearized inversion, used to infer the stellar mean density, was discussed by Reese et al. (2012) who applied it to several sets of observed data.

A conceptually very different technique for asteroseismic determination of stellar structure has been developed by Roxburgh & Vorontsov (Roxburgh & Vorontsov 2002, 2003b) (see also Roxburgh 2010). This is based on the fact that acoustic modes in the outer parts of the star can be characterized by a phase function which is independent of the degree of the mode. By imposing this constraint, at the observed frequencies, it is possible to determine the structure of the bulk of the star as parameterized in terms of a suitable variable, such as the density, with fitting parameters that are determined iteratively; it is assumed that in the relevant part of the star, $\Gamma_1 \simeq 5/3$. Thus the technique does not depend directly on a reference model, although an initial model is used as a starting point for the iteration. Applications to artificial data have yielded results remarkably close to the original model on which the data were based.

5   Concluding remarks

The advances in stellar inferences over the last few years, from the observations made by the CoRoT and *Kepler* missions have been remarkable, far beyond the expectations before the launch of these missions. In addition to the quality of the data, these advances have been based on the rapid development of techniques for the analysis of the data. It is probably fair to say that the asteroseismic community was not entirely prepared for the overwhelming quality, and quantity, of the data that we have obtained.

While striking results have been obtained, we are not yet quite at the point of challenging the modelling of stellar structure and evolution, with the exception of the evolution of stellar rotation where the theoretical basis is still highly

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[5]   In most cases the heavy-element abundance has a negligible effect on the thermodynamic state.

uncertain. Such tests and, one might hope, improved understanding of stellar internal physics are of course a key goal of asteroseismology. To reach this point the improvements in the data quality and quantity resulting from the extension of the missions will certainly be very helpful. Equally important, however, will be the continued development of the techniques used to analyse and interpret the observations. This includes a better statistical analysis of the results, which will be required to test whether existing models deviate in a significant manner from the observations. A key tool in this development will be the application of the techniques to artificial data; such tests were carried out before the launch of the missions (e.g., Monteiro et al. 2002; Mazumdar 2005) but have since largely been ignored in the flush of real data. Similarly, further work is needed to understand the differences between results of different stellar evolution and pulsation codes, as started in the ESTA project in preparation for the CoRoT mission (Monteiro 2008). Finally, it is probably time to consider the application of the inverse techniques discussed in Section 4, or at least to continue developing and testing these techniques on the basis of the understanding of the realistic data properties that has resulted from the observations. Also, the potential for inversion localized in the stellar core is undoubtedly improved for stars showing mixed modes, as was demonstrated by Lochard et al. (2004) in the corresponding case of inversion for rotation. This deserves further detailed study.

Beyond the present missions there are proposals for additional missions combining asteroseismology and the study of extra-solar planets, such as the Transiting Exoplanet Survey Satellite (TESS) under consideration by NASA and the PLAnetary Transits and Oscillations of stars mission (PLATO; Catala 2009) proposed to ESA. If selected, they will extend the asteroseismic investigations to more nearby stars in very large parts of the sky, although the duration of the observations will be shorter than in the case of longest timeseries obtained with *Kepler*. Also, the Stellar Observations Network group (SONG) network (e.g. Grundahl et al. 2011) will yield radial-velocity observations of stellar oscillations of even higher asteroseismic quality than the *Kepler* data, although for a much smaller sample of stars. Thus it is crucial to select the astrophysically most interesting targets for SONG, ideally based on the understanding of stellar properties obtained from the space missions. Here, also, the analysis of realistic artificial data is essential.

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