Mon. Not. R. Astron. Soc. **267,** 297–302 (1994)

The seismology of stellar cores: a simple theoretical description of the 'small frequency separations'

I. W. Roxburgh[1] and S. V. Vorontsov[1,2]★

[1] *Astronomy Unit, Queen Mary and Westfield College, Mile End Road, London E1 4NS*
[2] *Institute of Physics of the Earth, B. Gruzinskaya 10, Moscow 123810, Russia*

Accepted 1993 October 12. Received 1993 September 30; in original form 1993 July 1

ABSTRACT

We present a new theoretical description of the 'small frequency separations' $\delta\omega_{\ell,n}=\omega_{\ell,n}-\omega_{\ell+2,n-1}$ for high-frequency stellar p-modes of low degree, these separations being the observable quantities that are primarily sensitive to the structure of the deep stellar interior. The description is based on an integral representation of the phase shift of acoustic waves due to scattering off the stellar core, taking into account the effects of buoyancy and gravitational perturbations. The accuracy of the theoretical description is tested by comparing the predicted frequency separations with values determined by numerically solving the full set of eigenfrequency equations for a standard solar model and for simple zero-age and evolved models of a 3-$M_\odot$ main-sequence star with a convective core.

Key words: waves – methods: numerical – stars: interiors – stars: oscillations.

1 INTRODUCTION

The measurement of high-frequency acoustic oscillations provides a means of probing the internal structure of the Sun and stars. Low-degree p-modes are of particular interest since they penetrate deep into the stellar interior; their frequencies therefore contain information on the structure of the energy-generating core. (For reviews, see e.g. Gough & Toomre 1991; Libbrecht & Woodard 1991; Vorontsov 1992; Christensen-Dalsgaard 1993.) On stars other than the Sun, only low-degree modes are observable due to lack of spatial resolution.

The oscillation frequencies are governed by integral properties of the complete acoustic cavity in which the internal acoustic waves are trapped, producing the observable standing waves, or global p-modes. The acoustic cavity extends into the surface layers, and the sensitivity of the p-mode frequencies to the structure of the core is in general very low, the acoustic waves spending only a very small fraction of their traveltime across the cavity in the high-temperature stellar core.

However, the 'small frequency separations' $\omega_{\ell,n}-\omega_{\ell+2,n-1}$, where ℓ is the degree and n is the radial order, are known to be sensitive predominantly to the properties of the stellar core, being essentially free from uncertainties in the envelope structure (for discussions, see e.g. Gabriel 1989; Gough & Novotny 1990; Van Hoolst & Smeyers 1991). To utilize the diagnostic capability of these measurable quantities, it is highly desirable to have a convenient theoretical description that is capable of relating them to integral parameters of the seismic stratification of the stellar interior. The standard high-frequency asymptotic analysis (Tassoul 1980, 1990) is, however, known to produce poor results, even in higher approximations (Roxburgh & Vorontsov 1993, 1994), because of the inability of the standard asymptotic expansions to account accurately for the strong local effects of buoyancy forces and gravitational perturbations in the core. We here describe an alternative approach, which is essentially a generalization of the first Born approximation for the scattering by the stellar core of acoustic waves modified by buoyancy and gravity.

2 THEORETICAL DESCRIPTION

The complete fourth-order system of ordinary differential equations governing linear adiabatic oscillations of a spherically symmetric, self-gravitating star is (in spherical coordinates r, θ, φ)

$$\frac{\mathrm{d}\xi}{\mathrm{d}r}=\frac{h_2}{h_1}\left[\frac{\ell(\ell+1)}{\omega^2}-\frac{r^2}{c^2}\right]\eta-\frac{\ell(\ell+1)}{h_1\omega^2}P, \tag{1}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}r}=\frac{\omega^2}{r^2}\frac{h_1}{h_2}\left(1-\frac{N^2}{\omega^2}+\frac{4\pi G\rho_0}{\omega^2}\right)\xi+\frac{\ell(\ell+1)}{h_2r^2}S, \tag{2}$$

★ Permanent address.

$$\frac{dP}{dr}=\frac{4\pi G\rho_0 h_1}{r^2}\xi+\frac{\ell(\ell+1)}{r^2}S, \tag{3}$$

$$\frac{dS}{dr}=-\frac{4\pi G\rho_0 h_2}{\omega^2}\eta+\left(1+\frac{4\pi G\rho_0}{\omega^2}\right)P, \tag{4}$$

where the dependent variables are related to the radial displacement δr, the Eulerian pressure perturbation p' and the gravitational potential perturbation ψ' by

$$\delta r=\frac{h_1(r)}{r^2}\xi(r)\,Y_{\ell m}(\theta,\varphi)\exp(i\omega t), \tag{5}$$

$$p'=\rho_0(r)\,h_2(r)\,\eta(r)\,Y_{\ell m}(\theta,\varphi)\exp(i\omega t), \tag{6}$$

$$\psi'=-P(r)\,Y_{\ell m}(\theta,\varphi)\exp(i\omega t), \tag{7}$$

$$\ell(\ell+1)S(r)=r^2\frac{dP}{dr}-4\pi G\rho_0(r)\,h_1(r)\,\xi(r), \tag{8}$$

$$h_1(r)=\exp\int_0^r\frac{g(r)}{c^2(r)}dr,\qquad h_2(r)=\exp\int_0^r\frac{N^2(r)}{g(r)}dr, \tag{9}$$

where $\rho(r)$, $g(r)$, $c(r)$ and $N(r)$ are equilibrium distributions of density, gravitational acceleration, adiabatic sound speed, and Brunt–Väisälä frequency, and ω is the cyclical frequency.

The governing differential equations (1)–(4) are of the same form as used in some recent asymptotic analyses (Roxburgh & Vorontsov 1993, 1994, and references therein). In the Cowling approximation (when the gravitational perturbations are neglected), the equations reduce to the canonical form described in Unno et al. (1989, equations 15.5, 15.6). Gravitational perturbations enter the governing equations with two functions $P(r)$ and $S(r)$ defined by equations (7) and (8).

To analyse high-frequency, low-degree modes, we introduce the small parameter

$$\epsilon=\frac{\ell(\ell+1)}{\omega^2}. \tag{10}$$

Let the solutions of equations (1)–(4) in the zero-order approximation ($\epsilon=0$) be ξ_0, η_0, P_0 and S_0, respectively. These are solutions for radial ($\ell=0$) modes. As is well known, for such modes the oscillation equations reduce to second order; ξ_0 and η_0 are governed solely by equations (1) and (2); P_0 and S_0 do not enter these equations and can be calculated from ξ_0 and η_0 using equations (3) and (4) (with appropriate boundary conditions). If terms of order ϵ are retained in the analysis, only the zero-order terms P_0 and S_0 enter equations (1) and (2) for ξ and η. This property of the governing fourth-order system of differential equations permits a reduction to second order if terms of order ϵ^2 and higher are neglected.

We now introduce new variables t and ψ:

$$t=\int_0^r A(\omega;r)^{1/2}\frac{dr}{c},\qquad \psi=A(\omega;r)^{-1/4}B(r)^{1/2}\eta, \tag{11}$$

with

$$A(\omega;r)=1-\frac{N^2}{\omega^2}+\frac{4\pi G\rho_0}{\omega^2},\qquad B(r)=\frac{h_2 r^2}{h_1 c}, \tag{12}$$

and rewrite equations (1) and (2) formally as one second-order equation for $\psi(t)$:

$$\begin{aligned}\frac{d^2\psi}{dt^2}+\left[\omega^2-U_0-\ell(\ell+1)\frac{c^2}{r^2}\right]\psi&=\\ A^{1/4}B^{-1/2}\frac{d}{dt}\left[\frac{\ell(\ell+1)}{h_1 A}S\right]&-A^{-1/4}B^{-1/2}c\frac{\ell(\ell+1)}{h_1}P,\end{aligned} \tag{13}$$

where

$$U_0=A^{1/4}B^{-1/2}\frac{d^2}{dt^2}(A^{-1/4}B^{1/2}). \tag{14}$$

For conciseness we have suppressed the explicit dependence on $(\omega;r)$. In the following analysis the terms containing P and S on the right-hand side of equation (13) will be formally considered as an inhomogeneity.

Note that equation (13) will be used throughout the stellar interior except in the thin layer near the surface, where reflection of the trapped acoustic waves occurs due to the rapid variation of the seismic parameters on a scale short compared with the radial wavelength: in the surface layers, equation (13) can have singular points with $A(\omega;r)=0$. In the inner domain, where equation (13) will be applied, we assume $A(\omega;r)>0$ for the high-frequency acoustic modes.

We now estimate the two terms on the right-hand side of equation (13) to the first order in the small parameter ϵ. From equations (2) and (3), we obtain

$$\frac{dP}{dr}=\frac{4\pi G\rho_0 h_1}{r^2}\xi+O(\epsilon)=\frac{4\pi G\rho_0 h_2}{\omega^2 A}\frac{d\eta}{dr}+O(\epsilon). \tag{15}$$

When integrating this equation for the gravitational perturbations, we neglect, in the high-frequency approximation, logarithmic derivatives of ρ_0, h_2 and A compared with the radial wavenumber, and obtain

$$\begin{aligned}P&\simeq\frac{4\pi G\rho_0 h_2}{\omega^2 A}\eta+O(\epsilon)\\ &=\frac{4\pi G\rho_0 h_2}{\omega^2}A^{-3/4}B^{-1/2}\psi+O(\epsilon).\end{aligned} \tag{16}$$

The constant of integration is fixed by the condition of regularity of $P(r)$ as $r\to\infty$. In the low-density envelope, well outside the stellar core, the regular solution of Laplace's equation for the gravitational perturbations requires that $P(r)\to\text{constant}\times r^{-\ell-1}$, $S(r)\to-(\text{constant}/\ell)r^{-\ell}$: this condition is identically satisfied by solution (16) with constant $=0$, since $P(r)\to 0$ when $\rho_0(r)\to 0$.

From equations (4) and (16), we obtain

$$\frac{dS}{dr}\simeq\frac{4\pi G\rho_0 h_2}{\omega^2 A}\frac{N^2}{\omega^2}\eta+O(\epsilon). \tag{17}$$

When estimating the term containing S on the right-hand side of equation (13), we neglect, in the high-frequency approximation, logarithmic derivatives of h_1 and A compared with the radial wavenumber, and obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\ell(\ell+1)}{h_1 A}S\right] \simeq \ell(\ell+1)A^{-9/4}B^{-1/2} \times c\frac{h_2}{h_1}\frac{4\pi G\rho_0}{\omega^2}\frac{N^2}{\omega^2}\psi + O(\epsilon^2). \tag{18}$$

By neglecting terms of order ϵ^2 and higher, the fourth-order system of governing differential equations reduces to a second-order Schrödinger-type equation,

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2}+\left[\omega^2-\frac{\ell(\ell+1)}{t^2}-U_0(\omega;t) - \ell(\ell+1)\,U_2(\omega;t)\right]\psi=0, \tag{19}$$

with two frequency-dependent 'acoustic potentials',

$$U_0(\omega;t)=\frac{\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}\left[\left(1-\dfrac{N^2}{\omega^2}+\dfrac{4\pi G\rho_0}{\omega^2}\right)^{-1/4}\left(\dfrac{h_2 r^2}{h_1 c}\right)^{1/2}\right]}{\left(1-\dfrac{N^2}{\omega^2}+\dfrac{4\pi G\rho_0}{\omega^2}\right)^{-1/4}\left(\dfrac{h_2 r^2}{h_1 c}\right)^{1/2}}, \tag{20}$$

$$U_2(\omega;t)=\left[\left(1-\frac{N^2}{\omega^2}\right)^2+\frac{4\pi G\rho_0}{\omega^2}\right] \times\left(1-\frac{N^2}{\omega^2}+\frac{4\pi G\rho_0}{\omega^2}\right)^{-2}\frac{c^2}{r^2}-\frac{1}{t^2}. \tag{21}$$

At the origin ($r=0$, $t=0$), $c^2(r)$, $\rho_0(r)$ and $N^2(r)$ are expandable in even powers of r, thus ensuring the regularity of both potentials at $t=0$.

The remaining part of our analysis is just the first Born approximation for acoustic waves scattered by the potential $U_0+\ell(\ell+1)\,U_2$. We assume that both the potentials U_0 and U_2 are localized predominantly in the stellar core, and become small in the stellar envelope, where a simple high-frequency asymptotic description of wave propagation becomes adequate. The potentials computed for the standard solar model and for a 3-$M_\odot$ main-sequence star are shown in Figs 1, 4 and 6 (see later).

We place the terms involving U_0 and U_2 on the right-hand side of equation (19), and consider them formally as an inhomogeneity:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2}+\left[\omega^2-\frac{\ell(\ell+1)}{t^2}\right]\psi=[U_0(\omega;t)+\ell(\ell+1)\,U_2(\omega;\mathrm{t})]\,\psi. \tag{22}$$

The solutions of the homogeneous equation are spherical Bessel functions $j_\ell(\omega t)$ and $n_\ell(\omega t)$. [Spherical Bessel functions are here defined as $j_\ell(z)=(\pi z/2)^{1/2}J_{\ell+1/2}(z)$, $n_\ell(z)=(\pi z/2)^{1/2}Y_{\ell+1/2}(z)$, with $J_{\ell+1/2}$ and $Y_{\ell+1/2}$ being the standard Bessel functions, so that $j_0(z)=\sin(z),\ldots$.]

We now introduce Green's function,

$$G(\omega;t,t')=\begin{cases}\dfrac{1}{\omega}j_\ell(\omega t)\,n_\ell(\omega t'), & t<t',\\[2mm] \dfrac{1}{\omega}n_\ell(\omega t)\,j_\ell(\omega t'), & t>t',\end{cases} \tag{23}$$

as a particular solution of the inhomogeneous equation

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}G(\omega;t,t')+\left[\omega^2-\frac{\ell(\ell+1)}{t^2}\right]G(\omega;t,t')=\delta(t-t'), \tag{24}$$

where $\delta(x)$ is the Dirac delta function. The general solution ψ_ℓ of equation (22) is then

$$\psi_\ell(\omega;t)=C_1 j_\ell(\omega t)+C_2 n_\ell(\omega t) + \int_0^{t_0} G(\omega;t,t')[U_0(\omega;t')+\ell(\ell+1)\,U_2(\omega;t')]\,\psi_\ell(\omega;t')\,\mathrm{d}t', \tag{25}$$

where t_0 is large enough for the potentials to be neglected above $t=t_0$. The differential equation (22) is thus transformed to the integral equation (25). We now set $C_1=1$ to fix the norm of the solution and $C_2=0$ from the condition of regularity at $t=0$, and consider the potential $U_0+\ell(\ell+1)\,U_2$ as a small perturbation. In the linear approximation, the behaviour of the solution outside the core is

$$\psi_\ell(\omega;t)\to j_\ell(\omega t)+\frac{1}{\omega}n_\ell(\omega t) \times\int_0^{t_0}[U_0(\omega;t')+\ell(\ell+1)\,U_2(\omega;t')]\,j_\ell^2(\omega t')\,\mathrm{d}t', \qquad t>t_0. \tag{26}$$

To determine the asymptotic behaviour of the wavefunction ψ_ℓ in the stellar envelope, far enough from the core, we replace $j_\ell(\omega t)$ and $n_\ell(\omega t)$ with their asymptotic expansions at large argument, and obtain

$$\psi_\ell(\omega;t)\to A_\ell(\omega;t)\sin\left[\omega t-\frac{\pi}{2}\ell+\delta_\ell(\omega;t)\right] \tag{27}$$

with a slowly varying amplitude factor $A_\ell(\omega;\,t)$ and an 'internal phase shift'

$$\delta_\ell(\omega;t)\simeq\frac{\ell(\ell+1)}{2\omega t} -\frac{1}{\omega}\int_0^{t_0}[U_0(\omega;t')+\ell(\ell+1)\,U_2(\omega;t')]\,j_\ell^2(\omega t')\,\mathrm{d}t'. \tag{28}$$

In the thin surface layer, where the reflection of the acoustic waves occurs, the solutions of the wave equations are independent of degree ℓ, when ℓ is small [the acoustic waves become nearly vertical close to the surface due to low temperature and hence low values of c^2; quantitatively, we assume $\ell(\ell+1)c^2/(\omega^2 r^2)\ll 1$ in the surface layers]. The effects of the surface layers can thus be described as a 'surface phase shift', which is a function of frequency only (e.g. Roxburgh & Vorontsov 1993, 1994 and references therein).

Consider now two internal solutions (28) with the same frequency but with the degree ℓ differing by 2. Since both modes must satisfy the same outer boundary condition, the small difference between the 'internal phase shifts' δ_ℓ and $\delta_{\ell+2}$, which appears just below the surface layer, must be compensated by a small difference in the frequencies. The 'small frequency separation' is thus

$$\omega_{\ell,n}-\omega_{\ell+2,n-1}\simeq\frac{\delta_{\ell+2}(\omega;\,T)-\delta_\ell(\omega;\,T)}{T}, \tag{29}$$

with $T=t(\omega;\,R)$, where we have neglected a small variation of the 'surface phase shift' which can arise due to the small frequency variation between $\omega_{\ell,n}$ and $\omega_{\ell+2,n-1}$, and assumed that the acoustic depth of the reflecting surface layer is small compared with the stellar acoustic radius T. Close to the reflecting layers, the potential U_0 can increase again, but the frequency separations remain independent of the upper limit t_0 of the integral in (28) even when t_0 is in this region, because the contributions of these layers to the phase shifts cancel in the difference $\delta_{\ell+2}(\omega;\,t)-\delta_\ell(\omega;\,t)$ since $j^2_{\ell+2}(\omega t')$ and $j^2_\ell(\omega t')$ have the same asymptotic behaviour for large values of their arguments.

In the simplest leading-order high-frequency asymptotic limit, when we neglect U_0 compared with ω^2, neglect the effects of buoyancy and gravitational perturbations (by neglecting N^2 and $4\pi G\rho_0$ compared with ω^2) in U_2 and t, and replace j^2_ℓ with its mean value 1/2, expression (29) gives

$$\begin{aligned}\omega_{\ell,n}-\omega_{\ell+2,n-1}&\simeq\frac{2\ell+3}{\omega T}\left[\frac{1}{T}-\int_0^T\left(\frac{c^2}{r^2}-\frac{1}{t^2}\right)\mathrm{d}t\right]\\&=\frac{2\ell+3}{\omega T}\left[-\int_0^R\frac{\mathrm{d}c}{\mathrm{d}r}\frac{\mathrm{d}r}{r}+\frac{c(R)}{R}\right],\end{aligned} \tag{30}$$

with $T=\int_0^R \mathrm{d}r/c$. This is the well-known Tassoul (1980) result for small frequency separations.

3 NUMERICAL TESTS

We test the accuracy of both approximations (29) and (30) for the small frequency separations by comparing the predictions with exact values obtained by numerically solving the full eigenfrequency equations.

The acoustic potentials for the standard solar model are shown in Fig. 1. Solar model 1 of Christensen-Dalsgaard (1982) was used in the computations. The small frequency separations shown in Figs 2 and 3 are defined as

$$D_{n\ell}=\frac{1}{2\ell+3}\,\frac{\omega_{\ell,n}-\omega_{\ell+2,n-1}}{2\pi}. \tag{31}$$

Fig. 2 gives the results for an artificial experiment using the Cowling approximation where gravitational perturbations are neglected, and Fig. 3 the results for the complete problem. The asymptotic description is very poor in the Cowling approximation, due to the neglect of buoyancy forces. It looks better at higher frequencies when gravitational perturbations are included, but it is clear that such an improvement is no more than an accidental mutual compensation of the effects of buoyancy and gravity; a factor-of-2 error appears at lower frequencies, where gravitational perturbations dominate over buoyancy. The first Born approximation accounts for the effects of both buoyancy and gravity with reasonable accuracy.

Figs 4 and 5 show the acoustic potentials and the small frequency separations computed for a simple model of a zero-age 3-$M_\odot$ main-sequence star. The convective core of radius $r=0.15\,R$ has the same chemical composition as the radiative envelope. The discontinuity in the second derivative of the sound speed, produced by the transition from the adiabatic to the radiative temperature gradient at the core boundary, leads to the discontinuity in $U_0(\omega;\,t)$; $U_2(\omega;\,t)$ remains smooth, and the small frequency separations vary monotonically with frequency. The simple asymptotic

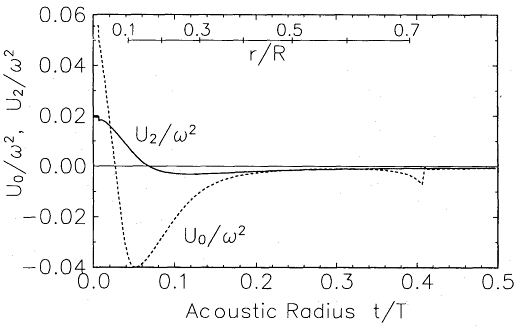

Figure 1. Acoustic potentials in the standard solar model, at frequency $\omega/2\pi=3$ mHz.

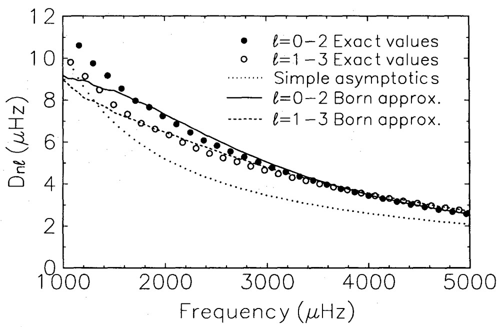

Figure 2. Small frequency separations computed for the standard solar model in the Cowling approximation.

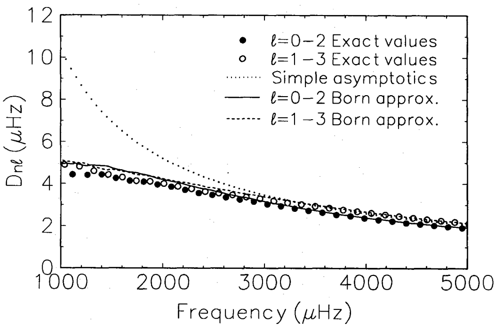

Figure 3. Small frequency separations computed for the standard solar model with gravitational perturbations taken into account.

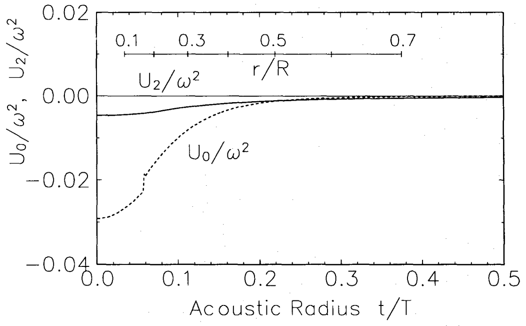

Figure 4. Acoustic potentials in a model of the 3-M$_\odot$ zero-age main-sequence star, at frequency $\omega/2\pi = 3$ mHz.

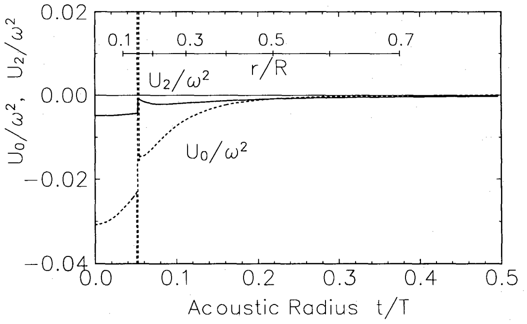

Figure 6. Acoustic potentials for the evolved 3-M$_\odot$ main-sequence star, at frequency $\omega/2\pi = 3$ mHz.

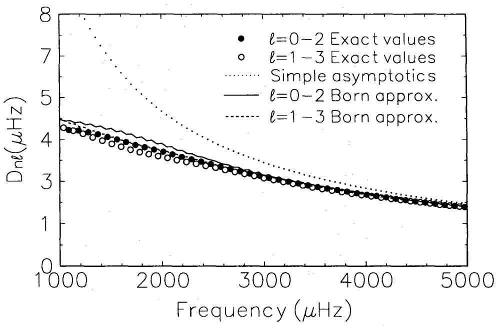

Figure 5. Small frequency separations for a model of the 3-M$_\odot$ zero-age main-sequence star, with gravitational perturbations taken into account.

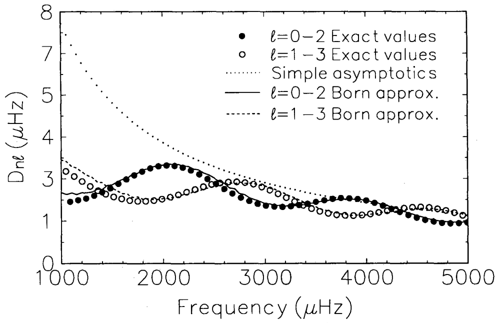

Figure 7. Small frequency separations for the evolved 3-M$_\odot$ main-sequence star, with gravitational perturbations taken into account.

(Tassoul) fit is very poor, whereas the first Born approximation reproduces the exact separations with reasonable accuracy.

Figs 6 and 7 show the results obtained for a simple model of a partially evolved 3-M$_\odot$ main-sequence star, which has a discontinuity in chemical composition across the core boundary. The discontinuity in density produces an integrable singularity in $U_0(\omega;\, t)$, and a discontinuity in $U_2(\omega;\, t)$. This discontinuity in density drastically affects the small frequency separations, leading to the prominent periodic variations with frequency. The period of these variations is determined by the acoustic radius of the core, being approximately $(T/t_{\rm core})\Delta\nu$, where $\Delta\nu \simeq 1/(2T)$ ('large frequency separation'). The amplitude of the periodic component is determined by the magnitude of this discontinuity. The first Born approximation reproduces the exact separations with reasonable accuracy.

4 CONCLUSIONS

The simple theoretical description of small frequency separations obtained using an integral representation of the acoustic phase shifts produces quite accurate results, even when this approximate perturbational-type analysis is limited to linear terms.

The diagnostic capability of the small frequency separations comes from the property that the acoustic potentials U_0 and U_2 are essentially localized in the core. Note that the sensitivity of the small separations to stellar structure is even more localized in the stellar centre than might be expected from the behaviour of the potentials plotted in Figs 1, 4 and 6. The eigenfunctions of both modes are nearly the same when t is large, and the contributions of U_0 to the phase shifts are cancelled (equation 29). (One immediate result is that a hump in U_0 near the base of the solar convection zone, $t \approx 0.4\,T$, does not contribute to the small separations.) A 'tail' in U_2 at high t is essentially $-1/t^2$ (equation 21) due to the small values of the sound speed in the envelope, and is independent of the structure of the star.

The only physical quantity that is related to the structure of the star outside the core, and which affects the small frequency separations, is the acoustic radius of the star $T = t(\omega;\, R)$. In our perturbational analysis, this quantity is like mode energy, and enters the frequency perturbations as a normalization factor. This quantity, however, is itself an observable, being measured by the 'large frequency separations'

$$\omega_{\ell,n+1} - \omega_{\ell,n} \simeq \frac{\pi}{T}. \tag{32}$$

The simple product of the small and the large frequency separations is thus sensitive only to the structure of the stellar core and can be used to study the structure of the core, free from the contaminating effects of the envelope.

ACKNOWLEDGMENTS

The authors thank J. Christensen-Dalsgaard for his solar model used in the computations. This work was supported in part by the UK SERC under grants GR/H 33466 and GR/H 33596.

REFERENCES

Christensen-Dalsgaard J., 1982, MNRAS, 199, 735

Christensen-Dalsgaard J., 1993, in Weiss W. W., Baglin A., eds, ASP Conf. Ser. Vol. 40, Inside the Stars. Astron. Soc. Pac., San Francisco, p. 483

Gabriel M., 1989, A&A, 226, 278

Gough D. O., Novotny E., 1990, Solar Phys., 128, 143

Gough D., Toomre J., 1991, ARA&A, 29, 627

Libbrecht K. G., Woodard M. F., 1991, Science, 253, 152

Roxburgh I. W., Vorontsov S. V., 1993, in Weiss W. W., Baglin A., eds, ASP Conf. Ser. Vol. 40, Inside the Stars. Astron. Soc. Pac., San Francisco, p. 535

Roxburgh I. W., Vorontsov S. V., 1994, MNRAS, in press

Tassoul M., 1980, ApJS, 43, 469

Tassoul M., 1990, ApJ, 358, 313

Van Hoolst T., Smeyers P., 1991, A&A, 248, 647

Vorontsov S. V., 1992, AZh, 69, 347 (SvA, 36, 175)

Unno W., Osaki Y., Ando H., Saio H., Shibahashi H., 1989, Nonradial Oscillations of Stars. University of Tokyo Press, Tokyo