Mon. Not. R. Astron. Soc. **356,** 671–679 (2005) doi:10.1111/j.1365-2966.2004.08487.x

The use of frequency-separation ratios for asteroseismology

H. Otí Floranes,[1⋆] J. Christensen-Dalsgaard[2⋆] and M. J. Thompson[3⋆†]

[1] *Facultad de Ciencias, Universidad de Cantabria, Avda. de los Castros s/n, 39005 Santander, Spain*
[2] *Institut for Fysik og Astronomi, Aarhus Universitet, Bygn. 520, Ny Munkegade, DK-8000 Aarhus C, Denmark*
[3] *Space and Atmospheric Physics Group, The Blackett Laboratory, Imperial College, London SW7 2BW*

Accepted 2004 October 7. Received 2004 August 31

ABSTRACT

The systematic patterns of separations between frequencies of modes of different degree and order are a characteristic of p-mode oscillations of stars. The frequency separations depend on the internal structure of the star and so measuring them in the observed oscillation spectra of variable stars gives valuable diagnostics of the interior of a star. Roxburgh & Vorontsov proposed using the *ratio* of the so-called small frequency separation to the large frequency separation as a diagnostic of the stellar interior, and demonstrated that this ratio was less sensitive than the individual frequency separations themselves to uncertain details of the near-surface structure. Here we derive kernels relating the frequency separation ratio to structure, and show why the ratio is relatively insensitive to the near-surface structure in terms of the very small amplitude of the kernels in the near-surface layers. We also investigate the behaviour of the separation ratio for stars of different masses and ages, and demonstrate the usefulness of the ratio in the so-called asteroseismic Hertzsprung–Russell diagram.

Key words: Sun: oscillations – stars: interiors – stars: variables: other.

1 INTRODUCTION

Helioseismology has proved the value of using the frequencies of resonant global modes for probing the internal structure of the Sun. The modes are excited by turbulent convection in the convective envelope of the Sun. Other stars with appreciable outer convective envelopes are expected to possess similar solar-like oscillations. Indeed there is now good evidence – predominantly from ground-based observations – for solar-like oscillations in a dozen such stars, including α Cen A (Bouchy & Carrier 2001), α Cen B (Carrier & Bourban 2003), Procyon (Martić et al. 1999; but see also Matthews et al. 2004), β Hyi (Bedding et al. 2001), η Boo (Kjeldsen et al. 1995) and ξ Hya (Frandsen et al. 2002): see, for example, the excellent review by Bedding & Kjeldsen (2003). Some efforts at seismological modelling of these stars using the observed frequencies have already been undertaken (e.g. Eggenberger et al. 2004).

The oscillations under consideration here are p modes, i.e. acoustic modes. The horizontal structure of each mode is described by a spherical harmonic of degree l ($l = 0, 1, 2, 3, \ldots$). Modes are also labelled by two other quantum numbers, namely the radial order n of the mode, which is essentially the number of nodes in the displacement eigenfunction from the centre of the star to its surface, and the (azimuthal) order of the spherical harmonic describing its horizontal structure. In the Sun, by virtue of its proximity, modes up to very high degrees are observed. A significant difference between asteroseismology and helioseismology is that, certainly for the foreseeable future, only oscillations on large horizontal scales (l=0,1,2,3) are likely to be detectable in solar-like stars other than the Sun. These are important modes to detect because the corresponding waves propagate all through the stellar interior and thus their frequencies contain information about the stars from their surface down to their core. Nuclear fusion in the core of a star largely drives its evolution and so probes of stellar cores are valuable for testing theories of stellar structure and evolution.

Asymptotically for large n, the low-degree p-mode frequencies of a near-spherical star have a rather regular pattern:

$$\nu_{n\,l} \simeq \frac{[n + (1/2)l + \epsilon]}{T} + \frac{1}{6} A l(l+1), \qquad (1)$$

where A, T and ϵ are constants: in particular T here is the sound travel-time across a diameter of the star (Tassoul 1980; Gough 1986). The last term is a rather small correction compared with the first term. It is apparent that modes of like l and with adjacent values of n are separated in frequency approximately regularly by an amount $1/T$; also that the frequencies of modes with degree l, order n and degree $l + 2$, order $n - 1$ differ by the small amount $(2l + 3)A/3$. This motivates the definition of the large frequency separation

$$\Delta_l \equiv \nu_{n\,l} - \nu_{n-1\,l} \qquad (2)$$

⋆E-mail: uc18008@correo.alumnos.unican.es (HOF); jcd@phys.au.dk (JCD); michael.thompson@sheffield.ac.uk (MJT)
†Present address: Department of Applied Mathematics, University of Sheffield, Sheffield S3 7RH.

between modes of like order, and the small frequency separation

$$d_{l\,l+2} \equiv \nu_{n\,l} - \nu_{n-1\,l+2} \tag{3}$$

between modes whose degrees differ by two. These are seismically useful in that Δ_0 and Δ_1 (for example) are global measures of the sound travel-time across the star, while d_{02} (for example) is sensitive in particular to the conditions in the evolving stellar core. Indeed, plotting the large separation versus the small separation for stellar models of different masses and ages yields the so-called asteroseismic Hertzsprung–Russell (H–R) diagram (Christensen-Dalsgaard 1984), whereby, if other physics is known, the large and small separations enable the stellar mass and age to be estimated.

For understanding and testing stellar evolution, the core is an extremely important region of a star. Unfortunately for asteroseismology, the more superficial layers of the star affect the mode frequencies considerably, because the sound speed is relatively low there and so acoustic waves spend more time propagating through the shallow layers than through the core. Waves corresponding to low-degree modes propagate almost vertically in the near-surface layers, independently of the degree l, so that the sensitivity of the mode to near-surface conditions is essentially a function only of frequency ν and independent of l. Thus the sensitivity to the near-surface layers of the small separation is relatively small, since this separation is formed from the difference of two nearly equal frequencies: none the less, some sensitivity to the near-surface structure remains.

Roxburgh & Vorontsov (2003, hereinafter RV03) proposed that a useful asteroseismic diagnostic that is even less sensitive to near-surface conditions in the star than the small separation is the *ratio* of small separation to large separation, e.g.

$$r_{02} \equiv d_{02}/\Delta_1, \tag{4}$$

and demonstrated the relative insensitivity of such ratios to near-surface structure by considering several ad hoc modifications to a model of the present Sun. This lack of sensitivity to the near-surface was explained by RV03 by demonstrating that the ratios are determined by partial phases for the core (a similar construct, but for the surface layers, was introduced for describing the sensitivity of mode frequencies to near-surface structure by Christensen-Dalsgaard & Pérez Hernández 1992). As an added advantage we note that both $d_{l\,l+2}$ and Δ_l approximately scale with stellar total mass M and surface radius R as $M^{1/2}R^{-3/2}$; evidently ratios such as r_{02} are independent of this scaling.

In the present work we demonstrate how the insensitivity of the ratios of small separations to large separations can be understood in terms of the properties of *kernels*. We further investigate the behaviour of the separation ratios for models of several stars of different mass and age, introducing artificial modifications to their near-surface structure in the manner of RV03 in order to see the effect on the frequency separations and frequency-separation ratios. We also consider the corresponding effect of changing the physical description of convective transport in a solar model. Finally we explore the properties of the asteroseismic H–R diagram constructed using the separation ratio r_{02} instead of the standard small separation d_{02}.

2 KERNELS FOR SEPARATION RATIO r_{nl}

Owing to the variational property of the frequencies of adiabatic oscillations, frequency differences $\delta\nu_{nl}$ between two sufficiently similar stellar models (or indeed two stars) are linearly related to the corresponding structural difference, which can typically be expressed in terms of differences $\underline{\delta X}$ in two structural variables, such as sound speed c and density ρ:

$$\delta\nu_{n\,l} = \int \underline{K}^{(n\,l)} \cdot \underline{\delta X}\,\mathrm{d}r \equiv \int \left[K_c^{(n\,l)}\frac{\delta c}{c} + K_\rho^{(n\,l)}\frac{\delta\rho}{\rho}\right]\mathrm{d}r, \tag{5}$$

where r is distance to the centre. The kernels $\underline{K}^{(n\,l)} \equiv [K_c^{(n\,l)}, K_\rho^{(n\,l)}]$ are determined by the structure and mode eigenfunctions of one of the models, chosen as reference (e.g. Gough & Thompson 1991). In the near-surface layers, the eigenfunctions for low-degree modes are essentially functions only of the mode frequency, not of its degree l. The kernels share this property. Hence we shall suppose that

$$\underline{K}^{(n\,l)}(r) \simeq \underline{K}(r;\nu_{n\,l}) \tag{6}$$

in the outer part of the interior of the star.

Now, for example, the small separation $d_{02} = \nu_{n\,0} - \nu_{n-1\,2}$ and so we can write down a kernel for the *difference* in d_{02} between two models:

$$\underline{K}_{d_{02}}^{(n)} = \underline{K}^{(n\,0)} - \underline{K}^{(n-1\,2)}. \tag{7}$$

(We shall henceforth suppress the superscript n on the difference kernels.) Given that the kernels depend only on ν in the outer part of the star, we can express this kernel in that region as

$$\begin{aligned}\underline{K}_{d_{02}}(r) &= \underline{K}(r;\nu_{n\,0}) - \underline{K}(r;\nu_{n-1\,2}) \\ &= \underline{K}(r;\nu_{n\,0}) - \underline{K}(r;\nu_{n\,0} - d_{02}) \\ &\simeq d_{02}\frac{\partial \underline{K}}{\partial \nu}.\end{aligned} \tag{8}$$

We may write down kernels for the difference in large separation, say in Δ_1, in like manner:

$$\underline{K}_{\Delta_1}(r) = \underline{K}(r;\nu_{n\,1}) - \underline{K}(r;\nu_{n-1\,1}) \simeq \Delta_1\frac{\partial \underline{K}}{\partial \nu} \tag{9}$$

in the outer part of the star.

Now $r_{02} = d_{02}/\Delta_1$ and so linearized perturbations (denoted by δ) of r_{02}, d_{02} and Δ_1 are related by

$$\delta r_{02} = \frac{1}{\Delta_1}(\delta d_{02} - r_{02}\delta\Delta_1) \tag{10}$$

and the kernel for differences in r_{02} is therefore

$$\underline{K}_{r_{02}} \equiv \frac{1}{\Delta_1}\left(\underline{K}_{d_{02}} - r_{02}\underline{K}_{\Delta_1}\right). \tag{11}$$

From this and from the near-surface approximations (8) and (9) for $\underline{K}_{d_{02}}$ and $\underline{K}_{\Delta_1}$, it follows that $\underline{K}_{r_{02}} \simeq 0$ in the near-surface layers of the star.

This property is illustrated in Fig. 1, which shows examples of kernels for d_{02} and r_{02} for a zero-age main-sequence model of solar mass. The original frequency kernels have substantially larger sensitivity near the surface than in the core, in accordance with the general sensitivity of the modes to stellar structure. For d_{02} this sensitivity is substantially reduced, confirming the asymptotic assertion that this quantity provides a measure of conditions in the stellar core; yet it retains substantial sensitivity also to the remainder of the star. As argued above, this sensitivity is very substantially reduced, particularly in the superficial layers, for r_{02}, the kernels for which are illustrated by the continuous lines.

3 SIMPLE MODIFICATIONS TO STELLAR MODELS

We have computed 'standard' evolutionary models of stars of different ages and masses. Models of the present Sun were based on

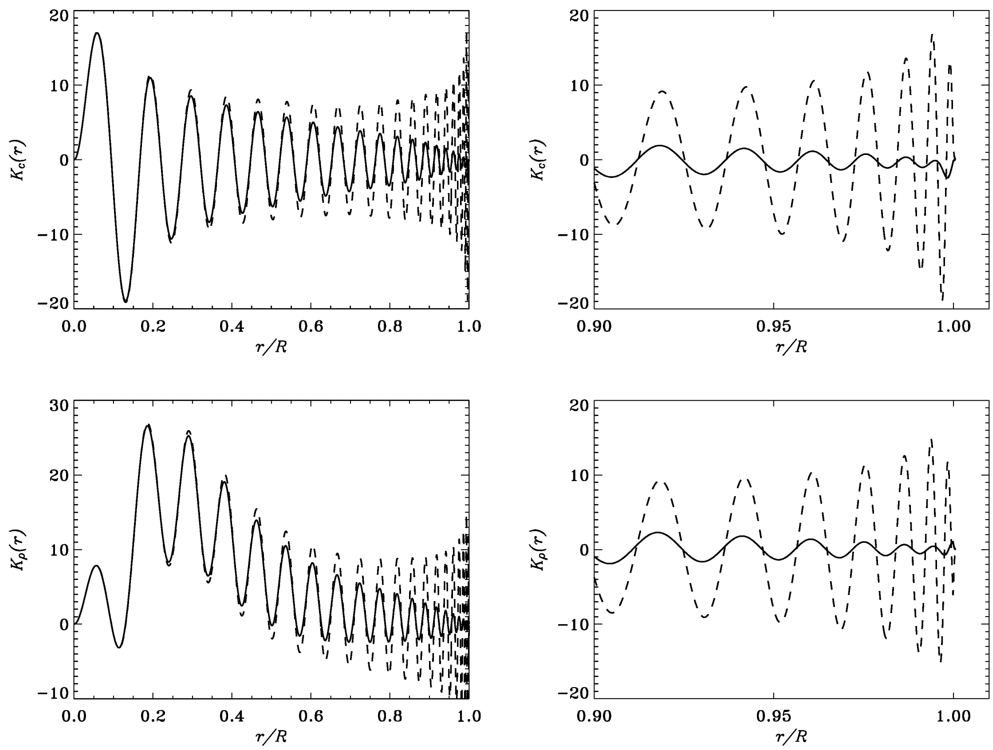

Figure 1. Kernels for the small separation d_{02} (dashed curves) and for the separation ratio r_{02} (continuous curves) as functions of fractional radius r/R, for $n = 20$, $l = 0$ and $n = 19$, $l = 2$ differences. The upper panels show the component of the kernels corresponding to sound speed, the lower panels show the component of the kernels corresponding to density. The panels on the right show in more detail the behaviour of the kernels in the near-surface layers.

Model S of Christensen-Dalsgaard et al. (1996), using the OPAL equation of state (Rogers, Swenson & Iglesias 1996) and opacity (Iglesias, Rogers & Wilson 1992), and including diffusion and settling of helium and heavy elements according to the formulation of Michaud & Proffitt (1993). The somewhat simpler models of other stars used instead the EFF equation of state (Eggleton, Faulkner & Flannery 1973) and neglected diffusion and settling. In order to investigate the effects on the frequency separations and separation ratios of changes to the near-surface structure, we have then constructed modified versions of these models by introducing one of the following three ad hoc modifications (RV03). (More details of how these modifications have been implemented are given in the appendix.)

Models A. In these models the value of the first adiabatic exponent γ_1 has been set equal to 5/3 everywhere in the model. This has a substantial effect only close to the surface, where γ_1 deviates considerably from 5/3 in the ionization zones of hydrogen and helium. The hydrostatic variables pressure (p), density (ρ), and mass (m) interior to r are unchanged as functions of r.

Models B. In these models the polytropic index μ has been modified to one given by $\mu = \mu_0 + \mu_1(r - r_{\mathrm{f}})$ in the region $r > r_{\mathrm{f}} \equiv 0.9\ R$. The parameters μ_0 and μ_1 are chosen such that the radius and mass of the models are the same as the original models, as is γ_1 at fixed p. The hydrostatic variables p, ρ and m are continuous at the matching point r_{f}.

Models C. In these models the stratification has been set to be that of an $\mu = 3/2$ polytrope with γ_1 equal to 5/3 throughout the outer region $r > 0.72\ R$ of the model. Consequently p, ρ and m are changed in this region: they are however continuous at $r = 0.72\ R$. The radii of Models C do not match the radii of the original models.

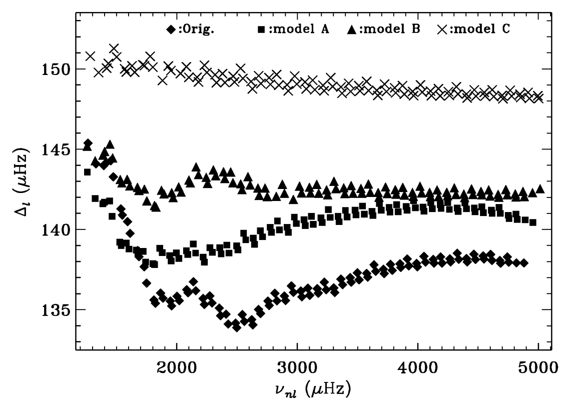

Figure 2. Large frequency separations Δ_l, $l = 0, \ldots, 3$, as a function of mode frequency, for the standard solar model (Model S) and for the three modified models (A, B, C) as described in the text.

Fig. 2 shows the large separations Δ_l between modes with $l = 0, \ldots, 3$ for the standard solar Model S and for the three modified models (Models A, B, C) based upon it. The large separations are generally like those exhibited by RV03 for their models, though not exactly the same. In particular, the separations for our Model B are somewhat different from those of their Model B. In fact their Model B did not have a linear variation in polytropic index as described in RV03, but rather had an envelope with a similarly imposed linear

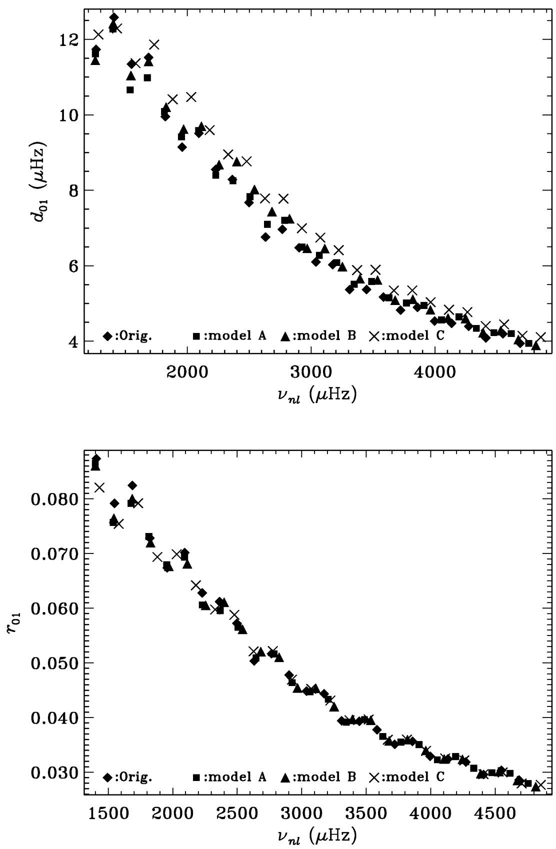

Figure 3. Upper panel: small frequency separation d_{01} as a function of mode frequency for the same four 1-M$_\odot$ models illustrated in Fig. 2. Lower panel: as for upper panel, but showing the scaled small separation r_{01}.

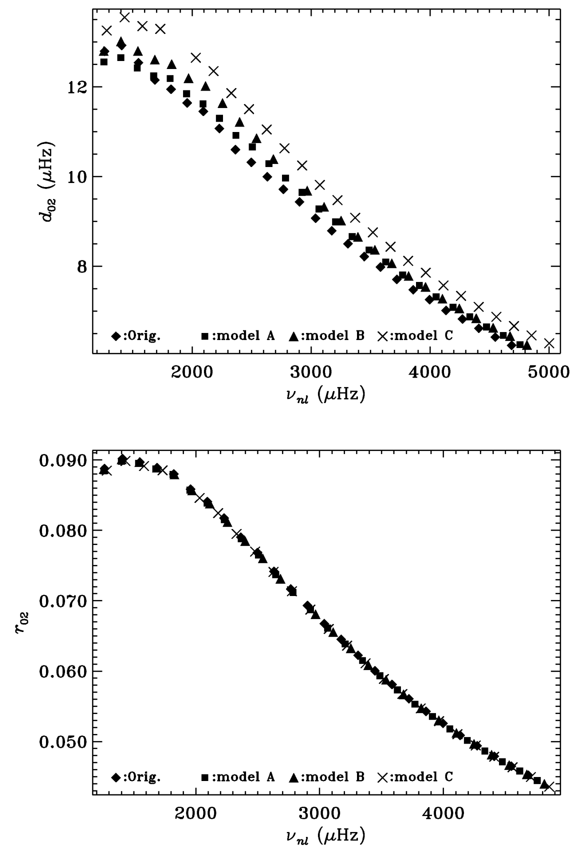

Figure 4. Upper panel: small frequency separation d_{02} as a function of mode frequency for the same four 1-M$_\odot$ models illustrated in Fig. 2. Lower panel: as for upper panel, but showing the scaled small separation r_{02}.

variation in $\nabla_\rho = \mathrm{d}\log\rho/\mathrm{d}\log p$ (I. Roxburgh, private communication).

The upper panel of Fig. 3 shows the small separation d_{01} defined as

$$d_{01} \equiv \frac{1}{2}\left(\nu_{n\,1} - 2\nu_{n\,0} + \nu_{n+1\,1}\right). \qquad (12)$$

This small separation depends only on the $l = 0$ and $l = 1$ modes: it has been argued that this may be advantageous over d_{02} if the visibility of $l = 2$ modes in stellar observations is poor. Its disadvantage is that it involves the combination of individual frequencies that are more widely separated in the spectrum, and so may be more sensitive to non-asymptotic effects such as arise from sharp transitions at convective boundaries. The lower panel of that figure shows the separation ratio $r_{01} \equiv d_{01}/\Delta_1$. It is evident that, as demonstrated by RV03, the ratios collapse on to nearly a single curve, demonstrating that they are less sensitive than the small separations to near-surface effects. (RV03 use a higher-order, five-point definition of d_{01}: we prefer the above three-point definition because it may be more suitable with real asteroseismic data containing observational noise.)

The analogous comparison of d_{02} and r_{02} is made in Fig. 4. Again, as demonstrated by RV03, taking the ratio reduces the small separation for all four models to essentially a single curve. Note that r_{01} is less smoothly varying than r_{02}: the small-scale variations in r_{01} come from the change in stratification at the base of the outer convective envelope of the models and might be used to make asteroseismic inferences about the nature and location of the base of the convective envelope.

We now consider main-sequence stars that differ from the Sun in both mass and age. First we consider a 0.85-M$_\odot$ star of age 14.55 Gyr (compared with the age of the Sun, 4.6 Gyr). This star is also more evolved than the Sun in terms of its central hydrogen abundance, having a central hydrogen abundance of 0.104 compared with its initial abundance of 0.693 and compared with the central abundance of the present-day solar model of 0.337. As for the solar case, we compute a set of three modified models (A, B, C) for the 0.85-M$_\odot$ star, with modifications as described earlier and in the appendix; note, in the case of Model C, that the modification is still confined to the outer convection zone, which in this case extends to $r = 0.664\,R$. The small separations and the corresponding ratios after division by the large separation are shown in Figs 5 and 6 for separations d_{01} and d_{02}, respectively. The reduction in sensitivity to the near-surface layers obtained by taking the separation ratios is very similar to that obtained in the solar case.

A rather different example is that of a zero-age 1.7-M$_\odot$ star. As before we have computed modified models A and C for this star. We were unable to obtain a satisfactory Model B in this case. Unlike the previous two examples, the model of such a star has no significant outer convective envelope. Consequently the outer part of the unmodified model does not at all closely resemble a $\mu = 3/2$ polytrope, and so Model C represents a very large modification of the stellar structure. This is apparent in Fig. 7: Model C has a significantly smaller radius. Consequently the acoustical diameter T is

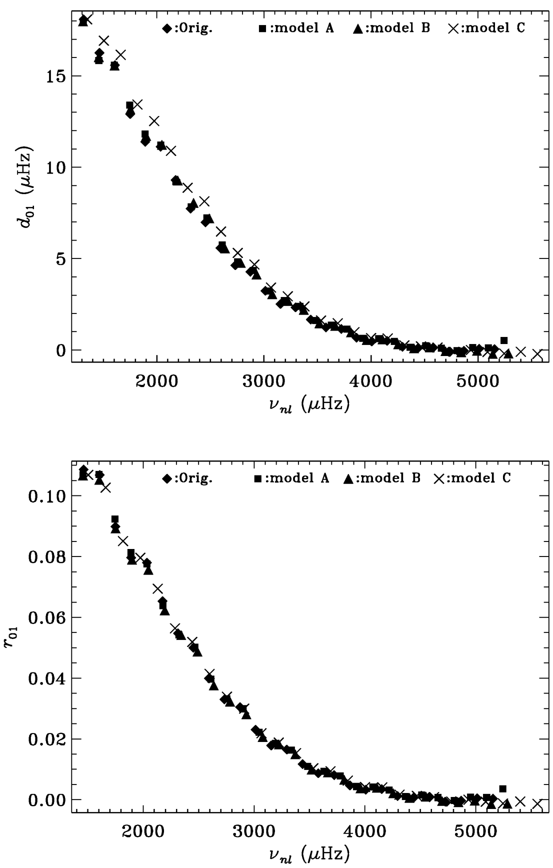

Figure 5. Upper panel: small frequency separation d_{01} as a function of mode frequency for a 14.55-Gyr 0.85-M$_\odot$ model, and for three models (A, B, C) modified as described in the text. Lower panel: as for upper panel, but showing the scaled small separation r_{01}.

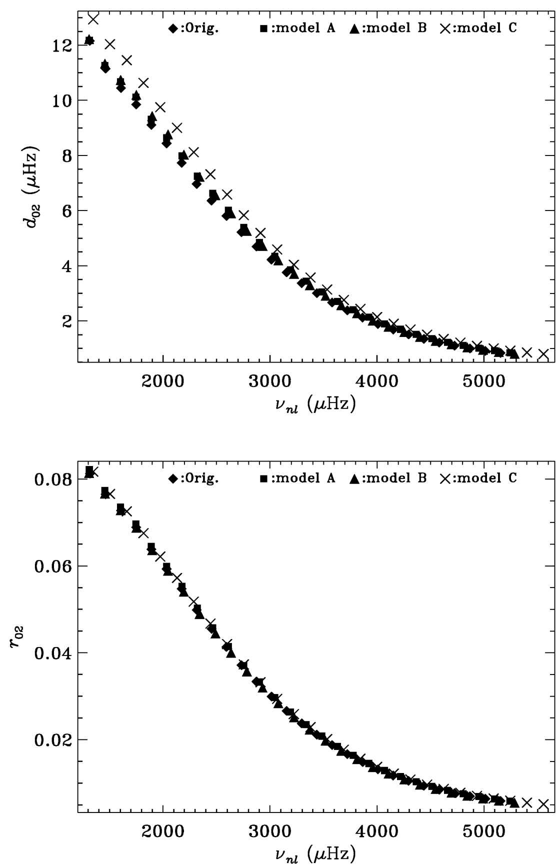

Figure 6. Upper panel: small frequency separation d_{02} as a function of mode frequency for a 14.55-Gyr, 0.85-M$_\odot$ model, and for three models (A, B, C) modified as described in the text. Lower panel: as for upper panel, but showing the scaled small separation r_{02}.

similarly reduced, and the large frequency separations, illustrated in Fig. 8, are very substantially increased. The separations are illustrated for the same mode orders in each case, but for Model C the frequencies are higher at fixed mode order and so too is the large frequency separation. This is a reflection of the shorter sound travel-time across the star for Model C compared with the unmodified star (or Model A) (cf. equation 1). Despite this very large difference in large separations, and in the frequencies for a given mode order, r_{02} as illustrated in the lower panel of Fig. 9 is essentially the same function of (dimensional) frequency. This reflects the fact that, as demonstrated by RV03, the ratio is determined by the internal phase shift, which is a function just of the core structure of the model.

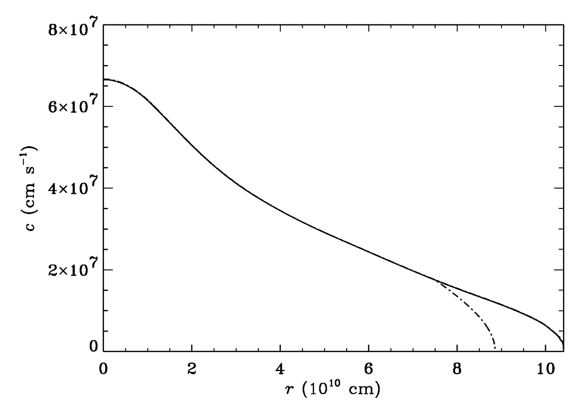

Figure 7. Sound speed as a function of radius in the zero-age 1.7-M$_\odot$ model (solid line) and in its Model C modification (dot–dashed line).

4 A MORE PHYSICALLY MOTIVATED NEAR-SURFACE MODIFICATION

The near-surface modifications introduced in Models A, B, C, following RV03, demonstrate well the decreased sensitivity of the separation ratios to the near-surface structure. However, they have no physical basis. When analysing asteroseismic stellar data we shall be confronted with real uncertainties regarding the surface regions of the stars. A significant source of uncertainty in stellar modelling is still the treatment of convection and so we now investigate the effect of modifying the convective treatment in a present-age solar model.

One model is Model S of Christensen-Dalsgaard et al. (1996), using the Böhm-Vitense (1958) mixing-length treatment. The second model uses identical physics, with the exception of treating convection with the formulation of Canuto & Mazzitelli (1991), based on a description of the spectrum of turbulence. Both models were calibrated to solar radius and luminosity at solar age. The effects on the model structure and frequencies of the difference in convection

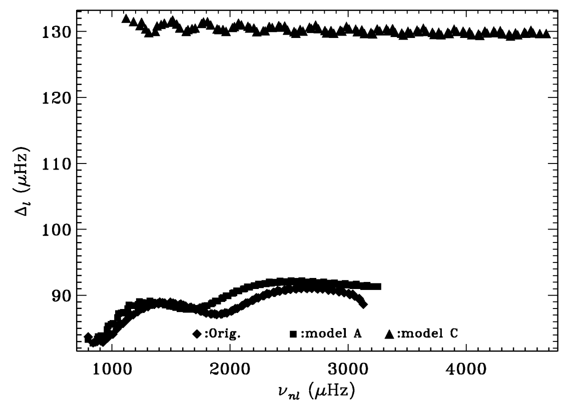

Figure 8. Large frequency separation Δ_l, $l = 0, \ldots, 3$, for a zero-age 1.7-$M_\odot$ model and for two models (A and C) modified as described in the text.

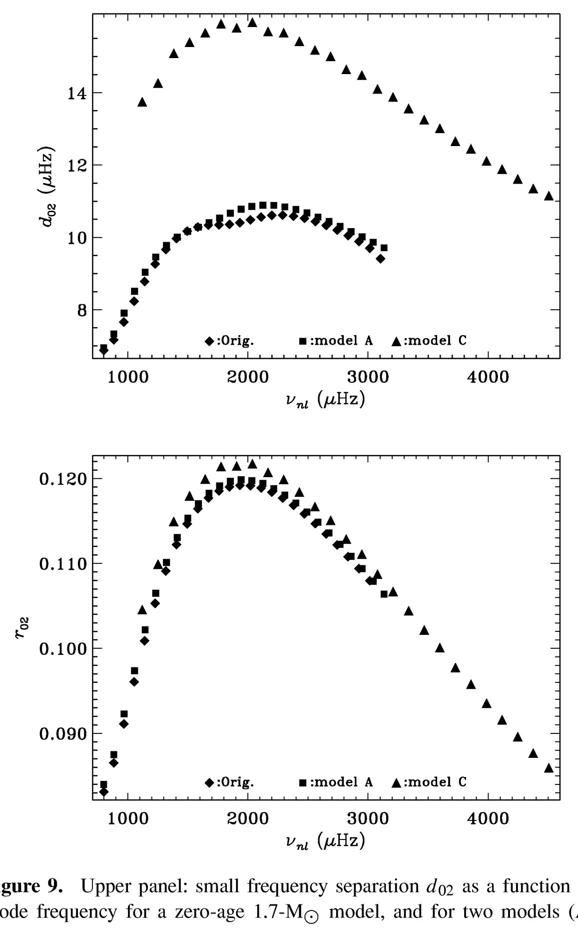

Figure 9. Upper panel: small frequency separation d_{02} as a function of mode frequency for a zero-age 1.7-$M_\odot$ model, and for two models (A, C) modified as described in the text. Lower panel: as for upper panel, but showing the scaled small separation r_{02}.

treatment were discussed by Monteiro, Christensen-Dalsgaard & Thompson (1996). The only significant structural changes caused by this change in convective treatment are in the near-surface layers ($r \gtrsim 0.995\ R$). Throughout the rest of the convective envelope the convection is highly efficient in both cases and the stratification is essentially adiabatic.

Fig. 10 (upper panel) compares the large frequency separation in the solar models with our standard mixing-length treatment of convection (solid curve) and with the Canuto-Mazzitelli (CM) mixing-length treatment (dashed curve). The modification, arising from the different near-surface sound speed in the two cases, is evident. In the lower panel, the dotted curve shows the *relative difference* between the d_{02} small separations in the standard and CM models. The differences are small, a few parts in 10^3. Once again, taking the ratio of the separations is very effective: the dot–dashed curve shows the relative difference between the r_{02} ratios for the two models, and the relative differences are reduced by an order of magnitude. This strongly suggests that the use of properly scaled quantities might be useful for investigations of subtle features of the solar core, reducing the effects of the remaining uncertainties in the modelling of the superficial layers. Also, the sensitivity of r_{02} to the convection treatment is likely to remain well below the likely errors in asteroseismic data in the foreseeable future.

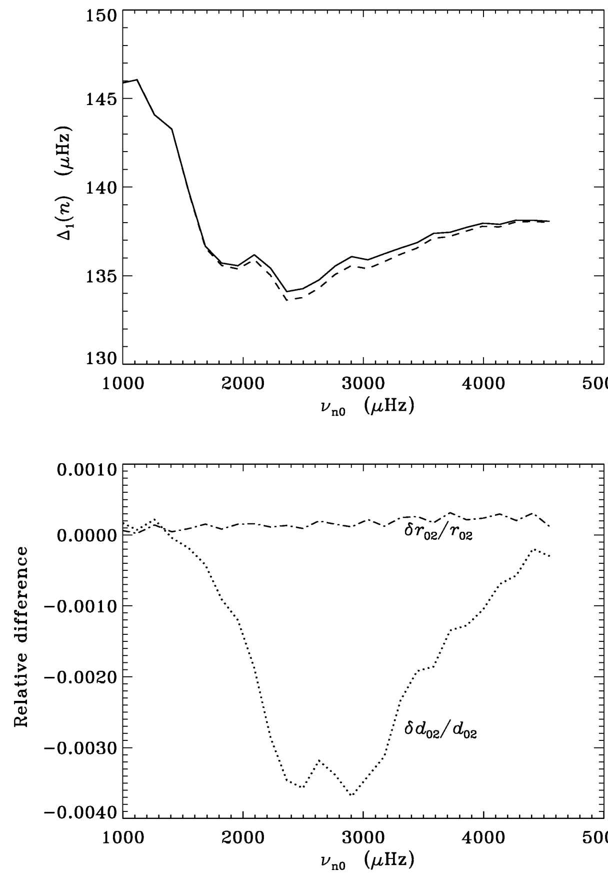

Figure 10. Upper panel: large frequency separations Δ_1 in a standard solar model (solid curve) and a model constructed using the CM treatment of convection (dashed curve). Lower panel: the relative difference between the small separation d_{02} in the sense of CM model minus standard model (dotted curve), and the corresponding relative difference between the separation ratio r_{02} between the two models (dot–dashed curve).

5 ASTEROSEISMIC DIAGNOSTIC DIAGRAMS

The 'standard' asteroseismic H–R diagram is a plot of some suitably chosen average of the large separation versus the small separation (Christensen-Dalsgaard 1984). An example is shown in the upper panel of Fig. 11. The averages were obtained in the manner of Scherrer et al. (1983): inspired by equation (1) a fit was carried out

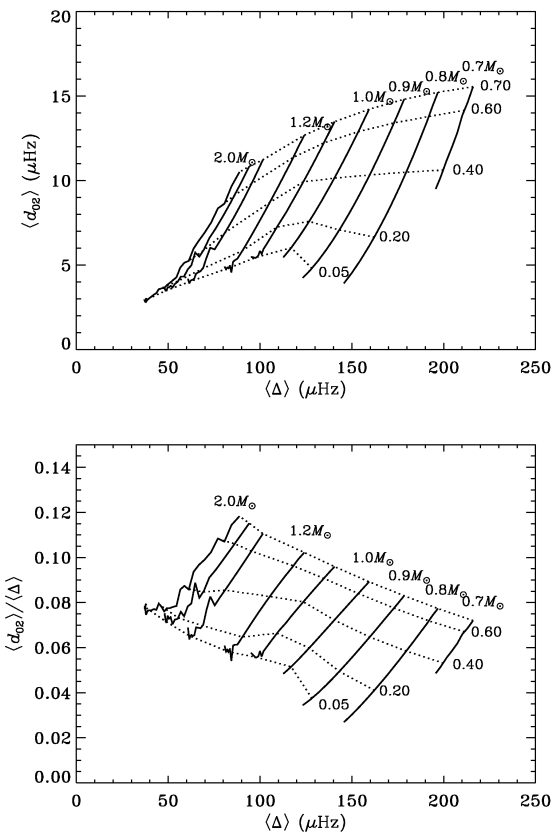

Figure 11. Upper panel: asteroseismic H–R diagram of average small separation against average large separation (see text), for main-sequence models of different masses and ages. All the models have the same input physics and the same initial compositions. Solid curves join models of the same mass; dotted curves join models with the same central hydrogen abundance and hence are the same fraction of the way through their main-sequence lifetimes. Lower panel: as above, but with the ratio of the average small separation to the average large separation on the ordinate axis, instead of simply the average small separation.

separately for each degree l to the frequencies up to second order in $n + l/2$, and the resulting coefficients, for $l = 0, \dots, 3$, were then fitted as linear functions of $l(l+1)$. These fits provide averages $\langle\Delta\rangle$ of the large separation and $\langle A\rangle$ of the coefficient in the second term of equation (1); the latter corresponds to an average $\langle d_{02}\rangle$ of d_{02}. In Fig. 11 solid curves show how a model of given mass evolves as it ages on the main sequence. As the star ages the sound speed decreases in its core, the increase in the mean molecular weight due to hydrogen burning dominating over the slight increase in temperature; also, its physical radius and acoustical radius increase. As a consequence the large and small separations both decrease. Dotted curves join up models that have the same central hydrogen abundance and hence are at the same fraction of the way through their main-sequence lifetimes. If other physical inputs are known, then the location of a main-sequence solar-like star in this diagram in principle allows its mass and age to be determined. We note however that for stars somewhat more massive than the Sun, i.e. around 2 $M_\odot$, there is a 'pile-up' so that the discrimination in the asteroseismic H–R diagram between stars of different masses is greatly reduced.

Inspired by RV03, the lower panel of Fig. 11 shows a similar asteroseismic H–R diagram but using the ratio $\langle d_{02}\rangle/\langle\Delta\rangle$ instead of the average small separation $\langle d_{02}\rangle$. This does not correspond strictly to the average of the individual ratios r_{02}, which include the frequency dependence of Δ_1, and hence probably does not to the same extent suppress the effect of the near-surface layers; however, at least it eliminates the overall scaling of the small separation with mass and radius of the star. Also, it does result in a more suitable diagnostic diagram: the layout of stars of different masses and ages within the diagram is similar to before, but with the advantage that lines of constant mass and lines of constant central hydrogen abundance are more nearly orthogonal. The near-degeneracy near masses of 2 $M_\odot$ is much reduced. Hence it is likely that the asteroseismic H–R diagram based on this ratio would provide a better discriminator of stellar masses and ages.

6 DISCUSSION AND CONCLUSION

A substantial problem in asteroseismic analyses of distant stars, for which only low-degree modes are available, is to isolate possibly subtle effects of the deep stellar interiors from the effects of the uncertain near-surface layers. An additional difficulty is the generally poor determinations of global stellar parameters, such as mass and radius. By taking ratios between small and large separations in the manner proposed by RV03 these problems are substantially reduced. In particular (as demonstrated by RV03) the resulting ratios are approximately determined by the internal phase shifts of the eigenfunctions, which depend on the structure of the stellar core but are essentially independent of the outer parts of the star. This is potentially of substantial importance to the study of the cores of older low-mass stars to determine their ages and test the age determinations of globular clusters – key goals of asteroseismology.

A powerful technique for analysing asteroseismic data is to use inverse analysis to determine corrections to an assumed reference model, from linearized relations between those corrections and the frequency differences between the star and the model (e.g. Basu, Christensen-Dalsgaard & Thompson 2002). We have demonstrated that the weight functions, or kernels, in such relations for the separation ratios have a strongly suppressed sensitivity to the near-surface layers. This suggests that it will be advantageous to carry out the inverse analysis for stars in terms of such combinations of the frequencies. Even in the solar case, where very extensive data on oscillation frequencies are available, the use of the separation ratios in inversions might improve the determination of subtle features in the solar core in the face of the substantial differences between the Sun and solar models near the surface.

To investigate the applicability of the technique we have considered several different models of main-sequence stars, with varying mass and age, in each case modifying the near-surface layers in manners similar to those applied to a solar model by RV03. In all cases we have found that the separation ratios are insensitive to such modifications. A particularly striking case, albeit admittedly somewhat artificial, is a modification of a 1.7-$M_\odot$ zero-age main-sequence model that substantially reduces the radius and hence introduces major changes to the frequencies. Even here, the separation ratios for the modified and original model were very similar. Thus it appears that such ratios can provide information about stellar cores even if the global properties of the star are quite uncertain.

The use for asteroseismic diagnostics of the small separation between neighbouring modes of different degree assumes that there are no large-scale asphericities in the star. The effect of such asphericities on the frequencies would depend on the latitude variation of the modes and hence on their degree, contributing to the frequency differences between the modes (e.g. Dziembowski & Goode 1997;

Gough 2003). An obvious source of such asphericities is the large-scale organization of stellar magnetic activity, such as seen in the Sun. It remains to be seen to what extent these effects can be eliminated through independent measures of activity or determination of the dependence of the frequencies on the azimuthal order (e.g. Gough 1993).

ACKNOWLEDGMENTS

We thank Ian Roxburgh for clarification of the nature of Roxburgh & Vorontsov's (2003) Model B. JCD and MJT are grateful to Michael Knölker (High Altitude Observatory) and Juri Toomre (University of Colorado) for their hospitality during the time when this work was being written up.

REFERENCES

Basu S., Christensen-Dalsgaard J., Thompson M. J., 2002, in Favata F., Roxburgh I. W., Galadí-Enríquez D., eds, ESA SP-485, Proc. 1st Eddington Workshop, Stellar Structure and Habitable Planet Finding. ESA Publications Division, Noordwijk, p. 249

Bedding T. R., Kjeldsen H., 2003, PASA, 20, 203

Bedding T. R. et al., 2001, ApJ, 549, L105

Böhm-Vitense E., 1958, Z. Astrophys., 46, 108

Bouchy F., Carrier F., 2001, A&A, 374, L5

Canuto V. M., Mazzitelli I., 1991, ApJ, 370, 295

Carrier F., Bourban G., 2003, A&A, 406, L23

Christensen-Dalsgaard J., 1984, in Mangeney A., Praderie F., eds, Space Research Prospects in Stellar Activity and Variability. Paris Observatory Press, Paris, p. 11

Christensen-Dalsgaard J., Berthomieu G., 1991, in Cox A. N., Livingston W. C., Matthews M., eds, Solar Interior and Atmosphere. Univ. of Arizona Press, Tucson, p. 401

Christensen-Dalsgaard J., Pérez Hernández F., 1992, MNRAS, 257, 62

Christensen-Dalsgaard J. et al., 1996, Sci, 272, 1286

Dziembowski W. A., Goode P. R., 1997, A&A, 317, 919

Eggenberger P., Charbonnel C., Talon S., Meynet G., Maeder A., Carrier F., Bourban G., 2004, A&A, 417, 235

Eggleton P. P., Faulkner J., Flannery B. P., 1973, A&A, 23, 325

Frandsen S. et al., 2002, A&A, 394, L5

Gough D. O., 1986, in Osaki Y., ed., Hydrodynamic and Magnetohydrodynamic Problems in the Sun and Stars. Department of Astronomy, Univ. of Tokyo, Tokyo, p. 117

Gough D. O., 1993, in Zahn J.-P., Zinn-Justin J., eds, Les Houches Session XLVII, Astrophysical Fluid Dynamics. Elsevier, Amsterdam, p. 399

Gough D. O., 2003, Ap&SS, 284, 165

Gough D. O., Thompson M. J., 1991, in Cox A. N., Livingston W. C., Matthews M., eds, Solar Interior and Atmosphere. Univ. of Arizona Press, Tucson, p. 519

Iglesias C. A., Rogers F. J., Wilson B. G., 1992, ApJ, 397, 717

Kjeldsen H., Bedding T. R., Viskum M., Frandsen S., 1995, AJ, 109, 1313

Martić M. et al., 1999, A&A, 351, 993

Matthews J. M., Kuschnig R., Guenther D. B., Walker G. A. H., Moffat A. F. J., Rucinski S. M., Sasselov D., Weiss W. W., 2004, Nat, 430, 51

Michaud G., Proffitt C. R., 1993, in Baglin A., Weiss W. W., eds, ASP Conf. Ser. 40, IAU Colloq. 137, Inside the Stars. Astron. Soc. Pac., San Francisco, p. 246

Monteiro M. J. P. F. G., Christensen-Dalsgaard J., Thompson M. J., 1996, A&A, 307, 624

Rogers F. J., Swenson F. J., Iglesias C. A., 1996, ApJ, 456, 902

Roxburgh I. W., Vorontsov S. V., 2003, A&A, 411, 215 (RV03)

Scherrer P. H., Wilcox J. M., Christensen-Dalsgaard J., Gough D. O., 1983, Solar Phys., 82, 75

Tassoul M., 1980, ApJS, 43, 469

APPENDIX A: COMPUTING THE MODIFIED MODELS

The variables that the Aarhus adiabatic pulsation code[1] (Christensen-Dalsgaard & Berthomieu 1991) uses to compute the frequencies of a stellar model are $a_1(x) \equiv q/x^3$, $a_2(x) \equiv V_g \equiv -\gamma_1^{-1}\,\mathrm{d}\log p/\mathrm{d}\log r$, $a_3(x) \equiv \gamma_1$ (the first adiabatic exponent), $a_4(x) \equiv \gamma_1^{-1}\,\mathrm{d}\log p/\mathrm{d}\log r - \mathrm{d}\log\rho/\mathrm{d}\log r$ and $a_5(x) \equiv U \equiv 4\pi\rho\, r^3/m$, where $x \equiv r/R$ and $q \equiv m/M$, R and M being the total radius and mass of the star.

As discussed in the text, models computed with consistent standard physical assumptions have been modified to produce models that differ in structure in the near-surface layers. The implementation of those modifications are as follows.

A1 Models A

The value of $\gamma_1 \equiv a_3$ has been set to 5/3 everywhere. The old value of V_g has been scaled by $\gamma_1/(5/3)$. Also (a technical point) the models used by the Aarhus package store $-(\gamma_1 p)^{-1}\,\mathrm{d}^2 p/\mathrm{d}x^2$ at the centre of the star: this has been scaled in the same way. The value of a_4 is not changed (nor are the values of a_1 and a_5).

A2 Models B

The modification to form Models B is to change the polytropic index μ in the outer layers $r > r_\mathrm{f} \equiv 0.9R$ with a linear function of radius $\mu_0 + \mu_1\,(r - r_\mathrm{f})$, where constant μ_0 is chosen to ensure continuity of the polytropic index with its value in the original model at the fitting radius r_f, and μ_1 is chosen to yield a model of the correct radius and mass. Models are recomputed with the revised polytropic index by integrating the equations

$$\frac{\mathrm{d}r}{\mathrm{d}p} = -\frac{r^2}{Gm\rho}, \tag{A1}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2\rho \tag{A2}$$

and

$$\frac{\mathrm{d}\log p}{\mathrm{d}r} = \left(1 + \frac{1}{\mu}\right)\frac{\mathrm{d}\log\rho}{\mathrm{d}r}. \tag{A3}$$

The modified models were integrated on the same pressure mesh as the original models, and the value of μ_1 was iterated until the original total radius R was obtained at the outermost pressure mesh point.

We note for completeness that RV03 used a slightly different formulation for their Model B (I. Roxburgh, private communication). They defined the structure of the outer layers in terms of

$$\nabla_\rho = \frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln p}, \tag{A4}$$

and replaced it for $r > r_\mathrm{f} = 0.9\,R$ by $\nabla_{\rho,0} + \nabla_{\rho,1}(r - r_\mathrm{f})$, imposing continuity of p, ρ and ∇_ρ at $r = r_\mathrm{f}$ and adjusting $\nabla_{\rho,1}$ to obtain a model with the original radius R.

A3 Models C

In these models the adiabatic exponent γ_1 is set to 5/3 and the polytropic index is set to $\mu = 3/2$ in the region $r > 0.72R$. Since

[1] Available at http://astro.phys.au.dk/~jcd/adipack.n/

μ is constant in this region, equation (A3) can be integrated to yield

$$p \propto \rho^{1+1/\mu}. \tag{A5}$$

Working on the pressure mesh of the original model, and with density continuous with the original model at $r = 0.72R$, this immediately yields ρ everywhere in the modified region. Equations (A1) and (A2) can then be integrated to yield the radius and mass variables as functions of p. The total radius of the model, taken to be at the outermost pressure mesh point, is not generally the same as in the original model.

This paper has been typeset from a TEX/LATEX file prepared by the author.