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Asteroseismic determination of helium abundance in stellar envelopes

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ABSTRACT

Intermediate degree modes of the solar oscillations have previously been used to determine the solar helium abundance to a high degree of precision. However, we cannot expect to observe such modes in other stars. In this work we investigate whether low degree modes that should be available from space-based asteroseismology missions can be used to determine the helium abundance, Y, in stellar envelopes with sufficient precision. We find that the oscillatory signal in the frequencies caused by the depression in Γ_1 in the second helium ionization zone can be used to determine the envelope helium abundance of low-mass main-sequence stars. For frequency errors of one part in 10^4, we expect errors σ_Y in the estimated helium abundance to range from 0.03 for 0.8-$M_\odot$ stars to 0.01 for 1.2-$M_\odot$ stars. The task is more complicated in evolved stars, such as subgiants, but is still feasible if the relative errors in the frequencies are less than 10^{-4}.

Key words: stars: abundances – stars: oscillations.

1 INTRODUCTION

The helium abundance of stars is often used to extrapolate back to obtain estimates of the primordial helium abundance, which is an important test of cosmological models. The helium content of the oldest stars cannot however, be measured directly – these are low mass stars, and their photospheres are not hot enough to excite helium lines that can be used to determine the helium abundance spectroscopically. Thus, the helium content of low-mass stars has to be derived from the evolution of heavier elements via stellar models. The evolution of stars, however, depends crucially on the helium content itself, making the knowledge of the helium abundance in stars very important. The estimated ages of globular clusters, which provide an important constraint on cosmological models, are known to depend sensitively on the efficiency of helium diffusion in the envelope (Deliyannis, Demarque & Kawaler 1990; Proffitt & VandenBerg 1991; Deliyannis & Demarque 1991; Chaboyer et al. 1992).

The helium abundance of the solar envelope has been successfully determined using helioseismic data (Gough 1984; Däppen et al. 1991; Basu & Antia 1995, etc.) These seismic determinations were based on the fact that the ionization of hydrogen and helium causes a distinct, localized depression in the adiabatic index, Γ_1, in the near-surface layers of the Sun. The first helium ionization zone is not very useful in making seismic inferences – it overlaps with the hydrogen ionization zone and is located very close to the stellar surface where there are significant uncertainties in seismic inversions. The second helium ionization zone, which is located below the highly superadiabatic layer of the convection zone, is not particularly sensitive to surface effects and hence is useful in determining the helium abundance. The depression of Γ_1 in this region can be directly related to the abundance of helium; this depression increases with increase in the helium abundance (see Fig. 1). The depression in Γ_1 causes a localized depression in the derivative of the sound speed in the region, and hence affects all acoustic modes that travel through that region. Thus, using helioseismic inversion techniques it is possible to determine the sound speed in the solar interior, which in turn could be calibrated to obtain the helium abundance in the solar envelope. All studies to determine the helium abundance in the Sun used intermediate degree modes to measure the sound speed in the He II ionization zone.

A number of current and proposed space-based asteroseismic missions such as Microvariablity and Oscillations of Stars (MOST; Walker et al. 2003), Convection Rotation and Planetary Transits (COROT; Baglin 2003), Measuring Oscillations in Nearby Stars (MONS; Kjeldsen, Christensen-Dalsgaard & Bedding 2003) and *Eddington* (Roxburgh & Favata 2003) are expected to measure the oscillation frequencies of several stars with sufficient accuracy to enable a more detailed study of stellar structure than has been possible so far. Ground-based observations have already started measuring the frequencies of many stars despite the difficulties of such measurements (Bouchy & Carrier 2002). Unfortunately, none of these observations can or will be able to observe the intermediate degree modes of oscillation that have been so useful in solar studies. These missions can, at most, hope to determine the frequencies of modes with degree $\ell \leqslant 3$. Therefore, it is important to devise a practical way for determining the helium content of stellar envelopes using only low degree modes.

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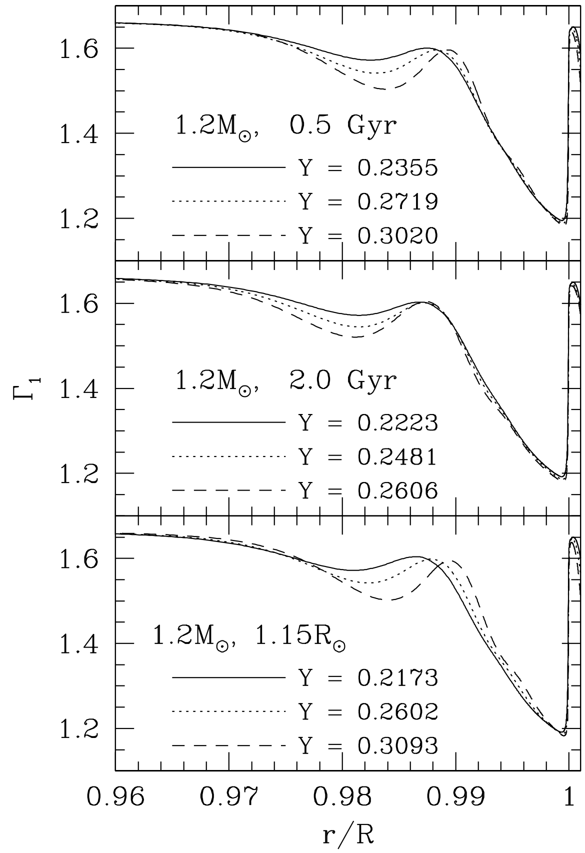

Figure 1. The adiabatic index, Γ_1, as a function of fractional radius for 1.2-$M_\odot$ models. The upper two panels are models with different *Y* evolved to the age indicated in the panels. The bottom panel shows Γ_1 for 1.2-$M_\odot$ models evolved until the surface is at a radius of 1.15 $R_\odot$. The lines of different types represent models with different helium abundances as marked in each panel.

Any localized feature in the sound speed inside a star, such as that caused by the change in the temperature gradient at the base of the convection zone, introduces an oscillatory term in frequencies as a function of radial order *n*, which is proportional to

$$\sin(2\tau_m\omega_{n,\ell}+\phi), \qquad \tau_m=\int_{r_m}^{R}\frac{\mathrm{d}r}{c}, \tag{1}$$

(e.g. Gough 1990). Here, τ_m is the acoustic depth of the localized feature, c is the speed of sound, r_m is the radial distance where the feature is located, $\omega_{n,\ell}$ is the angular frequency of a mode with radial order n and degree ℓ, and ϕ is a phase. This oscillatory signature has been extensively studied for the Sun in order to determine the extent of overshoot below the solar convection zone (Basu, Antia & Narasimha 1994; Monteiro, Christensen-Dalsgaard & Thompson 1994). The He II ionization zone also gives rise to a similar oscillatory signal, which can be extracted from frequencies of low degree modes (see, for example, Roxburgh & Vorontsov 2001). The present work is motivated by the possibility of using this signal in low degree seismic data from distant stars to study the properties of the He II ionization zone.

The depression in Γ_1 in the He II ionization zone is not a very sharp feature and has a finite width. However, we can show that it still gives rise to an oscillatory signal in the frequencies and that the amplitude of the oscillatory term caused by the depression in Γ_1 depends on the amount of helium present at the location of ionization. In principle, if this amplitude is measured, it can be compared against models to estimate the helium abundance in the stellar envelope. Monteiro & Thompson (1998) have studied the oscillatory signal arising from the He II ionization zone in stellar models. They have shown that, because of the finite width of this zone, the amplitude of oscillatory signal in frequencies is modulated by an additional factor

$$\sin^2(\omega\beta)/(\omega\beta), \tag{2}$$

where β is half the acoustic thickness of the dip in Γ_1. Because β is rather small, of the order of 100 s, this factor provides a slow modulation in amplitude of the oscillatory term. Miglio et al. (2003) have also examined the oscillatory signal due to He II ionization zone and pointed out the possibility of using it to determine the helium abundance.

Basu, Antia & Demarque (2004) examined several stellar models to study the possibility of determining helium abundance from the amplitude of the oscillatory signal. They calculated the amplitudes using the second differences of the frequencies. In this work we extend that study to a much larger sample of models to explore possible systematic errors in the measurement of helium abundance using only low degree modes. We also look at models with substantially different abundances, such as stars with Population II abundances. We also look at stars beyond the main sequence. The rest of the paper is organized as follows. We describe the measurement method in Section 2 and give details of the models we have used in this study in Section 3. The results (including those for solar low degree data) are presented in Section 4 and we discuss our conclusions in Section 5.

2 TECHNIQUE

The amplitude of the oscillatory signal due to the depression in Γ_1 in the He II ionization zone is very small and it may be desirable to amplify it for a proper measurement. We amplify the signal by taking the second difference of the frequencies,

$$\delta^2\nu_{n,\ell}=\nu_{n+1,\ell}-2\nu_{n,\ell}+\nu_{n-1,\ell}, \tag{3}$$

for modes with the same degree. We use modes of degrees $\ell =$ 0–3. In addition to amplifying the oscillatory signal, the process of taking second differences suppresses the dominant smooth trend of the frequencies to a large extent, making it easier to fit and measure the oscillatory term. The second differences are then fitted to the form used by Basu (1997), but without the degree-dependent terms (which are not relevant for low degrees), i.e.

$$\begin{aligned}\delta^2\nu=&\left(a_1+a_2\nu+\frac{a_3}{\nu^2}\right)\\&+\left(b_1+\frac{b_2}{\nu^2}\right)\sin(4\pi\nu\tau_{\rm He}+\phi_{\rm He})\\&+\left(c_1+\frac{c_2}{\nu^2}\right)\sin(4\pi\nu\tau_{\rm CZ}+\phi_{\rm CZ}).\end{aligned} \tag{4}$$

Here, the first term with coefficients a_1, a_2, a_3 defines the smooth part of the second difference, the second term is the oscillatory signal from the He II ionization zone, and the last term is the oscillatory signal from the base of the convection zone. In equation (4), the terms $\tau_{\rm He}$ and $\tau_{\rm CZ}$ are the acoustic depths of the He II ionization zone and the base of the convection zone, respectively. The terms $\phi_{\rm He}$ and $\phi_{\rm CZ}$ are the phases of the corresponding oscillatory signals. The parameters a_1, a_2, a_3, b_1, b_2, $\tau_{\rm He}$, $\phi_{\rm He}$, c_1, c_2, $\tau_{\rm CZ}$ and $\phi_{\rm CZ}$ are determined by least-squares fits to the second differences of the frequencies.

As mentioned earlier, the He II ionization zone actually has a small but finite width and the first derivative of the squared sound speed (after some scaling) can be better approximated by a triangular function rather than a discontinuity. To account for the extra modulation in the oscillatory term due to the finite width of the ionization zone (see equation 2), we have also fitted the oscillation with the modulating factor $\sin^2(\omega\beta)/(\omega\beta)$ (Monteiro & Thompson 1998) in the amplitude keeping β as a free parameter:

$$\delta^2\nu = \left(a_1 + a_2\nu + a_3\nu^2\right) + \left[\frac{b_1 \sin^2(2\pi\nu\beta)}{\nu\beta}\right] \sin(4\pi\nu\tau_{\mathrm{He}} + \phi_{\mathrm{He}}) + \left(c_1 + \frac{c_2}{\nu} + \frac{c_3}{\nu^2}\right) \sin(4\pi\nu\tau_{\mathrm{CZ}} + \phi_{\mathrm{CZ}}). \quad (5)$$

For most of the cases considered in this paper, β is found to be of the order of 100 s and hence, for the range of frequencies included in the fits, the term $\sin^2(2\pi\nu\beta)$ can be approximated reasonably well by a quadratic term as assumed in equation (4). Thus, although the fitting form in equation (4) had been empirically determined to fit the signature from the base of the solar convection zone using intermediate degree solar modes, it is expected to fit the signature from the He II ionization zone in low degree modes quite well.

To study the sensitivity of fitted amplitude and acoustic depth to the fitting form, we have tried one more variation where the oscillatory term due to He II ionization zone in equation (5) is modified to

$$\left(b_1 + \frac{b_2}{\nu} + \frac{b_3}{\nu^2}\right) \sin(4\pi\nu\tau_{\mathrm{He}} + \phi_{\mathrm{He}}). \quad (6)$$

This form is obtained by approximating the factor $\sin^2(2\pi\nu\beta)/(\nu\beta)$ in equation (5) by a quadratic in $1/\nu$. Because β is small, such an approximation is reasonably good. This form differs from equation (4) through the addition of a linear term in $1/\nu$ in amplitudes.

The fits of the three forms to frequencies generated by a 1.2-M$_\odot$, 1.2-R$_\odot$ model are shown in Fig. 2. Each fit is a composite function representing the smooth trend in the second differences, a dominant slow oscillation due to the second helium ionization zone, and a higher frequency oscillation arising from the base of the convection zone. Each panel in the figure shows two fits, one for each of the two extreme values of the initial helium abundance Y_0 that we have chosen for this study. As can be seen in the figure, the amplitude of the slower oscillation clearly increases with the amount of helium present in the envelope. We aim to calibrate this increase in the amplitude of the oscillatory signal from the He II ionization zone against the helium content. We perform such fits for each of the stellar models. Because the amplitude of the signal is a function of frequency, we use the fits to obtain the mean amplitude over the fitting interval. There is not much difference in the fits obtained using the three forms given by equations (4)–(6), as borne out by Fig. 2. This ensures that the amplitudes are not too sensitive to the particular fitting form chosen.

The frequencies of stellar oscillations roughly scale as $\sqrt{\bar{\rho}}$, where $\bar{\rho} \propto M/R^3$ is the mean density of the star. Because the models have different mean densities, it is convenient to scale the frequencies by the factor

$$f = \sqrt{\bar{\rho}/\bar{\rho}_\odot} \quad (7)$$

before taking the differences and fitting the signal. However, this requires a knowledge of M/R^3 for the stars to be studied, which may not be known for all stars that may be targets of asteroseismic study. This factor can actually be estimated from the seismic data

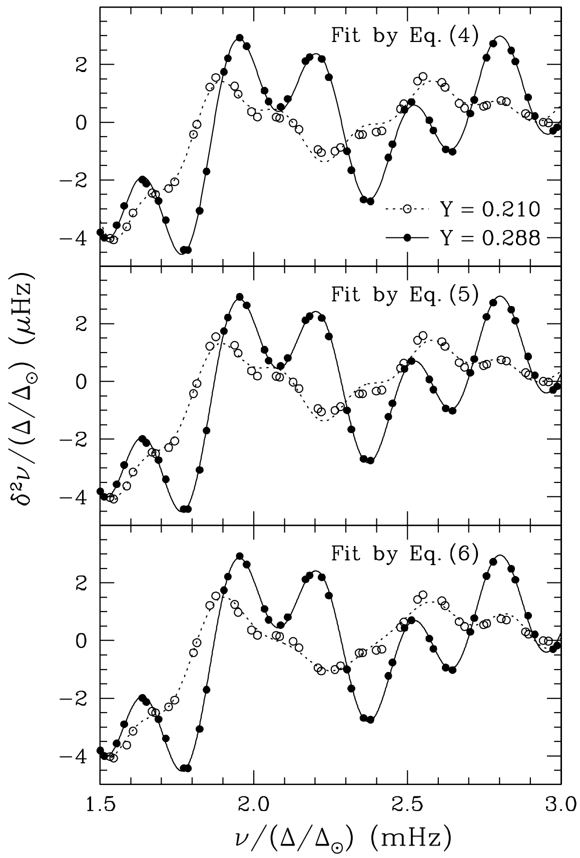

Figure 2. A sample of the fits to the second differences of the scaled frequencies. The points are the 'data', and the lines the fits to the points. The examples shown are for a 1.2-M$_\odot$ model evolved to a radius of 1.2 R$_\odot$.

using the large frequency separation,

$$\Delta_{n,\ell} = \nu_{n+1,\ell} - \nu_{n,\ell} \quad (8)$$

(see, for example, Mazumdar & Antia 2001; Christensen-Dalsgaard 2003). We find that the averaged large frequency separation scaled by the factor in equation (7) is around 135 μHz for all the main-sequence stellar models in the range of masses considered in this study. We therefore scale the frequencies using the average frequency separation to avoid the use of the potentially unknown factor M/R^3. We have chosen a frequency range of 1.5–3.0 mHz after scaling by $\Delta/\Delta_\odot$, where Δ is the mean large frequency separation and $\Delta_\odot = 135$ μHz is the value for the Sun. The choice of the frequency range is dictated by practical reasons. At higher frequencies the oscillatory signal due to the He II ionization zone becomes small. Furthermore, the observational error is expected to increase in higher frequency modes, as is the case for solar data. At lower frequencies, there is significant departure from the fitting forms assumed in this work because the low n modes cannot be approximated well by the asymptotic formulae used in obtaining the oscillatory form (Monteiro 1996).

It can be shown that the process of taking the second differences magnifies the amplitude of the oscillatory functions by a factor

$$4\sin^2(2\pi\tau\Delta), \quad (9)$$

where Δ is the mean large frequency separation. In order to obtain the real amplitude of the oscillatory signal, i.e. the amplitude of the oscillations in the frequencies, we divide the amplitude obtained for the second differences by this factor. This conversion back to frequencies is found to remove some variation in the amplitude between different models (Mazumdar & Antia 2001). All the amplitudes presented in this work are the amplitudes of the oscillatory function in the frequencies that we obtain by converting the fitted amplitudes in the second differences using the conversion factor in equation (9). We find that for the He II ionization zone, this factor is not very different from unity and hence the amplitudes are similar for both frequencies and the second differences of frequencies. This magnification is, however, quite large for the signal from the base of the convection zone, making it easier to fit the two oscillatory terms simultaneously in the second differences.

By fitting the oscillatory term in the second differences, it is possible to determine the characteristics of the He II ionization zone, like its acoustic depth and the thickness and amplitude of the depression in Γ_1 from the fitted parameters. We would expect the amplitude of the oscillatory signature to increase with helium abundance and hence it can be calibrated using stellar models with known Y. Miglio et al. (2003) have suggested that the area of the bump in Γ_1 should be a measure of helium abundance. The product of amplitude with half-width β provides a measure of this area. We find that it is difficult to estimate β reliably by fitting the oscillatory term. We find a large scatter in the results if the product of the amplitude with β is used, which makes it difficult to determine Y, and hence we do not use it in our study. Instead, we try to find a relation directly between the amplitude of oscillatory term and the helium abundance.

3 STELLAR MODELS

We use the technique described in the previous section to study the properties of the oscillatory signal in the frequencies of low degree modes and its relationship with helium abundance in the stellar envelope using a large number of stellar models. We restrict ourselves to stars with masses close to the mass of the Sun, namely, the range 0.8–1.4 $M_{\odot}$. The upper limit of the mass range is set by the fact that it is difficult to extract the oscillatory signal in more massive stars. This occurs because the acoustic depths of the base of the convection zone and that of the He II ionization zone are very similar in higher mass stars, and the two oscillatory signals, one from the convection zone base and the other from the helium ionization zone, interfere with each other. The lower mass limit is dictated by the fact that the amplitude of the oscillatory signal reduces with mass, although there is no particular difficulty in determining the signal as we are using exact model frequencies. In actual practice, the lower limit on the stellar mass that can be studied will be determined by the precision to which the frequencies can be determined from the future asteroseismic missions. Because observed frequencies will have uncertainties associated with them, we have examined the effect of frequency errors on the fits in Section 4. We study stellar models evolved to different ages on the main sequence for masses 0.8, 1, 1.2 and 1.4 $M_{\odot}$. We have also constructed a few stellar models which have evolved beyond the main sequence and are on the subgiant branch.

The stellar models were constructed using YREC, the Yale Rotating Evolution Code in its non-rotating configuration (Guenther et al. 1992). These models use the OPAL equation of state (EOS; Rogers & Nayfonov 2002), OPAL opacities (Iglesias & Rogers 1996), low-temperature opacities of Alexander & Ferguson (1994) and nuclear reaction rates as used by Bahcall & Pinsonneault (1992). The models take into account diffusion of helium and heavy elements, using the prescription of Thoul, Bahcall & Loeb (1994).

Most of the models have been constructed with Population I abundances. These models have an initial heavy element abundance $Z_0 = 0.022$ and were constructed with a mixing length parameter $\alpha = 2.1$. The initial heavy element abundance and the mixing length are the same as those used to construct calibrated standard solar models using YREC (Winnick et al. 2002). The value of the initial helium abundance Y_0 is varied: we use five values of Y_0 for the Population I models, $Y_0 = 0.24$, 0.26, 0.28, 0.30 and 0.32. Diffusion and gravitational settling of helium causes the helium abundance in the stellar envelope to change with time. Therefore, the helium content in the envelope of a star still evolving on the main sequence will decrease progressively with age from its initial (or zero-age main-sequence) helium abundance Y_0. Beyond the main sequence the dredge-up of helium begins to increase the helium abundance in the envelope. Thus, the amplitude of the He II signal will not reflect the initial abundance Y_0 of the models; it will depend on the abundance of helium in the stellar envelope at the stage we examine it. As a result we always use the actual, rather than the initial, helium abundance in the envelope to compare and calibrate the amplitude of oscillatory signal. To investigate systematic errors caused by model parameters, we have also constructed models with a slightly different value of the initial heavy element abundance ($Z_0 = 0.018$) and some models with a slightly different value of the mixing length parameter ($\alpha = 1.7$).

To test whether the method outlined in this paper also applies to stars of very different heavy element abundances, we have two other sets of models. One set with initial abundance Z_0 of 0.007 (the abundance of stars like τ Ceti) and another set with Population II abundance of $Z_0 = 0.001$. Models for these two sets were constructed with initial Y of 0.22, 0.24, 0.26, 0.28 and 0.30. For each of the stellar models, we calculate the frequencies of low degree modes and attempt to extract the oscillatory term as explained in the previous section.

From our experience of working with solar data, we expect that one of the important sources of uncertainty is the EOS which determines the width and the depth of the dip in Γ_1. The role of the EOS has been extensively studied for the solar case (Antia & Basu 1994; Richard et al. 1998; Basu 1998) by looking at models constructed with OPAL, the Mihalas, Hummer & Däppen (henceforth MHD) EOS (Däppen et al. 1988; Hummer & Mihalas 1988; Mihalas, Däppen & Hummer 1988) as well as other simpler EOS. Monteiro & Thompson (1998) have studied the influence of the EOS in the stellar case too and have found that the half-width β of the He II ionization zone is determined by the EOS. Helioseismic studies have shown that in the helium ionization zone, the OPAL EOS is a better representation of the stellar EOS (see, for example, Basu & Antia 1995; Basu & Christensen-Dalsgaard 1997; Basu, Däppen & Nayfonov 1999) and hence, in this work we restrict ourselves to models with the OPAL EOS, although we do study the effect of the EOS using a few 1-$M_{\odot}$ models constructed with the MHD EOS. This should give a reasonable estimate of the uncertainty associated with the EOS.

4 RESULTS

4.1 Population I main-sequence models

Fig. 3 shows the acoustic depth and the average amplitude of the oscillatory signal due to the He II ionization zone for the standard set of main-sequence models used in the study. It can be seen that the acoustic depth $\tau_{\rm He}$ is within a factor of 2 for all models, while the amplitude varies over a large range. It should be noted that all values of the acoustic depth and the amplitude listed in this paper are those obtained after scaling by the frequency separation. The actual

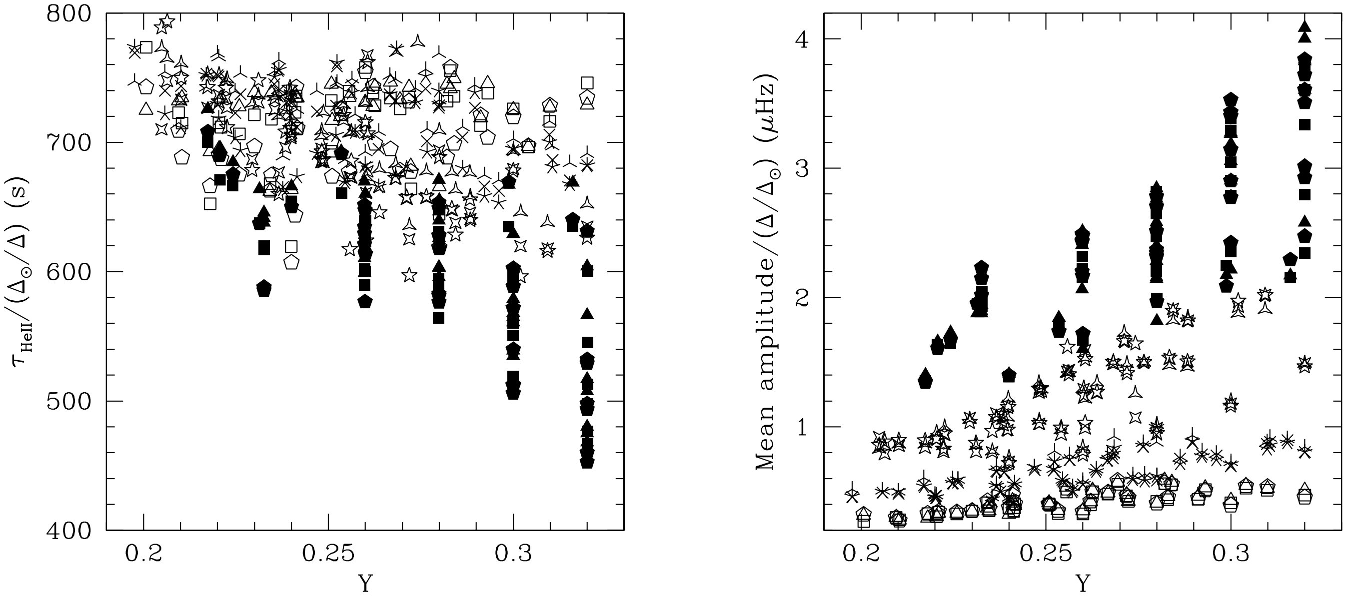

Figure 3. The scaled acoustic depth $\tau_{\rm He}$ and the mean amplitude of the oscillatory part in frequency due to the He II ionization zone in the scaled frequency range of 1.5–3.0 mHz are shown as a function of the helium abundance in the envelope. The open symbols represent 0.8-M$_\odot$ models, skeletal symbols show 1-M$_\odot$ models, stars denote 1.2-M$_\odot$ models and filled symbols show 1.4-M$_\odot$ models. For each model the results for the three independent fits using equations (4), (5) and (6) are shown by three-, four- and five-sided symbols, respectively.

values of $\tau_{\rm He}$ may show significant variation because of variation in M/R^3 or equivalently in Δ. On average, the amplitude appears to increase with Y, the helium abundance in the stellar envelope, but for a given helium abundance, the amplitude also increases with stellar mass. This figure combines the results obtained using different fitting forms and it can be seen that there is good agreement between different results, although for 1.4-M$_\odot$ models the difference tends to become large as the fits are not very good for many of these models. The mean difference in amplitudes obtained using two different fitting forms is found to be about 0.02 μHz for stellar models with masses 0.8 M$_\odot$, while for 1-M$_\odot$ and 1.2-M$_\odot$ models it is about 0.06 μHz. As we shall see later, these differences are smaller than the errors that may be expected to arise from uncertainties in observed frequencies. For 1.4-M$_\odot$ models the mean difference in amplitudes between two fitting forms is about 0.18 μHz, which is comparable to the errors expected to be caused by uncertainties in the observed frequencies. To show the variation of amplitude with Y more clearly, Fig. 4 shows the amplitude as a function of Y for each mass separately. There is considerable scatter around the mean trend because of factors such as age, and hence the evolutionary state.

The dependence of the amplitude on factors other than the helium abundance complicates the determination of the helium abundance of a given star. Our task is made easier if conventional observations combined with modelling give us a reasonable idea of the radius, R, and mass, M, of the star. If only the radius is known, we can estimate M/R^3 from the frequency separation, and combining that with the known radius will give an estimate of mass. Similarly, if only the stellar mass is known independently, we can use the frequency separation to estimate the stellar radius. This would enable us to make calibration models with a given radius and mass. Plots of the mean amplitude as a function of Y for models with given masses and radii are shown in Fig. 5. In this case, all we would need to do to determine Y is to determine the mean amplitude from the observations and interpolate. Models of the same mass but different initial helium abundances will have different ages when they have the same radius; as we shall see in Section 4.2, this does not affect the calibration process.

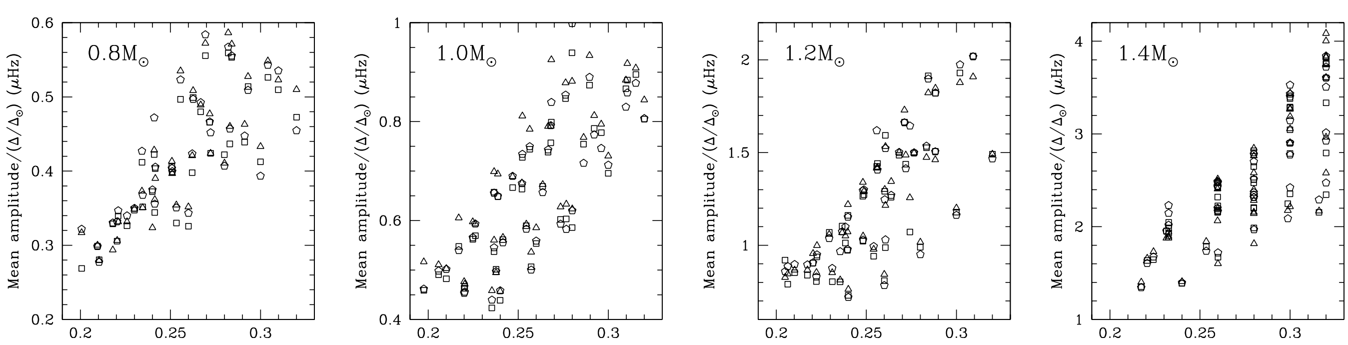

Figure 4. The mean amplitude of the oscillatory part in frequency due to the He II ionization zone is shown as a function of the helium abundance in the envelope. Each panel shows the result for models with a fixed mass. The triangles, squares and pentagons show the results using fitting functions (4), (5) and (6), respectively.

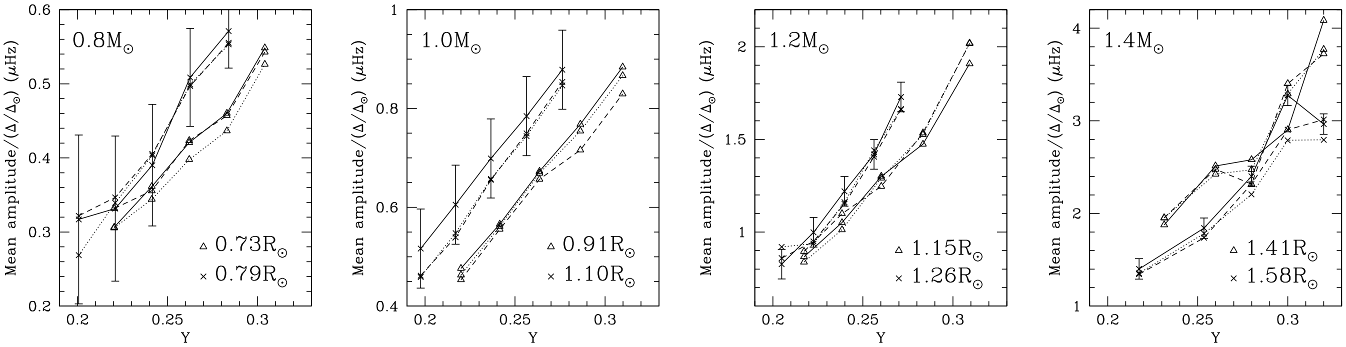

Figure 5. The mean amplitude of the oscillatory part in frequency due to the He II ionization zone is shown as a function of the helium abundance in the envelope for models with fixed mass and radius. The different masses and radii are marked in each panel. The solid, dotted and dashed lines show the results for fits using equations (4), (5) and (6), respectively. For sake of clarity, estimated errors are shown on only one set of points.

Fig. 5 also shows an estimate of the uncertainties in Y expected as a result of observational errors. Shown in the figure are the error bars on a few representative points, which are obtained assuming a constant relative error of one part in 10^4 in the frequencies. This level of error is not unrealistic given that even ground-based observations have almost reached this level of precision (see Bouchy & Carrier 2002). These error bars have been calculated using a Monte Carlo simulation where random errors with specified standard deviation were added to the exact frequencies before fitting the data. The results of the fits to the error-added frequencies were then used to estimate the variance in each fitted parameter to calculate the expected error. The precision to which we can use the amplitude to determine Y increases with increase in mass and age. From the error bars it appears that the expected error in Y would range from less than 0.01 for 1.2-$\mathrm{M}_\odot$ models to about 0.03 at 0.8 $\mathrm{M}_\odot$. To obtain better precision for 0.8-$\mathrm{M}_\odot$ stars we will need to measure the frequencies to a relative accuracy of better than 10^{-4}. The error in the scaled acoustic depth τ of the He II ionization zone is about 20–40 s except for 0.8-$\mathrm{M}_\odot$ models, where it is about 100 s. The above results were obtained assuming we had modes with degrees of 0, 1, 2 and 3. The uncertainty in the Y determination increases if $\ell = 3$ modes are unavailable, although the increase is very marginal and will not affect any of the results. Obviously, there would be additional errors because of uncertainties in our knowledge of stellar mass and radius. This error can be estimated by considering all models within the permitted range of mass and radius.

It should be noted that the points shown in the figures have been obtained using the exact model frequencies; only the extent of the error bars was obtained from the Monte Carlo simulations. From simulations it is found that when random errors are added to the frequencies, the estimated amplitudes increase by an amount comparable to the estimated errors. This effect has been seen earlier for helioseismic data (see, for example, Basu 1997). Therefore, for the purposes of calibration once real data are available, the calibration curve should be obtained after adding the random errors to the frequencies of the calibration models. We have not done that in this work as in that case the figures will depend to some extent on the estimated errors in frequencies.

To study the influence of the initial heavy element abundance, Z_0, and of the mixing length parameter, α, on the amplitude of oscillatory signal, we have constructed stellar models with different values Z_0 and α for stars with the same masses and radii shown in Fig. 5. Fig. 6 shows the results for 1-$\mathrm{M}_\odot$ stars that have the same radius (1 $\mathrm{R}_\odot$) but have different Z_0 and α. It can be seen that the variation in amplitude due to reasonable uncertainties in Z_0 and α are smaller than the variation due to Y, and also smaller than the error estimates for a relative error of 10^{-4} in frequencies.

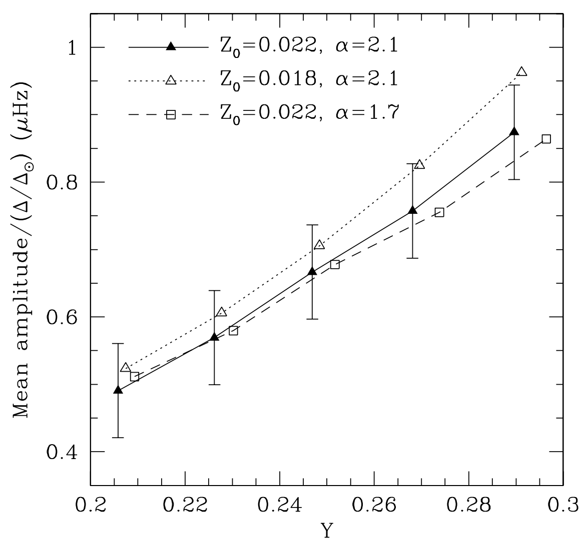

Figure 6. The mean amplitude of the oscillatory part in frequency due to the He II ionization zone is shown as a function of the helium abundance in the envelope for 1-$\mathrm{M}_\odot$ stars that have a radius of 1 $\mathrm{R}_\odot$. The results obtained for models with a slightly different initial heavy element abundance Z_0 and mixing length parameter α are compared. Representative errors in the amplitudes are indicated on one set of points. All results were obtained by using the function in equation (5) to fit the data.

In the above discussion, we have assumed that either the mass or the radius of the star is known independently. This is a reasonable assumption because most of the targets of asteroseismic missions are likely to be nearby stars, for which at least one of these quantities is known. However, if for some star neither the mass nor the radius is known, then from the frequency separation we can only estimate the ratio M/R^3. To examine the implication in this case, we show in Fig. 7 the amplitude as a function of Y for all models in a given bin of the mean large frequency separation Δ. It can be seen that there is some scatter because of variation in mass and radius, but in most cases the trend of increasing amplitude with Y is very clear and can be calibrated to calculate the helium abundance. Of course, the systematic errors will be larger in this case.

4.2 Results for the Sun

In order to test this technique we can try to estimate the helium abundance in the solar envelope using only the low degree modes.

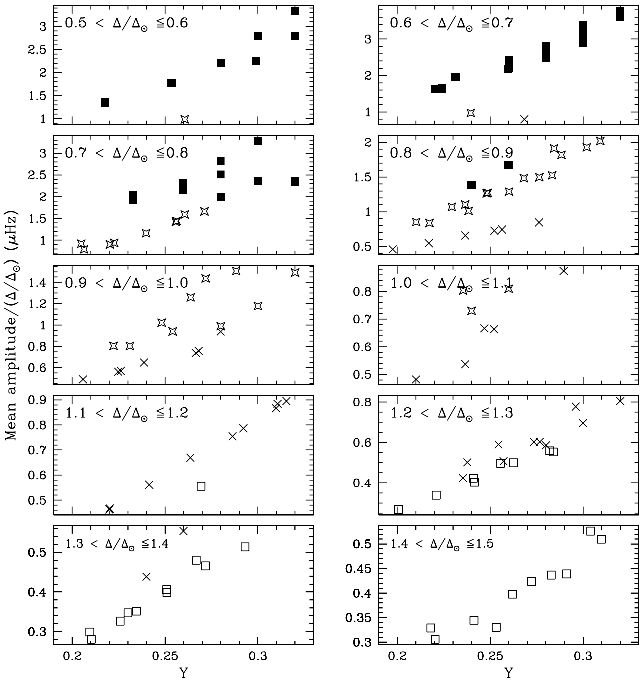

Figure 7. The mean amplitude of the oscillatory part in frequency due to the He II ionization zone is shown as a function of the helium abundance in the envelope. Each panel shows the results for a given bin in the ratio of the large frequency separation $\Delta/\Delta_\odot$. For clarity, only results obtained using equation (5) are shown. Open squares, crosses, stars and filled squares display stellar models with masses 0.8, 1, 1.2 and 1.4 M$_\odot$, respectively.

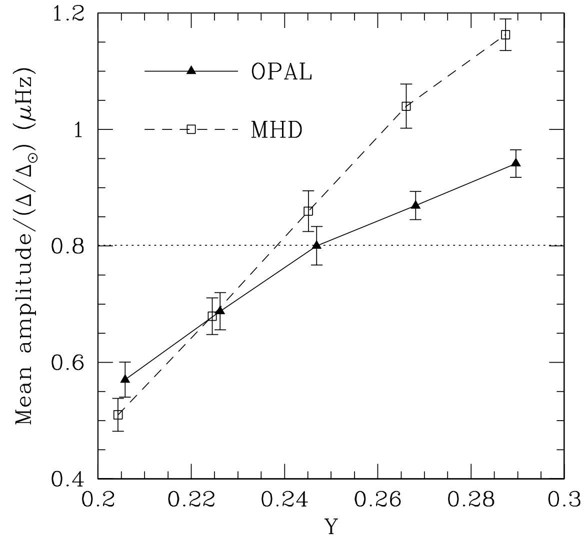

Figure 8. The mean amplitude of the oscillatory part in frequency due to the He II ionization zone is shown as a function of the helium abundance in the envelope for models with $M = 1$ M$_\odot$ and $R = 1$ R$_\odot$. The horizontal line shows the results obtained using observed frequencies from the MDI instrument for the Sun. The two lines show the results using OPAL and MHD EOS as marked in the figure. These results are obtained using the function in equation (4) to fit the data.

We use the frequencies obtained from the first 360 d of observations by the Michelson Doppler Imager (MDI) instrument on board the Solar and Heliospheric Observatory (SOHO) spacecraft Schou et al. (1998). The observed frequencies with degrees of $\ell \leqslant 3$ and frequencies in the range $1.5 \leqslant \nu \leqslant 3.0$ mHz were fitted to obtain the mean amplitude of the oscillatory signal due to the He II ionization zone, and the result is shown by the dotted horizontal line in Fig. 8. To infer the helium abundance in the Sun we have used 1-M$_\odot$, 1-R$_\odot$ models for the calibration. It should be noted that while these models have the same mass and radius as the Sun, these calibration models are not solar models, the models have different ages and luminosities. To the frequencies of each model we added 50 realizations of the observed errors, and for each realization we determined the amplitude of the oscillations. We have plotted the mean amplitude for each model as a function of the envelope helium abundance of the model in Fig. 8. All these fits use the same set of modes that was used to extract the oscillatory signal from the observed data. To study the sensitivity of results to uncertainties in the EOS, the figure also shows the results of a set of models constructed with the MHD EOS. The result of the observations is shown as the horizontal line and we can see that we obtain a helium abundance of 0.239 ± 0.005 with the MHD models and 0.246 ± 0.006 with the OPAL models. Similar results have been obtained using frequencies from GOLF (Global Oscillations at Low Frequencies). These results agree with the results obtained using intermediate degree solar p-modes (e.g. Antia & Basu 1994; Perez Hernandez & Christensen-Dalsgaard 1994; Basu & Antia 1995; Basu 1998; Richard et al. 1998). This exercise therefore demonstrates that it is indeed possible to determine helium abundance using only low degree modes.

4.3 Stellar models beyond the main sequence

All the stellar models that we have considered so far are on the main sequence. We have constructed a few models which have evolved beyond the main sequence to see how we might determine the envelope helium abundance for evolved stars. For these stars, many non-radial modes show a mixed character where the same mode behaves like a gravity mode in the core and like an acoustic mode in the envelope. The analysis that leads to the calculation of the oscillatory signal in frequencies is not valid for such modes. In fact, even calculating the large frequency separation from the calculated frequencies is difficult for such stars. However, radial modes are assured to be purely acoustic in nature throughout the star, and hence, for such stars, we restrict our analysis to radial modes ($\ell = 0$) only. Thus, the number of modes available for fitting is small and we do not expect to be able to calculate all parameters in equations (4), (5) or (6) reliably. Because the amplitude of the oscillatory signal due to the base of the convection zone is generally much smaller than that due to the He II ionization zone, we only fit the signal due to the He II ionization zone. Sample fits are shown in Fig. 9. The fits are reasonably good, although not as good as those in Fig. 2, hence we expect larger systematic errors. Because the amplitude of the He II signal increases with the reduction in frequency, we have extended the fitting interval by including modes with frequencies of 1–3 mHz. The enhanced range increases the number of modes available as well as the mean amplitude of the signal, thus providing better characterization of the signal. When compared to main-sequence stars, we expect larger uncertainties arising from errors in frequency because of the fewer number of modes in these stars. Nevertheless, Monte Carlo simulations with comparable error in frequencies show that the estimated error in mean amplitude for evolved stars is not too much larger than that for main-sequence stars.

Fig. 10 shows the amplitude of the signal as a function of Y for models with a fixed mass and radius. Error bars corresponding to frequency errors of one part in 10^4 are also shown for a few

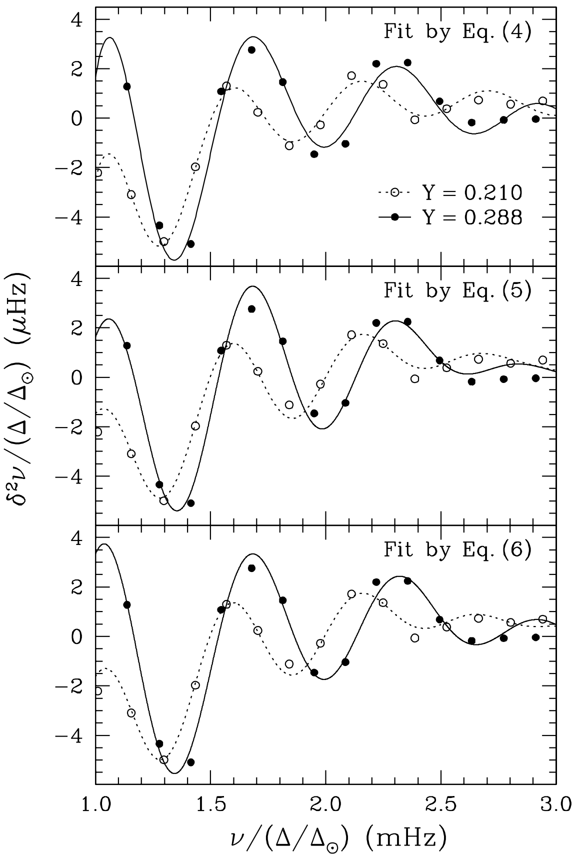

Figure 9. A sample of the fits to the second differences of the scaled frequencies using only $\ell = 0$ modes for models evolved beyond the main sequence. The points are the 'data', and the lines the fits to the points. The examples shown are for a 1.2-$M_\odot$ model evolved to a radius of 1.8 $R_\odot$.

typical models. As expected, the amplitude increases with Y and it is possible to determine Y from known amplitude. We find again that these amplitudes are not particularly sensitive to small variations in Z_0 and α. However, in this case the differences due to Z_0 and α are larger than those for main-sequence models. As can be seen, despite the errors, we should be able to determine the helium abundance in the envelope of these stars to a precision of about 0.01–0.02.

4.4 Results for models with low heavy element abundances

Stars with very low heavy element abundances evolve quite differently from stars with near-solar heavy element abundances. This is because the convection zone of low-Z stars is shallower at the same effective temperature (Deliyannis et al. 1990; Proffitt & Michaud 1991). We do not expect this to affect the helium signature; nevertheless, we confirm this using models of low Z_0. We have constructed two sets of models: models with $Z_0 = 0.007$ and models with $Z_0 = 0.001$. We have both main-sequence and subgiant models with $Z_0 = 0.007$, but only subgiants with $Z_0 = 0.001$ because Population II models evolve very quickly compared to stars of the same mass with higher Z. Hence, we do not expect to see many main-sequence stars. As with the $Z_0 = 0.022$ models, we use $\ell = 0$, 1, 2 and 3 modes for the main-sequence stars and only $\ell = 0$ modes for the subgiants.

Fig. 11 shows the amplitude as a function of Y for $Z_0 = 0.007$ models. Again we can see that the amplitude increases with Y. The amplitudes are, however, slightly different from those of $Z_0 = 0.022$ models, and hence for such large differences in Z_0 it will be better to make different calibration models. Fig. 11 also shows the results for Population II models. Again, the amplitude increases with Y. The amplitudes for these evolved models are much larger than those for the main-sequence models studied in Section 4.1. Nevertheless, to obtain good results for Y, we would need lower errors in frequencies. We expect to be able to observe only lower mass Population II stars, and this implies that the expected amplitude of the oscillation signal will be small, making errors larger.

5 CONCLUSIONS

We have presented results of our investigation into how low degree oscillation frequencies may be used to determine the helium abundance in stars with solar-type oscillations. We have demonstrated that, in principle, it is possible to use this technique to estimate the

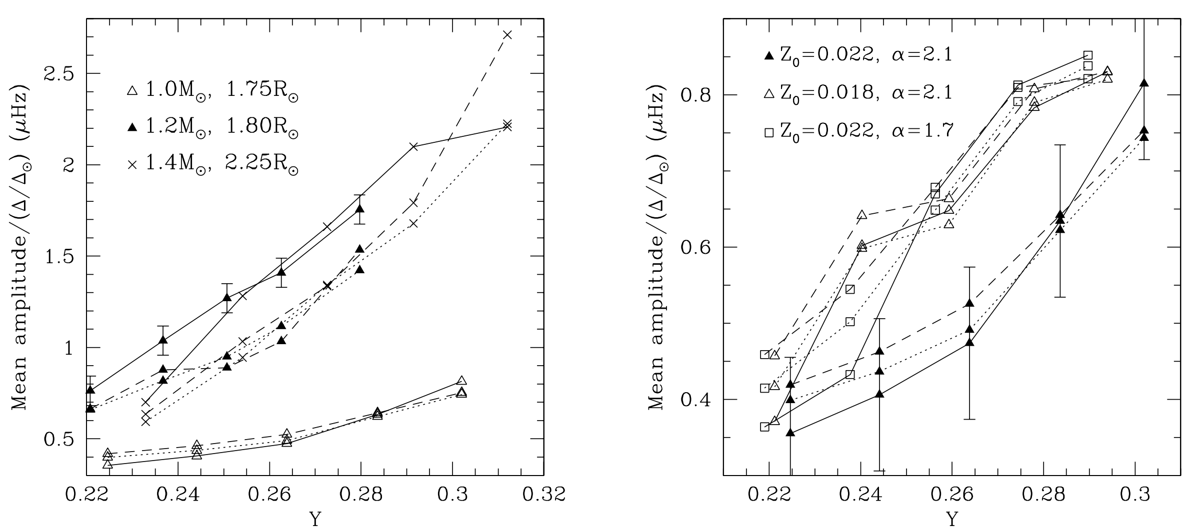

Figure 10. The mean amplitude of the oscillatory part in frequency due to the He II ionization zone is shown as a function of the helium abundance in the envelope for evolved stellar models with a fixed mass and radius. The solid, dotted and dashed lines show the results for fits using equations (4), (5) and (6), respectively. The left panel shows the results for a few different masses and radii as marked in the figure, while the right panel shows the results for $M = 1$ $M_\odot$, $R = 1.75$ $R_\odot$ models with different Z_0 and α. For the sake of clarity, the error estimates are shown on only one set of points.

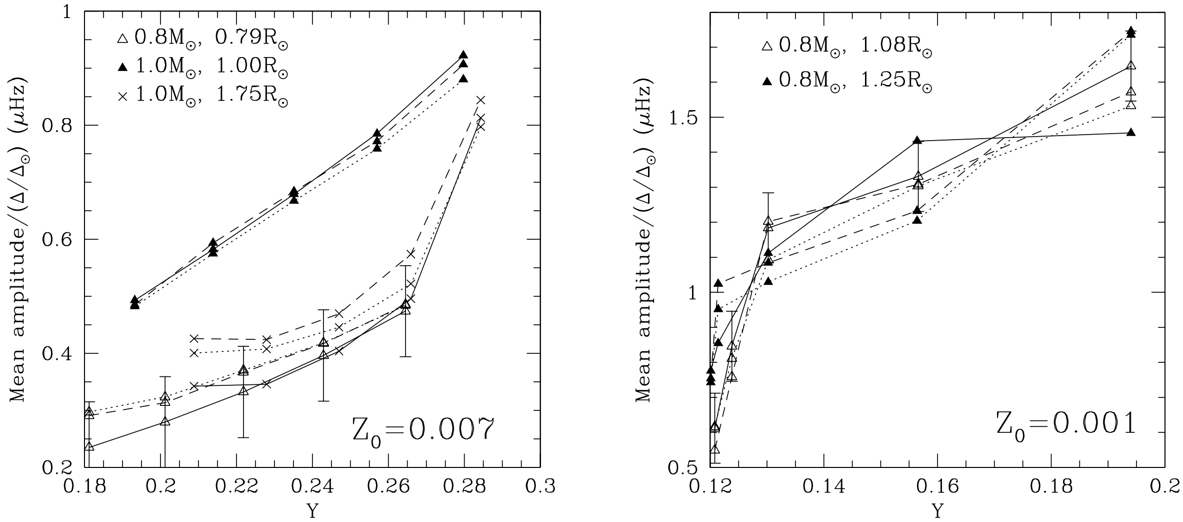

Figure 11. The mean amplitude of the oscillatory part in frequency due to the He II ionization zone is shown as a function of the helium abundance in the envelope for stellar models with fixed mass and radius, constructed with low heavy element abundance. This figure shows the results for stellar models with initial low heavy element abundance, $Z_0 = 0.007$ (left panel) and $Z_0 = 0.001$ (right panel). The solid, dotted and dashed lines show the results for fits using equations (4), (5) and (6), respectively. For the sake of clarity, the error estimates are shown on only one set of points.

helium abundance in stellar envelopes using the frequencies of low degree modes of oscillation.

We find that the oscillatory signal in the frequencies due to the helium ionization zone can be used to determine the helium abundance of a low-mass main-sequence star, provided either the radius or the mass is known independently. The precision to which we may be able to determine the helium abundance increases with increase in mass. Using reasonable error estimates, it appears that a precision of 0.01–0.02 in Y is possible in most cases. The amplitude of the oscillatory term is not particularly sensitive to small variations of the initial heavy element abundance, Z_0, and mixing length parameter, α. For stars with masses of about 1.4 $M_\odot$ and larger, it is difficult to fit the oscillatory signal reliably, and hence the uncertainties are larger. This difficulty is most probably due to the fact that these stars have shallow convection zones and there is not much difference in the acoustic depths of the He II ionization zone and that of the base of the convection zone, thus leading to interference between the two terms. For low-mass stars, the amplitude of the oscillatory term is small, leading to large errors in determining the helium abundance. Thus, the best results are obtained for stars with masses close to and larger than the solar mass. Using solar data, we have shown that we can indeed determine the envelope helium abundance using low degree modes.

Even if the mass and radius for a star are not known independently, we can use the large frequency separation Δ to estimate M/R^3 and, in that case also, a reasonably tight correlation is found between Y and amplitude of the oscillatory signal, thus allowing us to determine Y, although with larger errors.

For stellar models that are evolved off the main sequence, it is not possible to use non-radial modes, however by using only radial modes it is possible to fit the oscillatory term arising from the He II ionization zone, which can then be used to measure the helium abundance. A more detailed study of stars in different post-main-sequence phases is required to understand the oscillatory signal from such stars. Models with much lower heavy element abundances behave in a manner similar to those with near-solar heavy element abundances as far as the helium signature is concerned.

Although most of the present work uses exact frequencies from stellar models, we have successfully applied this method to solar low degree data. We have also estimated the expected errors assuming frequency errors of one part in 10^4, a level that is expected to be achieved by future space- and ground-based observations. This opens up the possibility of using asteroseismic data to obtain the helium abundance in stellar envelopes, leading to a better understanding of the evolution of stellar populations in our galaxy.

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