

Template for VoronoiFVM notebook

V1.0, 2024-11-17

```
1 begin
2   using VoronoiFVM # The VoronoiFVM package
3   using ExtendableGrids # Manage grids, create rectangular grids
4   using SimplexGridFactory, Triangulate # Create grids with general geometry
5   using GridVisualize, CairoMakie # Visualization
6   CairoMakie.activate!(type="png")
7   GridVisualize.default_plotter!(CairoMakie)
8   using PlutoUI # Sliders etc.
9 end
```

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Grid Creation

We create grids in domains Ω which have disjoint boundary parts Γ_1, Γ_2 .

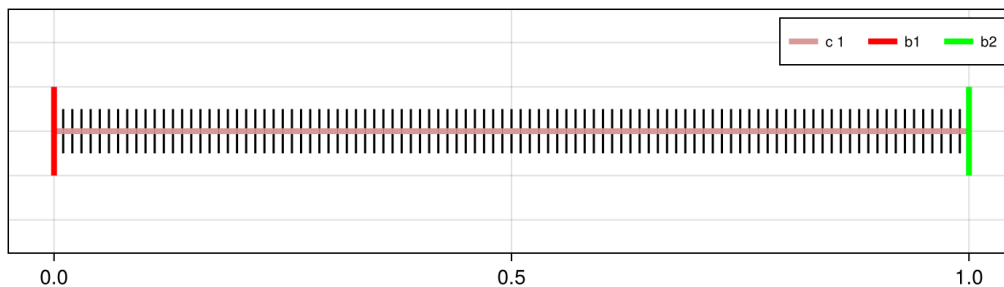
1D Grid

```
X = 0.0:0.01:1.0
```

```
1 X=range(0,1,length=101)
```

```
grid1d = ExtendableGrids.ExtendableGrid{Float64, Int32}
          dim =      1
          nnodes =   101
          ncells =   100
          nbfaces =    2
```

```
1 grid1d=simplexgrid(X)
```



```
1 gridplot(grid1d, size=(700,200), legend=:rt)
```

2D rectangular grid

```
Y = 0.0:0.01:0.1
```

```
1 Y=range(0,0.1, length=11)
```

```

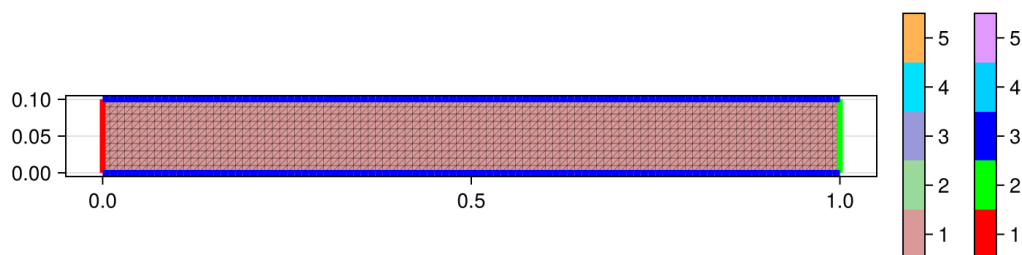
ExtendableGrids.ExtendableGrid{Float64, Int32}
  dim =      2
  nnodes = 1111
  ncells = 2000
  nbfaces = 220

```

```

1 begin
2   grid2d=simplexgrid(X,Y)
3   bfacemask!(grid2d,[0,0], [1,1], 3, allow_new=false)
4   bfacemask!(grid2d,[0,0], [0,1], 1)
5   bfacemask!(grid2d,[1,0], [1,1], 2)
6 end
7

```



```

1 gridplot(grid2d, size=(700,200), linewidth=0.1)

```

2D general grid

```

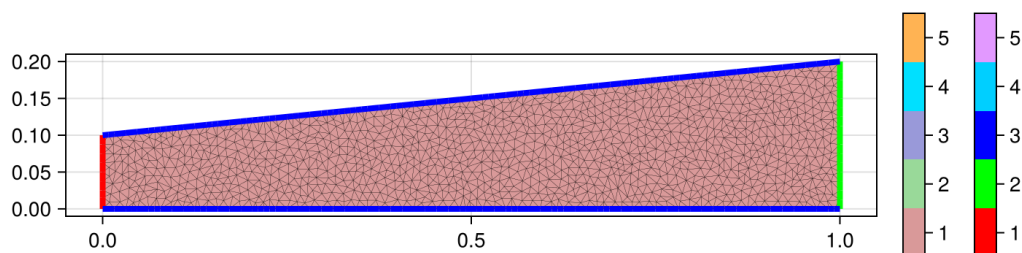
ExtendableGrids.ExtendableGrid{Float64, Int32}
  dim =      2
  nnodes = 1259
  ncells = 2329
  nbfaces = 187

```

```

1 begin
2   builder = SimplexGridBuilder(; Generator = Triangulate)
3
4   p1 = point!(builder, 0, 0)
5   p2 = point!(builder, 1, 0)
6   p3 = point!(builder, 1, 0.2)
7   p4 = point!(builder, 0, 0.1)
8
9   facetregion!(builder, 3)
10  facet!(builder, p1, p2)
11  facet!(builder, p3, p4)
12  facetregion!(builder, 1)
13  facet!(builder, p1,p4)
14  facetregion!(builder, 2)
15  facet!(builder, p2,p3)
16
17  options!(builder; maxvolume = 1.0e-4)
18
19  ggrid2d = simplexgrid(builder)
20
21
22 end

```



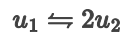
```

1 gridplot(ggrid2d, size=(700,200), linewidth=0.1)

```

Problem description

Regard the following system of reacting species with the reaction



in the time interval $[0, t_{end}]$

$$\begin{aligned}\partial_t u_1 - \nabla \cdot D_1 \nabla u_1 + R(u_1, u_2) &= 0 \\ \partial_t u_2 - \nabla \cdot D_2 \nabla u_2 - R(u_1, u_2) &= 0 \\ R(u_1, u_2) &= kp + u_1 - km - u_2^2 \\ u_1|_{t=0} = u_2|_{t=0} &= 0 \\ u_1|_{\Gamma_1} &= 1 \\ D_2 \partial_n u_2|_{\Gamma_1} &= 0 \\ D_1 \partial_n u_1|_{\Gamma_2} &= 0 \\ u_2|_{\Gamma_2} &= 0\end{aligned}$$

myflux (generic function with 1 method)

```
1 function myflux(y,u, edge, data)
2     (;D_1, D_2)= data
3     y[1]= D_1*(u[1,1]- u[1,2])
4     y[2]= D_2*(u[2,1]- u[2,2])
5 end
```

myreaction (generic function with 1 method)

```
1 function myreaction(y, u, node, data)
2     (;kp, km) = data
3     R=kp*u[1] - km*u[2]^2
4     y[1]=R
5     y[2]=-R
6 end
7
```

mystorage (generic function with 1 method)

```
1 function mystorage(y,u, node, data)
2     y[1]=u[1]
3     y[2]=u[2]
4 end
5
```

mybc (generic function with 1 method)

```
1 function mybc(y,u, bnode, data)
2     boundary_dirichlet!(y,u,bnode, species=1, value=1, region=1)
3     boundary_dirichlet!(y,u,bnode, species=2, value=0, region=2)
4 end
5
```

mydata = ▶ (kp = 1.0, km = 0.1, D_1 = 1, D_2 = 0.1)

```
1 mydata= (kp=1.0, km =0.1, D_1=1, D_2=0.1)
```

mygrid = ExtendableGrids.ExtendableGrid{Float64, Int32}

```
    dim =      2
    nnodes =   1259
    ncells =   2329
    nbfaces =   187
```

```
1 mygrid=ggrid2d
```

mysystem =

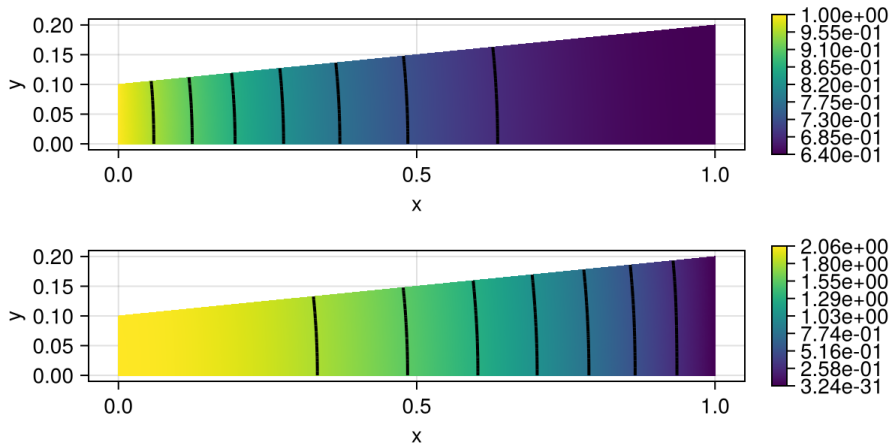
```
VoronoiFVM.System{Float64, Float64, Int32, Int64, Matrix{Int32}}{
    grid = ExtendableGrids.ExtendableGrid{Float64, Int32}(dim=2, nnodes=1259, ncells=2329, nbfaces=187),
    physics = Physics(data=@NamedTuple{kp::Float64, km::Float64, D_1::Int64, D_2::Float64}, flux=myflux, storage=mystorage, reaction=myreaction, breaction=mybc, num_species = 2)
```

```
1 mysystem=VoronoiFVM.System(mygrid; flux=myflux, reaction=myreaction,
    storage=mystorage, breaction=mybc, data=mydata, species=[1,2])
```

Stationary solution

```
mysol = 2×1259 VoronoiFVM.DenseSolutionArray{Float64, 2}:
 1.0      0.640308      0.639961      ...      0.640139      0.640587      0.640777      0.640427
 2.06278  4.23622e-31  3.24012e-31      ...      0.0808262      0.169523      0.192866      0.145453
```

```
1 mysol=solve(mysystem)
```



```
1 let
2   vis=GridVisualizer(size=(600,300),layout=(2,1), legend=:rt)
3   scalarplot!(vis[1,1],mygrid, mysol[1,:], label="u_1")
4   scalarplot!(vis[2,1],mygrid, mysol[2,:], label="u_2")
5   reveal(vis)
6 end
```

Transient solution

```
tend = 10
```

```
1 tend=10
```

```
myinival =
2×1259 VoronoiFVM.DenseSolutionArray{Float64, 2}:
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
```

```
1 myinival=unknowns(mysystem, inival=0)
```

```

mysol =
t: 64-element Vector{Float64}:
 0.0
 0.001
 0.0010999999999999999
 0.0012199999999999978
 0.0013639999999999963
 0.0015367999999999944
 0.0017441599999999922
  ⋮
 6.0509416850392705
 6.999044549788927
 7.749283412341695
 8.499522274894463
 9.249761137447232
10.0
u: 64-element Vector{Matrix{Float64}}:
 [1.0 0.0 ... 0.0 0.0; 0.0 0.0 ... 0.0 0.0]
 [1.0 3.179554553889352e-14 ... 6.549149473352493e-14 4.583693869516127e-14; 0.0007961571
 [1.0 3.532838393210388e-14 ... 7.276832748169428e-14 5.0929931883512466e-14; 0.00087457
 [1.0 4.014589083193617e-14 ... 8.269128122919793e-14 5.787492259490045e-14; 0.000967672
 [1.0 4.689940517749544e-14 ... 9.660196405280116e-14 6.761089088189293e-14; 0.001078578
 [1.0 5.6696572990202296e-14 ... 1.1678187143713848e-13 8.173463597907736e-14; 0.0012110
 [1.0 7.152878102316569e-14 ... 1.4733280106623088e-13 1.031169711080399e-13; 0.00136947
  ⋮
 [1.0 0.633752815899025 ... 0.6342201060700844 0.6338714022418487; 1.9553106929208774 4.
 [1.0 0.63605933054252665 ... 0.6365274112354683 0.6361781332608476; 1.9943507826733005 4
 [1.0 0.6373502688656448 ... 0.6378188149917686 0.6374692276648308; 2.0156534312501324 4
 [1.0 0.6382560286235159 ... 0.6387248734531001 0.6383750766527738; 2.0303324345889306 4
 [1.0 0.638887819387511 ... 0.6393568673921701 0.6390069283169636; 2.0404427704759933 4.
 [1.0 0.6393267051792006 ... 0.6397958917865866 0.639445855758309; 2.047404444557198 4.2

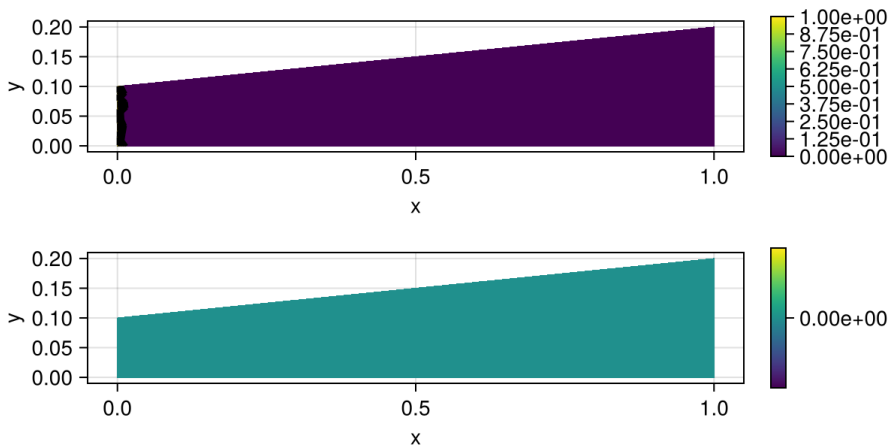
```

```

1 mysol=solve(mysystem; inival=myinival, times=(0,tend), Δt=1.0e-4,
  force_first_step=true)

```

0.0



```

1 let
2   u=mysol(myt)
3   vis=GridVisualizer(size=(600,300),layout=(2,1), legend=:rt)
4   scalarplot!(vis[1,1],mygrid, u[1,:], label="u_1")
5   scalarplot!(vis[2,1],mygrid, u[2,:], label="u_2")
6   reveal(vis)
7 end

```