From Newton's Cooling to Quant Mean Reversion (OU), Half-Life & Trading

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Data: water-cooling experiment (Δt=2 min, 0–30 min, Tamb=25 °C)

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1) Problem Statement (Physics → Quant)

Estimate the exponential time constant τ of a physical cooling process and show the one-to-one mapping to quant mean-reversion (OU/AR(1)), half-life, and EWMA decay λ. Verify the model with two fits (linearized and nonlinear least squares), then translate the parameters into OU notation and a toy mean-reversion strategy.

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2) Model

Newton's cooling (physics)

$$\frac{dT}{dt} = -k\,(T - T_\infty), \qquad T(t) = T_\infty + (T_0 - T_\infty)e^{-t/\tau}, \;\; \tau = \frac{1}{k}.$$

Let $\Delta T(t) = T(t) - T_\infty \Rightarrow \ln\Delta T(t) = \ln\Delta T_0 - \frac{t}{\tau}$.

OU / AR(1) (quant)

$$dX_t = -k(X_t - \mu)\,dt + \sigma\,dW_t \quad \Leftrightarrow \quad X_{t+\Delta} - \mu = \phi\,[X_t - \mu] + \varepsilon_t, \;\; \phi = e^{-k\Delta} = e^{-\Delta/\tau}.$$

Half-life $t_{1/2} = \frac{\ln 2}{k} = \tau \ln 2$. In discrete filters, EWMA decay \$ \$ and span $\approx \frac{1}{1-\lambda}$.

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3) Data & Fits (Physics)

- Ambient $T_\infty = 25.0^\circ$C; samples every 2 min for 30 min.
- Linearized fit (ambient fixed): slope $m = -0.1033$ min^{-1} ⇒ τ ̂= 9.682 mi; 95% CI [9.01, 10.35]; $R^2 = 0.972$, RMSE 1.592 °C.
- Nonlinear LS fit (ambient estimated): T_∞ 22.30 °C, τ ̂= 13.650 min, 95% CI [12.49, 14.81]; $R^2 = 0.998$, RMSE 0.411 °C.

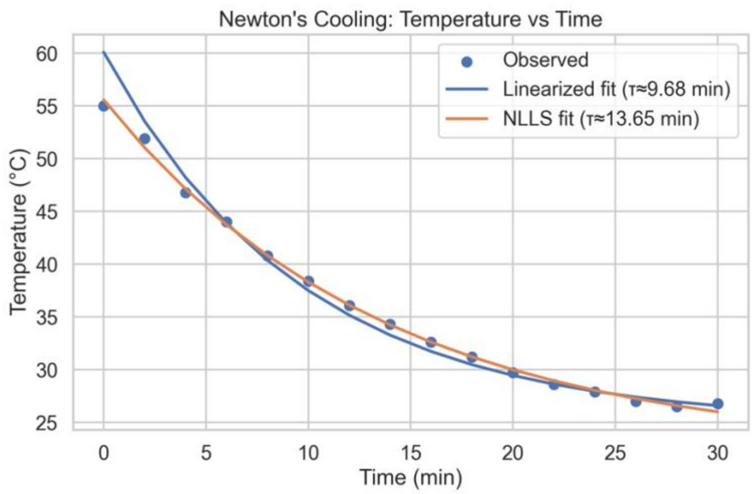

Figure 1: Temperature vs time with both fits overlaid. The NLLS fit with free T_∞ explains the early points better (lower RMSE).

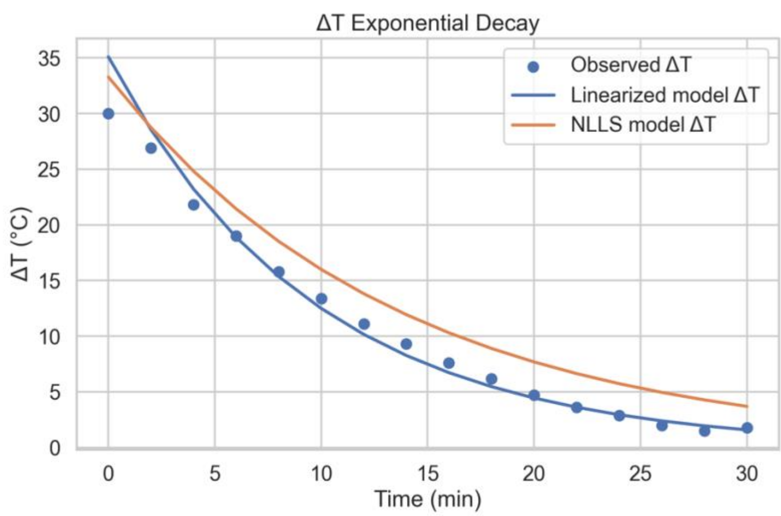

Figure 2: Exponential decay of ΔT.

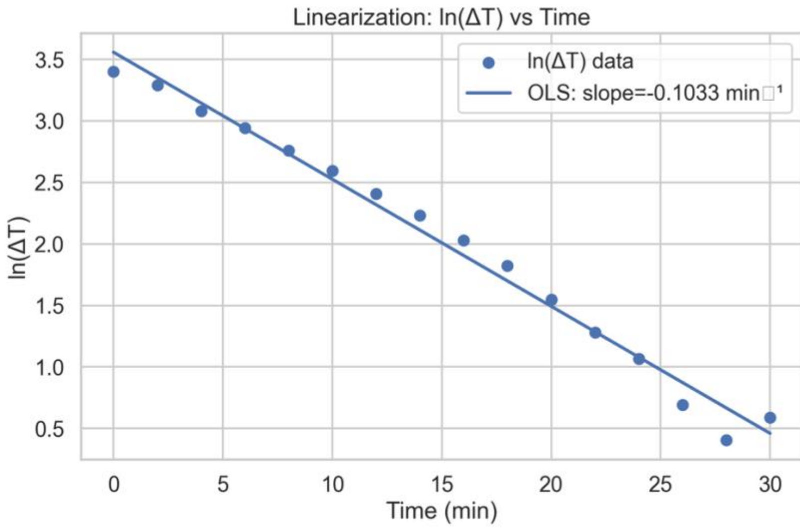

Figure 3: *Linearization* $\ln \Delta T$ *vs time; slope* $m = -1/\tau$.

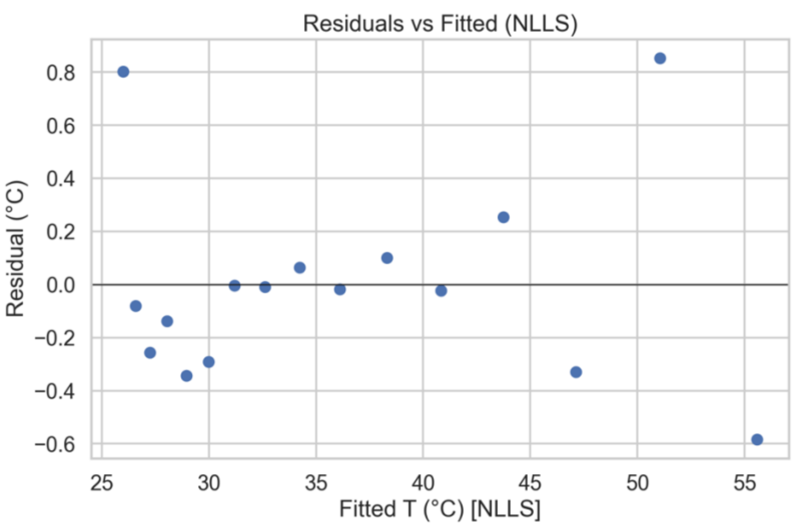

Figure 4: *Residuals vs fitted for NLLS—no major structure, supporting the exponential model.*

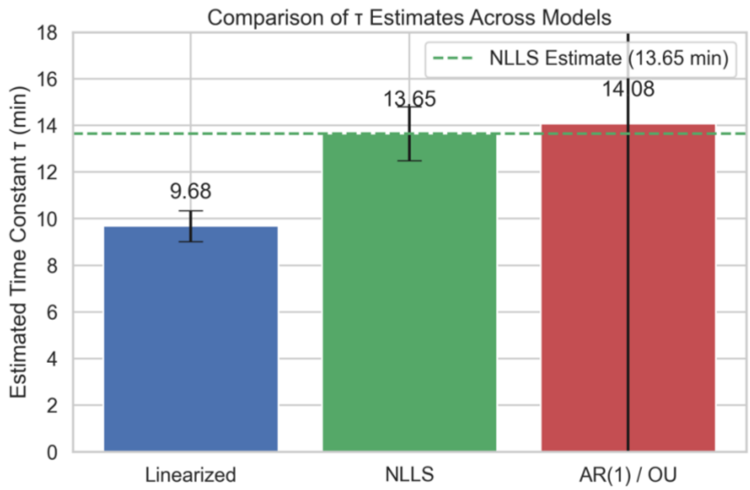

Figure 9: *Side-by-side comparison of τ: Linearized (9.68), NLLS (13.65), AR(1)/OU (14.08) with error bars. The "physics with free T_∞" and the "quant AR(1)" land in the same neighborhood.*

> Why the two physics fits differ**:** fixing $T_\infty = 25°\text{C}$ biases early points; letting T_∞ float (NLLS) attributes part of the gap to ambient mis-specification and increases τ. That's expected with short windows and sensor noise.

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4) Quant Mapping (Treat ΔT as a "spread")

Using $\Delta = 2$ min and AR(1) on ΔT:

- $\hat{\phi}$ = 0.8676 ± 0.0164 ⇒ $\hat{k}$ = 0.07104 min^{-1}, $\hat{\tau}$ = 14.078 min, half-life = 9.758 min.
- EWMA: = 0.8676, span ≈ 7.6 samples (≈ 15.2 min).
- OU diffusion: $\sigma_{\text{diff}} \approx 0.428$, stationary std $\sigma_{\text{stat}} \approx 1.136$ (in °C units here).

AR(1)/OU impulse response ϕ^h. Shocks decay geometrically at rate ϕ.

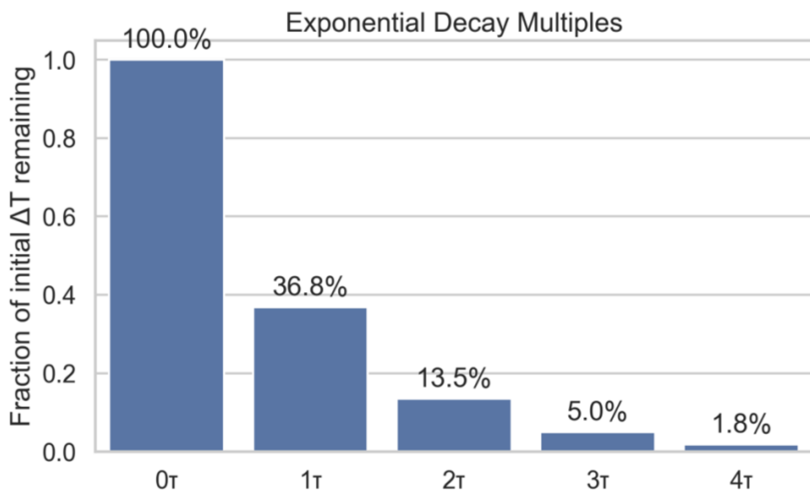

Figure 5: *Decay fractions after multiples of τ*

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5) Educational Demo: Synthetic OU & Toy Strategy

(Solely illustrative; ignores cointegration, financing, costs.)

- Synthetic OU: τ = 20 min, $\phi = 0.951$.
- Simple z-score reversion with entry 1σ, exit 0.1σ, cap at 3 half-lives produced smooth cum-PnL in simulation.
- Don't read the absolute Sharpe (≈ 33 in "spread units") as tradable PnL—it reflects toy units and zero costs; the point is *mechanics* (signal, half-life, holding discipline).

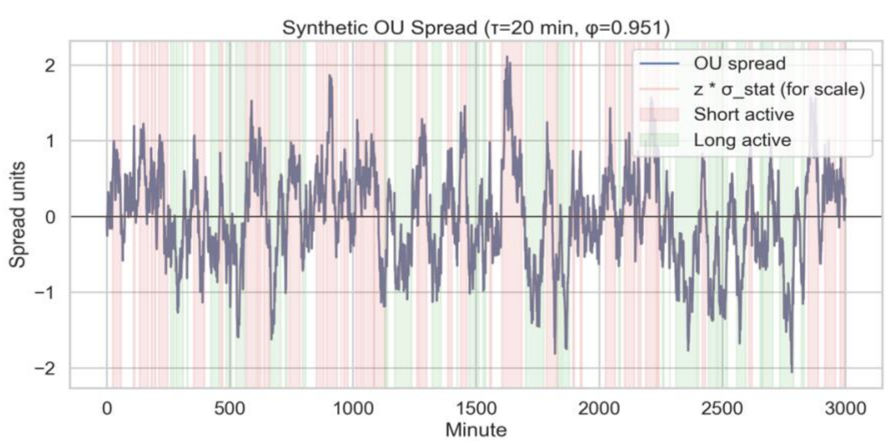

Figure 7: *Synthetic OU path with long/short regime shading; z-score shown for scale.*

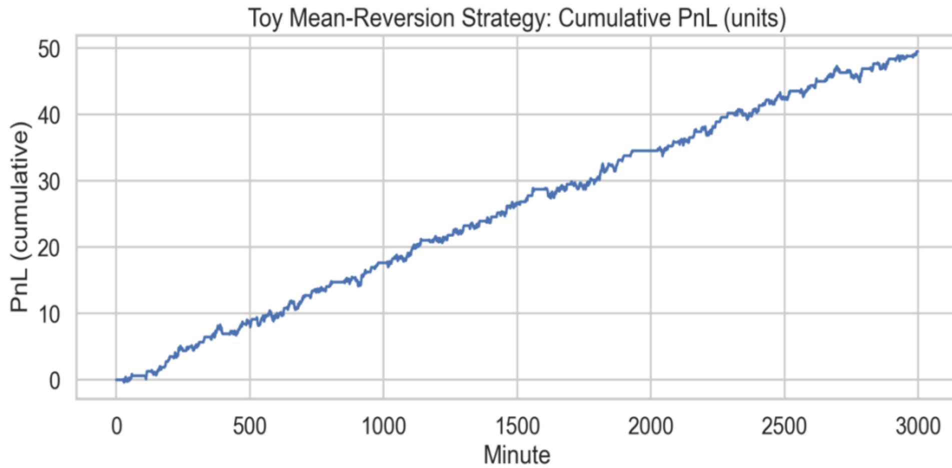

Figure 8: *Cumulative "PnL in spread units" of the toy strategy*.

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6) Takeaways

- The same $e^{-t/\tau}$ behind cooling underpins mean reversion in markets..
- Estimation matters: freeing nuisance parameters (like T_∞) or switching to AR(1)/OU changes $\hat{\tau}$, but the order of magnitude is stable across methods—a good sign of model robustness.
- For real trading, replace ΔT with a cointegrated spread, re-estimate ϕ, τ, σ, and add costs/risk controls.

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7) Key references

1. **Stochastic Calculus:** Shreve, S. E. (2004). *Stochastic Calculus for Finance II: Continuous-Time Models*. Springer.
2. **Time Series:** Brockwell, P. J., & Davis, R. A. (2016). *Introduction to Time Series and Forecasting* (3rd ed.). Springer.
3. **Quantitative Trading:** Chan, E. P. (2013). *Algorithmic Trading: Winning Strategies and Their Rationale*. Wiley.

A full, categorized bibliography is available in the accompanying Jupyter Notebook.

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7) Repository & Full Code

The full, reproducible Jupyter notebook—including all diagnostics, plots, and a toy trading strategy simulation—is available for verification in Github.

Appendix: Reproducibility Summary (key numbers)

- **Linearized (fixed T_∞)**: $m = -0.10328$, τˆ =9.682 min; 95% CI [9.01, 10.35]; $R^2 = 0.972$, RMSE 1.592 °C.
- **NLLS (free T_∞)**: $T_\infty = 22.302$°C; τˆ =13.650 min; 95% CI [12.49, 14.81]; $R^2 = 0.998$, RMSE 0.411 °C.
- **AR(1)/OU ($\Delta = 2$ min)**: $\hat{\phi}$=0.8676±0.0164, $\hat{k}$=0.0710, $\hat{\tau}$=14.078 min; half-life 9.758 min; $\lambda \approx 0.868$, span ≈ 7.6; $\sigma_{\text{diff}} \approx 0.428$, $\sigma_{\text{stat}} \approx 1.136$.
