

# mioni\_rng

August 21, 2021

```
[1]: import numpy as np
import pandas as pd
from mioni_helper import load_mioni_data
from random_helper_functions import get_bitstring
```

```
[2]: import sys

import matplotlib.pyplot as plt

sys.path.append('../..')
from plotting.matplotlib_setup import configure_latex, savefig, \
    ↪set_size_decorator, savefig, thinner_border

tex_dir, images_dir = 'porocilo/main.tex', 'porocilo/images'

configure_latex(style=['science', 'notebook'], global_save_path=images_dir)

%config InlineBackend.figure_format = 'pdf'
```

## 1 Meritve

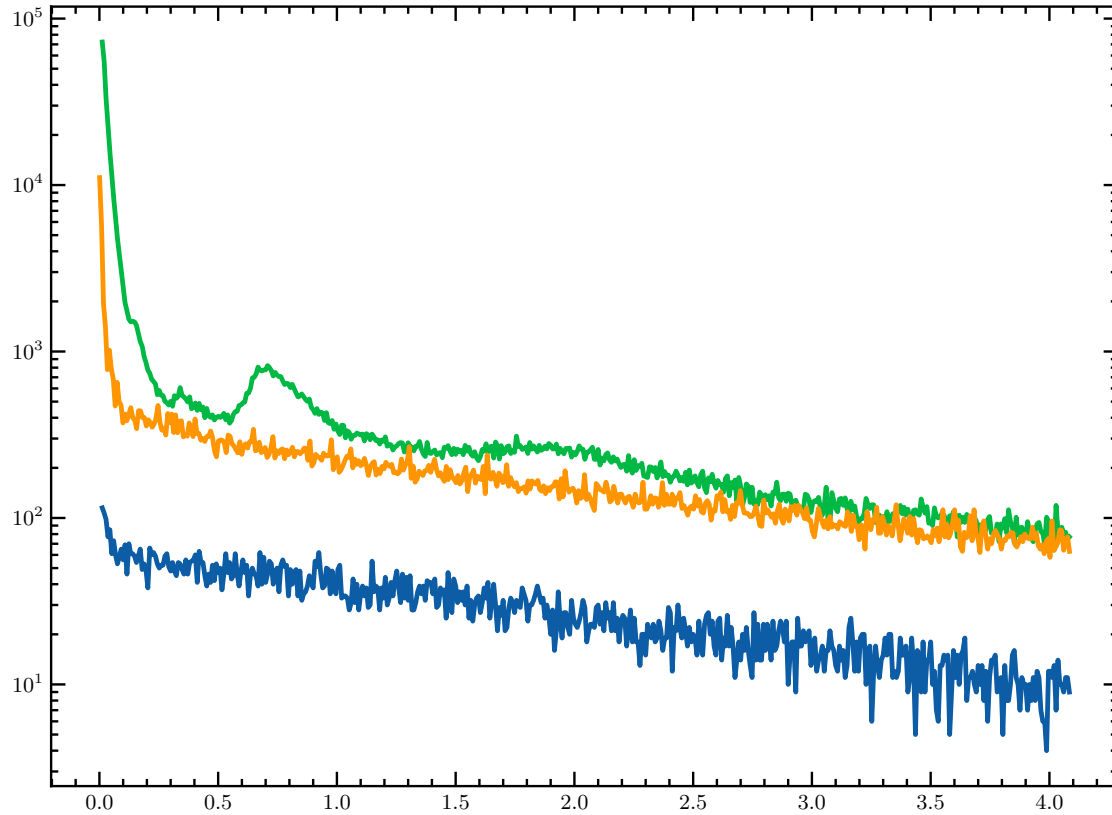
```
[3]: dfs = load_mioni_data()
```

```
[4]: for df in dfs:
    ind = df[np.abs(df['N']) < 1].index
    df.drop(ind, inplace=True)

    ind = df[df['us'] < 0].index
    df.drop(ind, inplace=True)
```

```
[5]: plt.yscale('log')

for df in dfs:
    plt.plot(df['us'], np.abs(df['N']))
```



```
[6]: dfs = [dfs[0], dfs[2]]
```

```
[7]: dfs[0] = dfs[0][(len(dfs[0]) - len(dfs[1])):]
```

```
[8]: fig, ax = set_size_decorator(plt.subplots, fraction=0.5, ratio='4:3')(1, 1)

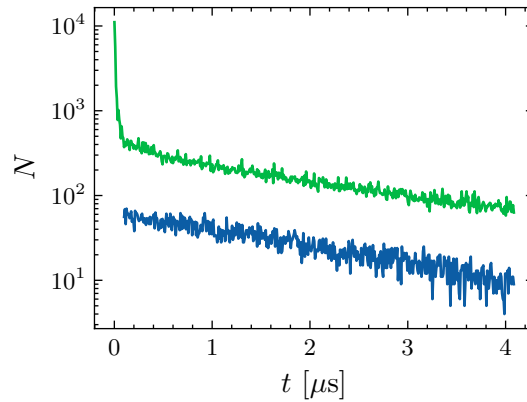
ax.set_yscale('log')

for df in dfs:
    ax.plot(df['us'], np.abs(df['N']), lw=1)

ax.set_xlabel(r'$t$ [$\mu s$]')
ax.set_ylabel('$N$')

# savefig('mioni_meritve')
```

```
[8]: Text(0, 0.5, '$N$')
```



## 2 Noise

```
[9]: from scipy.optimize import curve_fit
```

```
def lin_func(t, a, b):
    return a * t + b
```

```
[10]: start_fit, end_fit = 80, -1
```

```
ks = []
```

```
for df in dfs:
    xdata = df['us'].values[start_fit:end_fit]
    ydata = np.log(df['N'].values[start_fit:end_fit])

    popt, pcov = curve_fit(lin_func, xdata, ydata)

    k, k_err = popt[0], np.sqrt(np.diag(pcov))[0]
    ks.append(popt)
```

```
[11]: fig, ax = set_size_decorator(plt.subplots, fraction=0.5, ratio='4:3')(1, 1)
```

```
cc = 0
cs = [['C0', 'C2'], ['C1', 'C3']]
for k, df in zip(ks, dfs):
    xdata = df['us'].values[start_fit:end_fit]
    ydata = np.log(df['N'].values[start_fit:end_fit])

    ax.plot(xdata, ydata, lw=1, c=cs[cc][0])
    ax.plot(xdata, lin_func(xdata, *k), lw=1, c=cs[cc][1])

    cc += 1
```

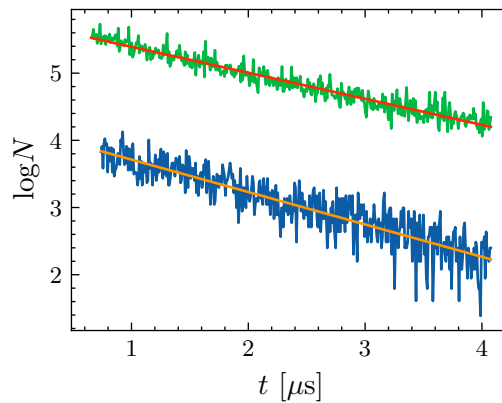
```

ax.set_xlabel(r'$t$ [$\mu$s]')
ax.set_ylabel('log$N$')

# savefig('mioni_fit')

```

```
[11]: Text(0, 0.5, 'log$N$')
```



```

[12]: noise = []
      ys = []

      for k, df in zip(ks, dfs):
          xdata = df['us'].values[start_fit:end_fit]
          ydata = np.log(df['N'].values[start_fit:end_fit])

          n = ydata - lin_func(xdata, *k)

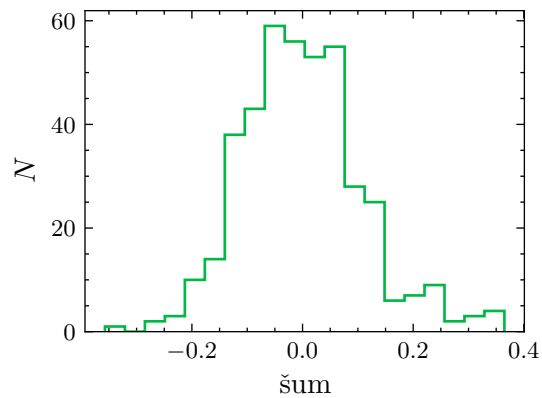
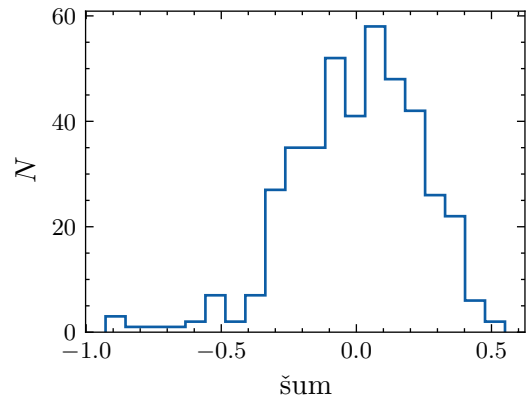
          noise.append(n)

```

```

[13]: for i, n in enumerate(noise):
      fig, ax = set_size_decorator(plt.subplots, fraction=0.5, ratio='4:3')(1, 1)
      # plt.plot(range(len(n)), n)
      ax.hist(n, bins=int(np.sqrt(len(n))), histtype='step', lw=1, color=f'C{i}')
      ax.set_xlabel(r'$\sum$')
      ax.set_ylabel('$N$')
      # savefig(f'mioni_sum_{i}')

```



### 3 ECDF: does not work!!!

<https://stackoverflow.com/questions/24788200/calculate-the-cumulative-distribution-function-cdf-in-python>

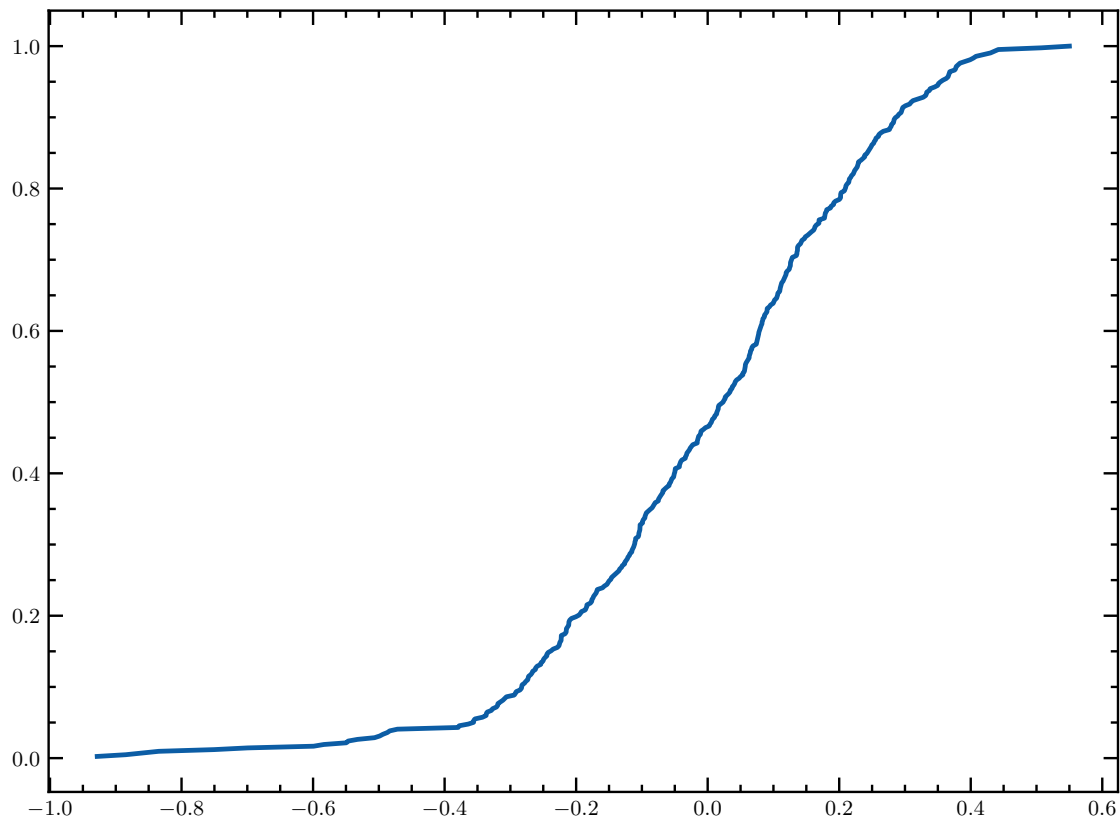
[https://en.wikipedia.org/wiki/Probability\\_integral\\_transform](https://en.wikipedia.org/wiki/Probability_integral_transform)

```
[14]: from statsmodels.distributions.empirical_distribution import ECDF

      ecdf = ECDF(noise[0])
      x, y = ecdf.x, ecdf.y
```

```
[15]: plt.plot(x, y)
```

```
[15]: [<matplotlib.lines.Line2D at 0x7fb457806400>]
```



```
[16]: from scipy.stats import norm
```

```
[ ]: def ecdf(a):
    x, counts = np.unique(a, return_counts=True)
    cusum = np.cumsum(counts)

    inv = np.argsort(a)

    cusum = cusum / cusum[-1]

    return x, cusum, inv
```

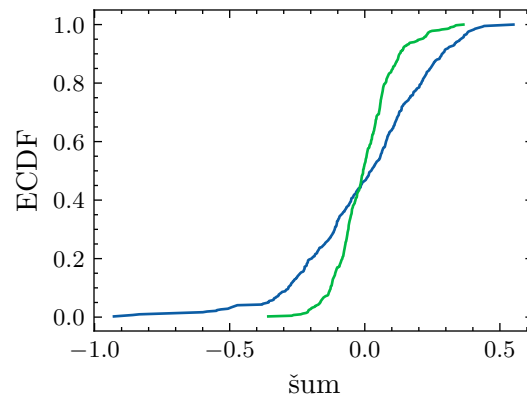
```
[ ]: x1, y1, inv1 = ecdf(noise[0])
     x2, y2, inv2 = ecdf(noise[1])
```

```
[19]: fig, ax = set_size_decorator(plt.subplots, fraction=0.5, ratio='4:3')(1, 1)

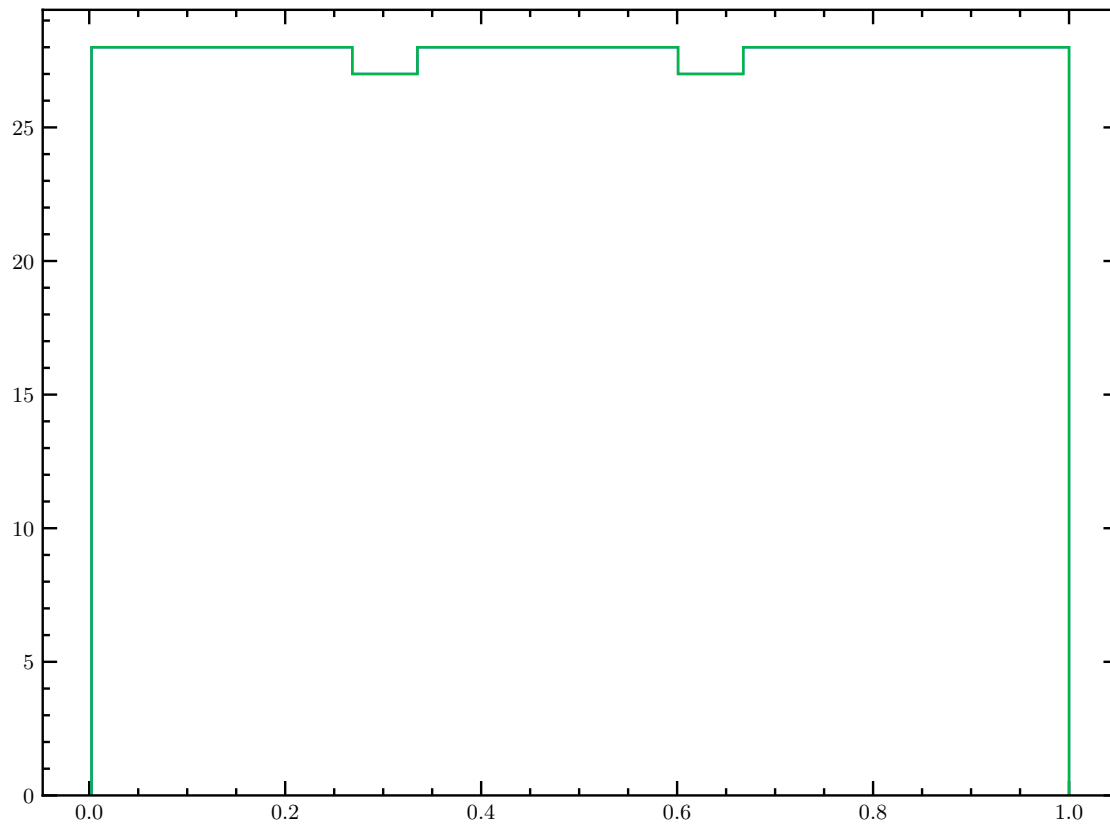
     ax.plot(x1, y1, lw=1)
     ax.plot(x2, y2, lw=1)
     ax.set_xlabel('šum')
     ax.set_ylabel('ECDF')
```

```
# savefig('mioni_ecdf')
```

```
[19]: Text(0, 0.5, 'ECDF')
```



```
[20]: plt.hist(y1, bins=15, histtype='step')  
plt.hist(y2, bins=15, histtype='step')  
plt.show()
```



```
[21]: from NIST_tests import RNG_test
      from random_helper_functions import float32_to_bin, split_to_arr, \
      ↪ bin_str_to_matrix
```

```
[22]: from numba import njit

@njit
def binary_tree_walk(arr):
    bits = np.zeros(len(arr)).astype(np.int16)
    for i, a in enumerate(arr):
        if a > 0.5:
            bits[i] = 1
    return bits
```

```
[23]: rng1 = y1[inv1]
      rng2 = y2[inv2]

      rng = np.concatenate((rng1, rng2))
      rng = rng[rng != 1]
```

```
[24]: bits = float32_to_bin(rng2, cut=9)
      bits = ''.join(bits)

      result1 = RNG_test(bits, short_df=True)
      result1
```

```
100%|
      | 16/16 [00:00<00:00, 29.96it/s]
```

```
[24]:      0      1      2      3      4      5      6      7      8      9      10     11  \
p  0.73  0.85  0.86  0.19  0.37  0.90  0.15  0.20  nan  0.82  0.00  0.00

      12     13     14
p  0.38  0.06  0.14
```

```
[25]: bits = binary_tree_walk(rng2).astype(str)
      bits = ''.join(bits)

      result2 = RNG_test(bits, short_df=True)
      result2
```

```
100%|
      | 16/16 [00:00<00:00, 326.15it/s]
```

```
[25]:      0      1      2      3      4      5      6      7      8      9      10     11     12  \
p  1.00  0.20  0.84  0.71  nan  0.25  0.34  nan  nan  nan  0.30  1.00  0.69
```



```

      13      14
p 0.69 0.43

```

CDF  $\sim U(0, 1) \Rightarrow \text{argsort} / \max(\text{argsort}) \sim U(0, 1) \Leftrightarrow \text{argsort}, \text{linspace}(0, 1)[\text{argsort}] \sim U(0, 1)$

```

[26]: rngs = [rng1, rng2]

res = []
b = []
for rng in rngs:
    bits = get_bitstring(rng, length=32, cut=30)
    bits = ''.join(bits)
    b.append(bits)

    test_res = RNG_test(bits, short_df=True)
    res.append(test_res)

for rng in rngs:
    bits = binary_tree_walk(rng).astype(str)
    bits = ''.join(bits)
    b.append(bits)

    test_res = RNG_test(bits, short_df=True)
    res.append(test_res)

```

```

100%|
      | 16/16 [00:00<00:00, 304.40it/s]
100%|
      | 16/16 [00:00<00:00, 297.10it/s]
100%|
      | 16/16 [00:00<00:00, 338.44it/s]
100%|
      | 16/16 [00:00<00:00, 316.95it/s]

```

```

[27]: df = pd.concat([i for i in res])
df.index = [f'$p_{i}$' for i in range(1, len(df)+1)]
df.columns = [i + 1 for i in range(len(df.columns))]

```

```

[28]: df

```

```

[28]:
      1      2      3      4      5      6      7      8      9     10     11     12  \
$p_1$ 0.94 0.12 0.21 0.71 nan 0.33 0.02 nan nan nan 0.10 1.00
$p_2$ 0.94 0.12 0.37 0.84 nan 0.12 0.45 nan nan nan 0.74 1.00
$p_3$ 1.00 0.45 0.01 0.28 nan 0.81 0.00 nan nan nan 0.00 1.00
$p_4$ 1.00 0.20 0.84 0.71 nan 0.25 0.34 nan nan nan 0.30 1.00

```

	13	14	15
\$p_1\$	0.54	0.21	0.26
\$p_2\$	0.31	0.01	0.76
\$p_3\$	0.34	0.93	0.40
\$p_4\$	0.69	0.69	0.43

```
[29]: fig, ax = set_size_decorator(plt.subplots, fraction=0.5, ratio='4:3')(1, 1)

bits_arr = split_to_arr(b[0])
m = bin_str_to_matrix(bits_arr)
ax.matshow(m, cmap='Greys_r')
ax.axis('off')
# savefig('mioni_matrika_dobra', save_format='png', dpi=1000)
```

```
[29]: (-0.5, 27.5, 27.5, -0.5)
```



## 4 Reverse Box–Muller transform

```
[30]: from scipy.optimize import curve_fit

def gauss(x, a, b, c):
    """c...sigma, b...mu, a...normalization"""
    return a * np.exp(-((x - b)**2) / (2 * c**2))

def make_hist_and_fit(dist, func):
    n_bins = int(np.sqrt(len(dist)))
    y, x = np.histogram(dist, bins=n_bins)
    x = x[1:]

    popt, pcov = curve_fit(gauss, x, y)
    return popt, x, y
```

```
def get_Z(dist):
    popt, _, _ = make_hist_and_fit(dist, gauss)

    sigma, mu = popt[2], popt[1]
    Z = (dist - mu) / sigma

    # Z_popt, _, _ = make_hist_and_fit(Z, gauss) # debug

    return Z
```

```
[31]: import sympy as smp
```

```
[32]: X1, X2, U1, U2 = smp.symbols('X1, X2, U_1, U_2', real=True)
R, theta = smp.symbols(r'R, \theta', real=True)
```

```
[33]: x1 = R * smp.cos(theta)
x1
```

```
[33]:  $R \cos(\theta)$ 
```

```
[34]: x2 = R * smp.sin(theta)
x2
```

```
[34]:  $R \sin(\theta)$ 
```

```
[35]: eq1 = smp.Eq(X1, x1)
eq1
```

```
[35]:  $X_1 = R \cos(\theta)$ 
```

```
[36]: eq2 = smp.Eq(X2, x2)
eq2
```

```
[36]:  $X_2 = R \sin(\theta)$ 
```

```
[37]: res = smp.solve([eq1, eq2], [R, theta], dict=True)
res
```

```
[37]: [{R: (-X1**2 + X1*sqrt(X1**2 + X2**2) - X2**2)/(X1 - sqrt(X1**2 + X2**2)),
\theta: -2*atan((X1 - sqrt(X1**2 + X2**2))/X2)},
{R: -(X1**2 + X1*sqrt(X1**2 + X2**2) + X2**2)/(X1 + sqrt(X1**2 + X2**2)),
\theta: -2*atan((X1 + sqrt(X1**2 + X2**2))/X2)}]
```

```
[38]: R_, theta_ = res[1][R], res[1][theta]
```

```
[39]: R_
```

```
[39]:
```

$$-\frac{X_1^2 + X_1\sqrt{X_1^2 + X_2^2} + X_2^2}{X_1 + \sqrt{X_1^2 + X_2^2}}$$

[40]: theta\_

$$-2 \operatorname{atan}\left(\frac{X_1 + \sqrt{X_1^2 + X_2^2}}{X_2}\right)$$

[41]: eq1 = smp.Eq(R, R\_)  
eq1

$$R = -\frac{X_1^2 + X_1\sqrt{X_1^2 + X_2^2} + X_2^2}{X_1 + \sqrt{X_1^2 + X_2^2}}$$

[42]: eq2 = smp.Eq(theta, theta\_)  
eq2

$$\theta = -2 \operatorname{atan}\left(\frac{X_1 + \sqrt{X_1^2 + X_2^2}}{X_2}\right)$$

[43]: eq1 = eq1.subs(R, smp.sqrt(-2 \* smp.log(U1)))  
eq1

$$\sqrt{2}\sqrt{-\log(U_1)} = -\frac{X_1^2 + X_1\sqrt{X_1^2 + X_2^2} + X_2^2}{X_1 + \sqrt{X_1^2 + X_2^2}}$$

[44]: U1\_sol = smp.solve(eq1, U1)[0] # U1  
U1\_sol

$$e^{-\frac{(X_1^2 + X_1\sqrt{X_1^2 + X_2^2} + X_2^2)^2}{2(X_1 + \sqrt{X_1^2 + X_2^2})^2}}$$

[45]: eq2 = eq2.subs(theta, 2 \* smp.pi \* U2)  
eq2

$$2\pi U_2 = -2 \operatorname{atan}\left(\frac{X_1 + \sqrt{X_1^2 + X_2^2}}{X_2}\right)$$

[46]: U2\_sol = smp.solve(eq2, U2)[0] # U2  
U2\_sol

$$-\frac{\operatorname{atan}\left(\frac{X_1 + \sqrt{X_1^2 + X_2^2}}{X_2}\right)}{\pi}$$

[47]: U1\_f = smp.lambdify([X1, X2], U1\_sol)  
U2\_f = smp.lambdify([X1, X2], U2\_sol)

```
[48]: Z1, Z2 = get_Z(noise[0]), get_Z(noise[1])
```

```
n1 = U1_f(Z1, Z2)
```

```
n2 = U2_f(Z1, Z2)
```

```
from scipy import signal
```

```
n2 = n2 + 0.5
```

```
[49]: from stat_tests import chi2_test, ks_test
```

```
[50]: n = [n1, n2]
```

```
tests = []
```

```
for i in n:
```

```
    tests.append([chi2_test(i, n_bins=20), ks_test(i)])
```

```
[51]: tests
```

```
[51]: [[(array([[30.99521531, 0.04042166]]), 37.56623478662507),  
      (array([[0.04322233, 0.40424711]]), [0.07917726627266543])],  
      [(array([[3.81722488e+01, 5.64228228e-03]]), 37.56623478662507),  
      (array([[0.08006591, 0.00887653]]), [0.07917726627266543])]]
```

```
[52]: alpha = 0.01
```

```
for c, i in enumerate(n):
```

```
    fig, ax = set_size_decorator(plt.subplots, fraction=0.5, ratio='4:3')(1, 1)
```

```
    ax.hist(i, histtype='step', color=f'C{c+2}', bins=20)
```

```
    chi2 = tests[c][0][0][0][0]
```

```
    ks = tests[c][1][0][0][0]
```

```
    critical_value_chi2 = tests[c][0][1]
```

```
    critical_value_ks = tests[c][1][1][0]
```

```
    print(critical_value_ks, critical_value_chi2)
```

```
    an1 = f'\n$\chi^2$ = {chi2:.2f}, $d$ = {ks:.2e}'
```

```
    an2 = f'\n$\chi^2_*$ = {critical_value_chi2:.2f}, $d_*$ =
```

```
→{critical_value_ks:.2e}'
```

```
    an3 = r' pri $\alpha$ = {}'.format(alpha)
```

```
    if c == 1:
```

```
        an = an1 + an2 + an3
```

```
    else:
```

```

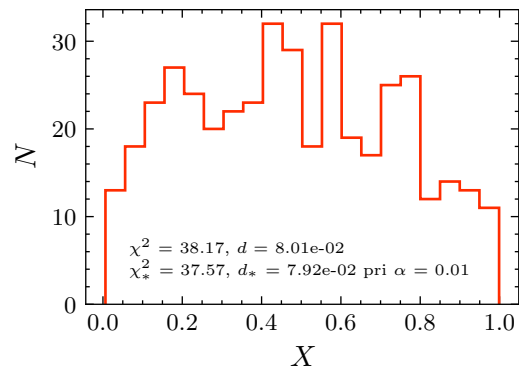
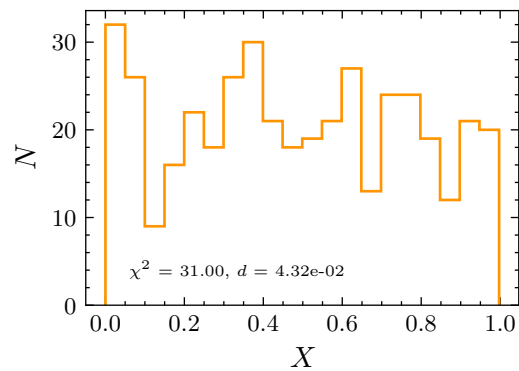
an = an1

ax.annotate(an, xy=(0.1, 0.1), xycoords='axes fraction', fontsize=6)

ax.set_xlabel('$X$')
ax.set_ylabel('$N$')
savefig(f'box_muller_uniform_{c}')
```

```

0.07917726627266543 37.56623478662507
0.07917726627266543 37.56623478662507
```



```

[53]: # bits = get_bitstring(n1, length=32)
# bits = ''.join(bits)
bits = binary_tree_walk(n1).astype(str)
bits = ''.join(bits)

res1 = RNG_test(bits, short_df=True)
```

100%|

| 16/16 [00:00<00:00, 334.50it/s]

```
[54]: # bits = get_bitstring(n2, length=32)
# bits = ''.join(bits)
bits = binary_tree_walk(n2).astype(str)
bits = ''.join(bits)

res2 = RNG_test(bits, short_df=True)
```

100%|

| 16/16 [00:00<00:00, 348.55it/s]

```
[55]: r = np.concatenate((n1, n2))

# bits = get_bitstring(r, length=32)
# bits = ''.join(bits)
bits = binary_tree_walk(r).astype(str)
bits = ''.join(bits)

res3 = RNG_test(bits, short_df=True)
```

100%|

| 16/16 [00:00<00:00, 312.06it/s]

```
[56]: res = [res1, res2, res3]

df = pd.concat([i for i in res])
df.index = [f'$p_{i}$' for i in range(1, len(df)+1)]
df.columns = [i + 1 for i in range(len(df.columns))]
```

```
[57]: df
```

```
[57]:
```

	1	2	3	4	5	6	7	8	9	10	11	12	\
\$p_1\$	0.33	0.00	0.22	1.00	nan	0.27	0.34	nan	nan	nan	0.22	1.00	
\$p_2\$	0.04	0.00	0.05	0.22	nan	0.49	0.00	nan	nan	nan	0.70	1.00	
\$p_3\$	0.03	0.00	0.02	0.93	nan	0.36	0.00	nan	nan	nan	0.01	1.00	
	13	14	15										
\$p_1\$	0.19	0.00	0.54										
\$p_2\$	0.03	0.69	0.84										
\$p_3\$	0.03	0.01	0.36										

```
[58]: # test
a = np.random.normal(size=100000)
b = np.random.normal(size=100000)

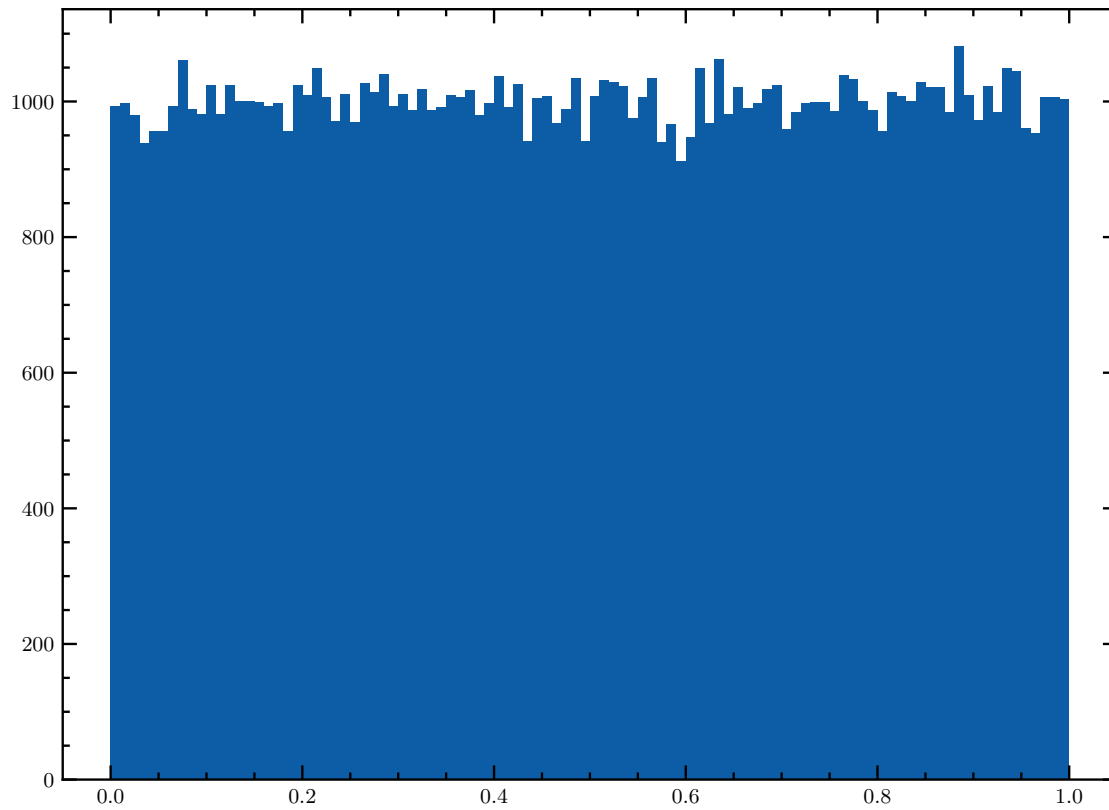
Z1, Z2 = get_Z(a), get_Z(b)

n1 = U1_f(Z1, Z2)
```

```
n2 = U2_f(Z1, Z2)

n2 = n2 + 0.5

plt.hist(n1, bins=100)
plt.show()
```

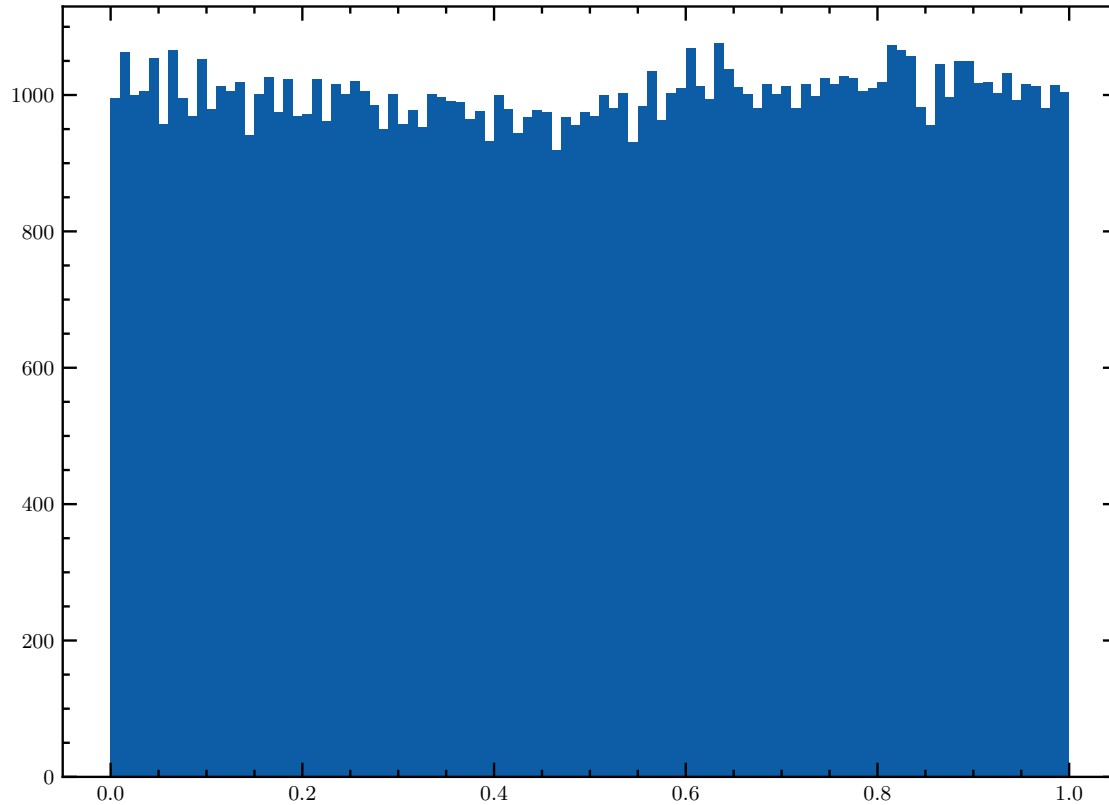


```
[59]: print(ks_test(n1))
      print(chi2_test(n1, n_bins=100))
```

```
(array([[0.0023873 , 0.61799027]]), [0.005145321961471824])
(array([[90.45      , 0.71846975]]), 135.80672317102676)
```

```
[60]: plt.hist(n2, bins=100)
      plt.show()
```





```
[62]: print(ks_test(n2))
      print(chi2_test(n2, n_bins=100))
```

```
(array([[7.13637944e-03, 7.50552966e-05]]), [0.005145321961471824])
(array([[107.834      ,  0.255555649]]), 135.80672317102676)
```

```
[63]: bits = get_bitstring(n1, length=32)
      bits = ''.join(bits)

      RNG_test(bits)
```

100%|

| 16/16 [00:16<00:00, 1.06s/it]

```
[63]:
```

	test	p
0	Frequency Test (Monobit)	0.05
1	Frequency Test within a Block	0.42
2	Run Test	0.81
3	Longest Run of Ones in a Block	0.41
4	Binary Matrix Rank Test	0.59
5	Discrete Fourier Transform (Spectral) Test	0.19
6	Non-Overlapping Template Matching Test	0.83

7	Overlapping Template Matching Test	0.41
8	Maurer's Universal Statistical test	0.74
9	Linear Complexity Test	0.52
10	Serial test	0.73
11	Approximate Entropy Test	0.02
12	Cummulative Sums (Forward) Test	0.08
13	Random Excursions Test	0.79
14	Random Excursions Variant Test	0.57

```
[64]: bits = get_bitstring(n2, length=32)
bits = ''.join(bits)

RNG_test(bits)
```

100%|

| 16/16 [00:17<00:00, 1.07s/it]

	test	p
0	Frequency Test (Monobit)	0.80
1	Frequency Test within a Block	0.54
2	Run Test	0.94
3	Longest Run of Ones in a Block	0.77
4	Binary Matrix Rank Test	0.65
5	Discrete Fourier Transform (Spectral) Test	0.75
6	Non-Overlapping Template Matching Test	0.60
7	Overlapping Template Matching Test	0.88
8	Maurer's Universal Statistical test	0.87
9	Linear Complexity Test	0.90
10	Serial test	0.21
11	Approximate Entropy Test	0.75
12	Cummulative Sums (Forward) Test	0.91
13	Random Excursions Test	0.61
14	Random Excursions Variant Test	0.25

```
[ ]:
```