Binary Search Trees

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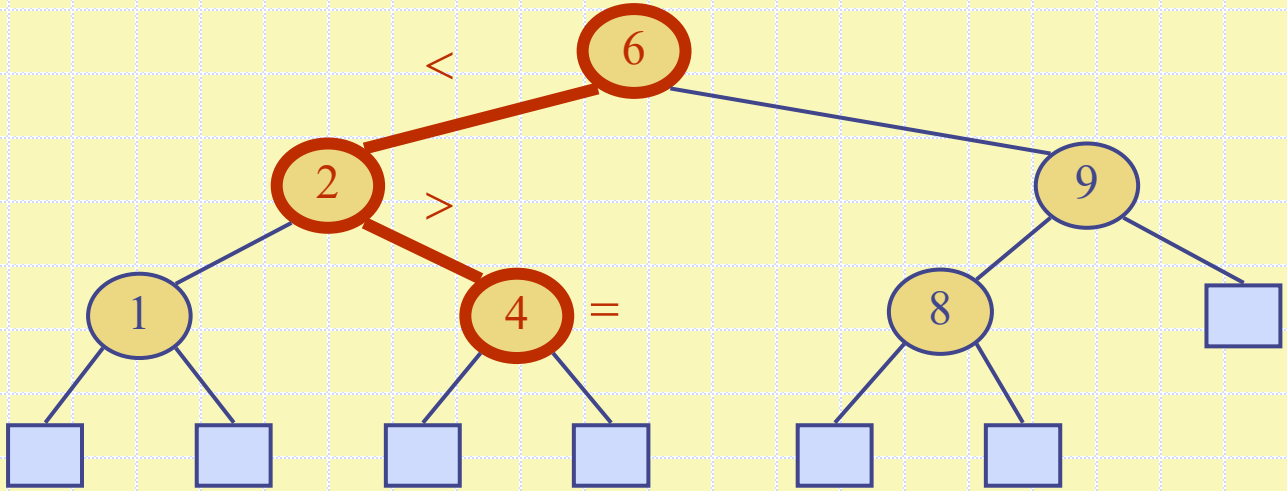
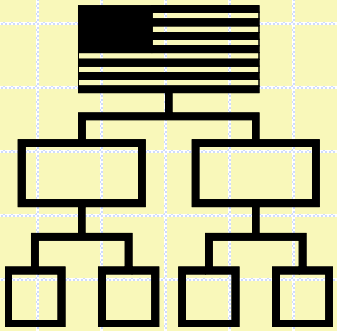
Data Structures and Algorithms in Java, 5th edition. John Wiley& Sons, 2010. ISBN 978-0-470-38326-1.
Data Structures and the Java Collections Framework by William J. Collins, 3rdedition, ISBN 978-0-470-48267-4.
Both books are published by Wiley.

Coverage

- Section 10.1: Binary Search Trees

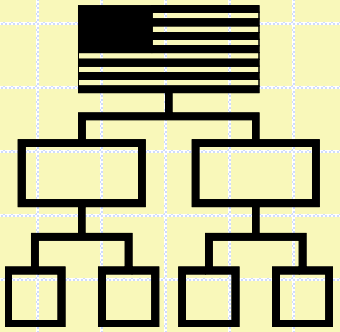


Binary Search Trees

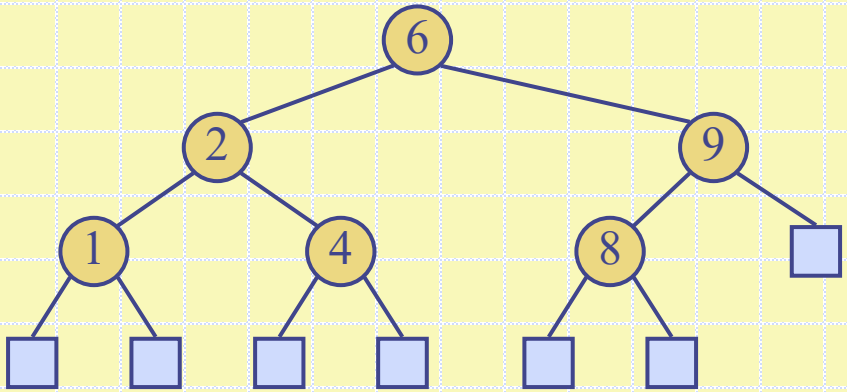


- A binary search tree is a binary tree storing keys (or key-value entries) at its internal nodes and satisfying the following property:
  - Let u, t, and w be three nodes such that u is in the left subtree of t and w is in the right subtree of t. We have $key(u) \leq key(t) \leq key(w)$
- External nodes do not store items. They are there to guarantees that the tree is a proper binary tree, which simplifies other operations.

Binary Search Trees



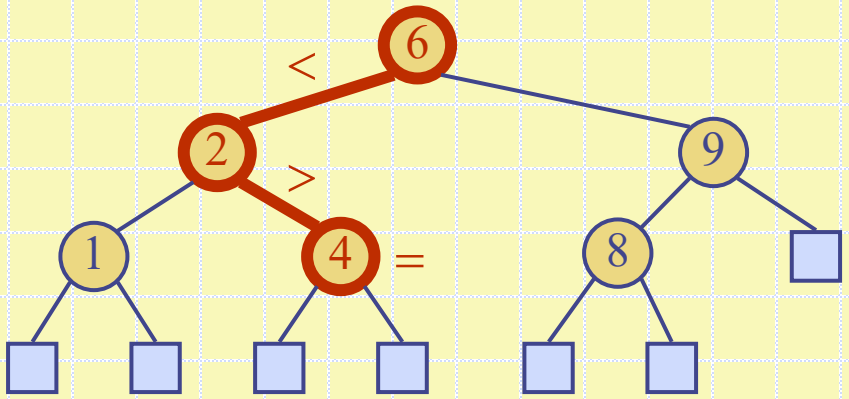
- An inorder traversal of a binary search trees visits the keys in increasing order



Search

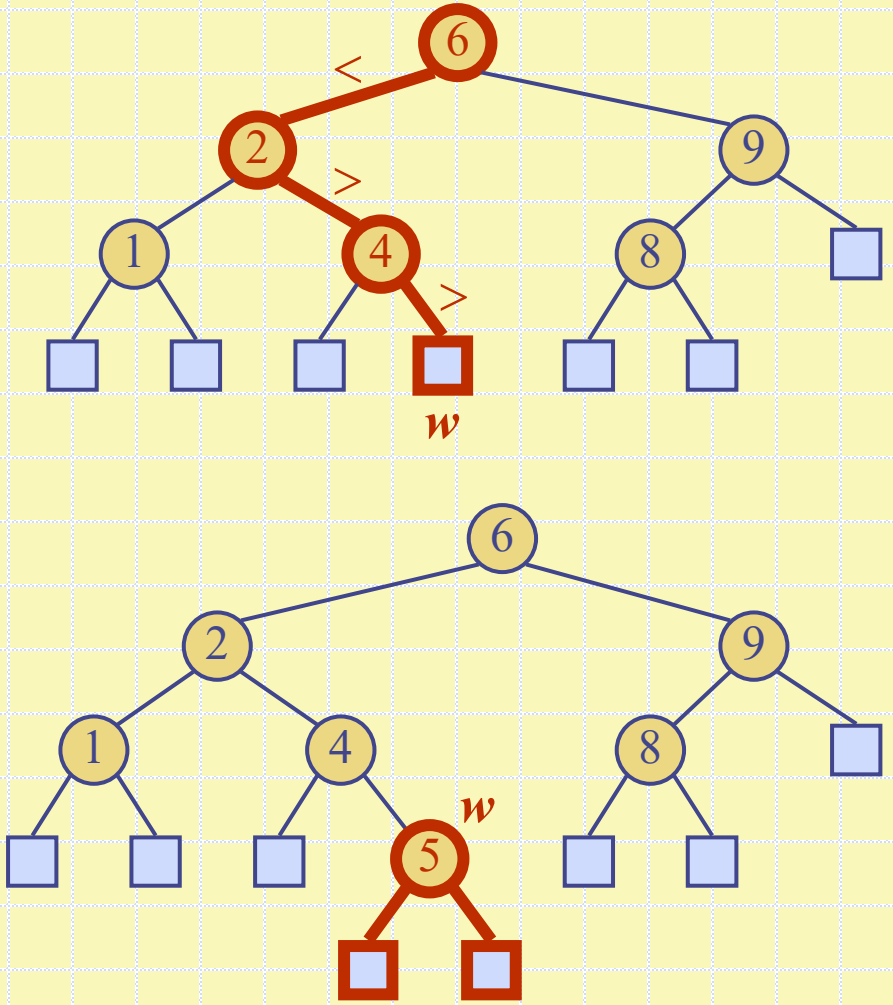
- To search for a key k, we trace a downward path starting at the root
- The next node visited depends on the comparison of k with the key of the current node
- If we reach a leaf, the key is not found
- Example: get(4):
  - Call TreeSearch(4,root)
- The algorithms for floorEntry and ceilingEntry are similar

```
Algorithm TreeSearch(k, w)
    if T.isExternal (w)
        return null
    if k < key(w)
        return TreeSearch(k, T.left(w))
    else if k = key(w)
        return w
    else { k > key(w) }
        return TreeSearch(k, T.right(w))
```



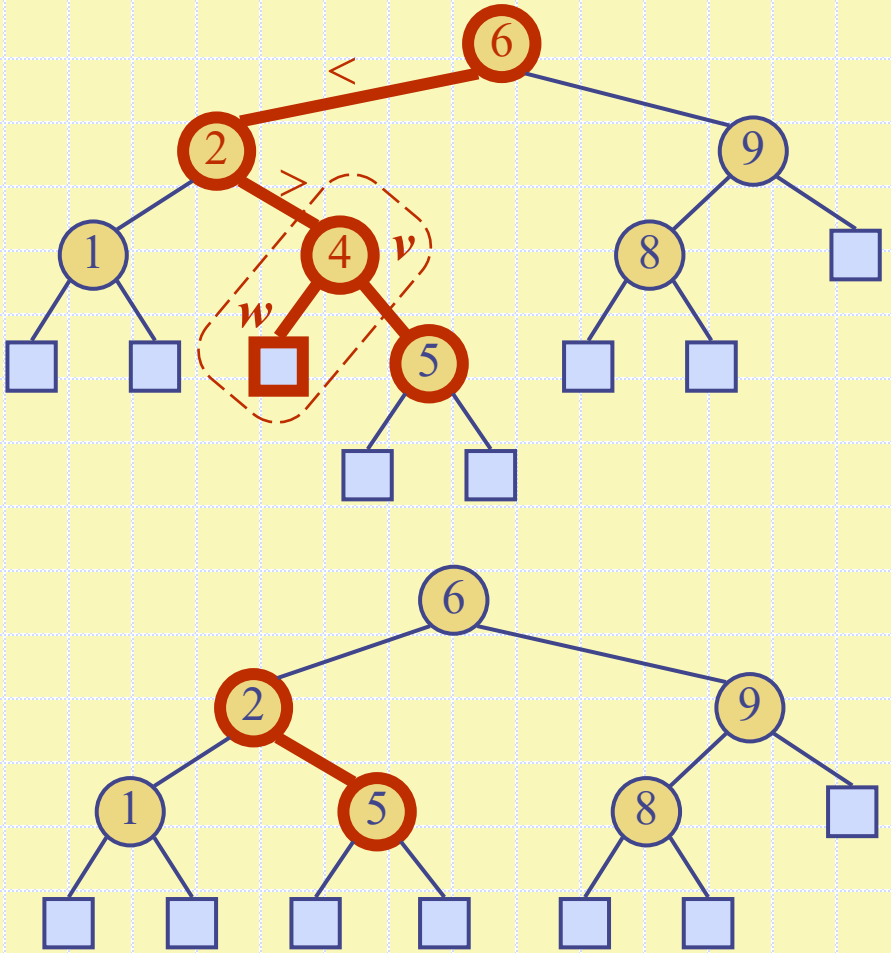
Insertion

- To perform operation put(k, o), we search for key k (using TreeSearch)
- Assume k is not already in the tree, and let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node
- Example: insert (5)



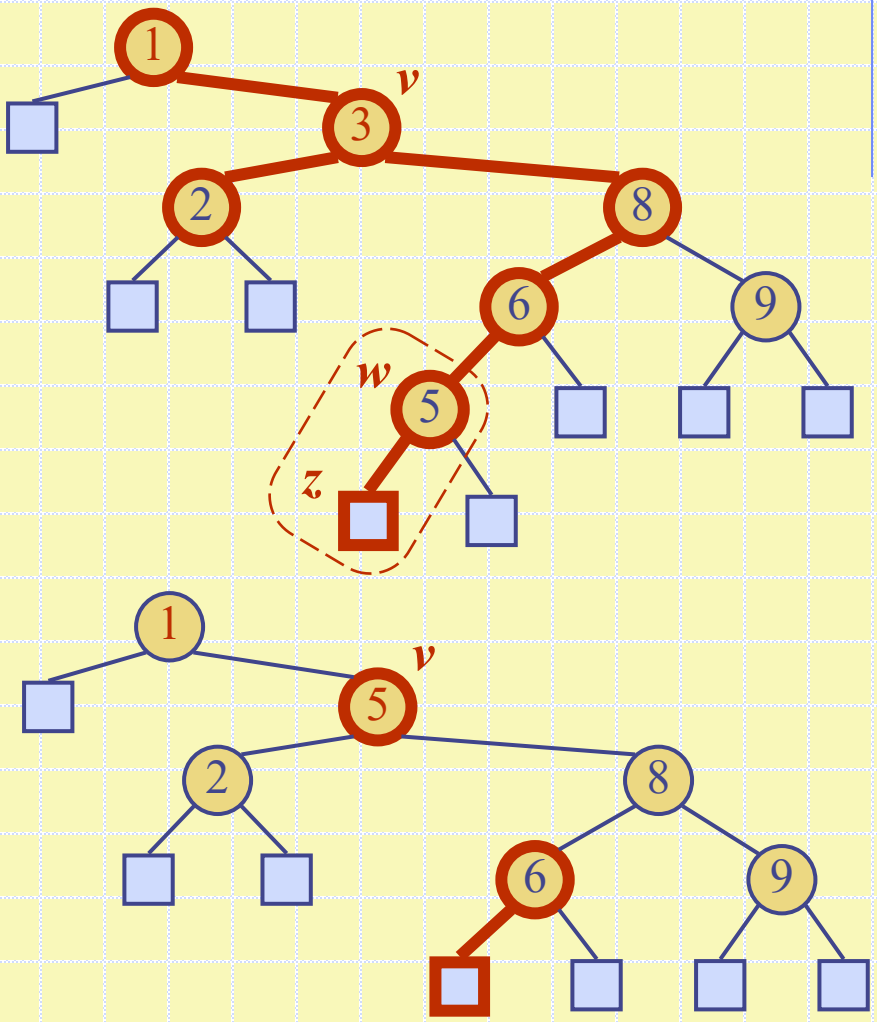
Deletion

- To perform operation remove(k), we search for key k
- Assume key k is in the tree, and let v be the node storing k
- If node v has a leaf child w, we remove v and w from the tree with operation removeExternal(w), which removes w and its parent
- Example: remove (4)



Deletion (cont.)

- We consider the case where the key k to be removed is stored at a node v whose children are both internal
  - we find the internal node w that follows v in an inorder traversal
  - we copy $key(w)$ into node v
  - we remove node w and its left child z (which must be a leaf) by means of operation removeExternal(z)
- Example: remove(3)



Performance

- Consider an ordered map with n items implemented by means of a binary search tree of height h
  - the space used is $O(n)$
  - methods get, floorEntry, ceilingEntry, put and remove take $O(h)$ time
- The height h is $O(n)$ in the worst case and $O(\log n)$ in the best case