Stacks

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Data Structures and Algorithms in Java, 5th edition. John Wiley& Sons, 2010. ISBN 978-0-470-38326-1.
Data Structures and the Java Collections Framework by William J. Collins, 3rdedition, ISBN 978-0-470-48267-4.
Both books are published by Wiley.

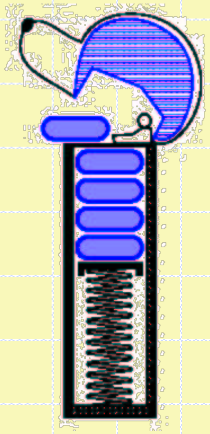
Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction/model of a data structure.
- An abstract data type is defined indirectly, only by the operations that may be performed on it. An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

Abstract Data Types (ADTs)

- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

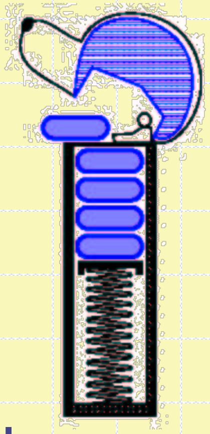
The Stack ADT



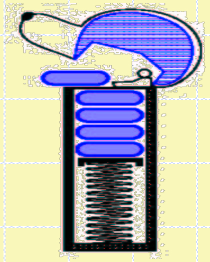
- The Stack ADT stores arbitrary objects.
- Insertions and deletions follow the ***last-in first-out (LIFO)*** scheme. Think of a spring-loaded plate dispenser.
- Formally, a stack is an ADT that supports the following main operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Examples: operations of "Back" button on a browser or "undo" on text editors.

The Stack ADT

- Secondary stack operations include:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored



The Stack ADT



- The following table shows a series of stack operations and their effects on an initially empty stack of integers:

Operation	Output	Stack Contents
push(5)	--	[5]
push(2)	--	[5, 2]
push(8)	--	[5, 2, 8]
pop()	8	[5, 2]
isEmpty()	false	[5, 2]
top()	2	[5, 2]
pop()	2	[5]
pop()	5	[]
pop()	*"error"*	[]

The Java Built-in *Stack* Class

- Because of its importance, Java has a built-in class for the stack (java.util.Stack).
- The class has various methods, including:
  - push(), pop(), peek(), empty(), and size().
- pop() and peek() throw EmptyStackException if operations are attempted on an empty stack. .
- While this class is convenient, it is very important to know how to design and implement a Stack class from scratch.

Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception.
- Exceptions are said to be "thrown" by an operation that cannot be executed.
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty.
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException.

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

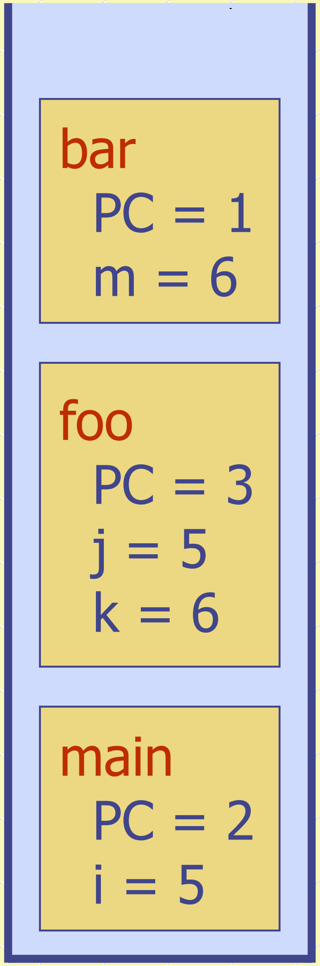
Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack.
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack.
- Allows for recursion.

```
main() {
   int i = 5;
   foo(i);
   }

foo(int j) {
   int k;
   k = j+1;
   bar(k);
   }

bar(int m) {
   …
   }
```

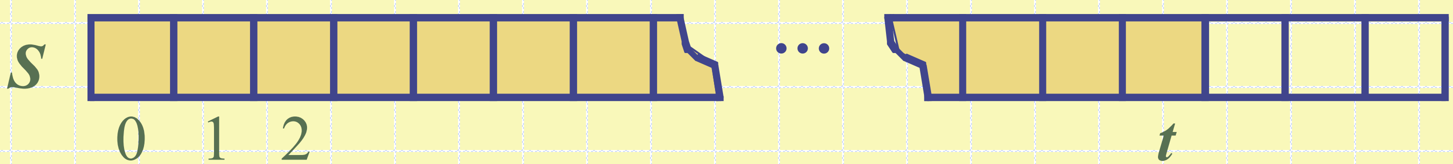


Array-based Stack

- A simple way of implementing the Stack ADT uses an array.
- We add elements from left to right.
- A variable keeps track of the index of the top element.
  - Initialized to -1 when stack is created.

Algorithm *size*()
  return $t + 1$

Algorithm *pop*()
  if *isEmpty*() **then**
    throw *EmptyStackException*
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$



Array-based Stack (cont.)

- The array storing the stack elements may become full.
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - We need to define this class; it is not intrinsic to the Stack ADT

Algorithm *push*(o)
  if $t = S.length - 1$ **then**
    throw *FullStackException*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$

Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an *implementation-specific* exception

Array-based Stack in Java*

```
public class ArrayStack<E>
      implements Stack<E> {

   // holds the stack elements
   private E S[ ];

   // index to top element
   private int top = -1;

   // constructor
   public ArrayStack(int capacity) {
       S = (E[]) new Object[capacity]);
   }
```

```
   public E pop()
          throws EmptyStackException {
     if isEmpty()
       throw new EmptyStackException
              ("Empty stack: cannot pop");
       E temp = S[top];
       // facilitate garbage collection:
       S[top] = null;
       top = top – 1;
       return temp;
   }

…  (other methods of Stack interface)
```

* Notice that this is not a built-in Java implementation

Example use in Java

```
/** A non-recursive generic method for reversing an array */
public static <E> void reverse(E[] a){
        Stack<E> s = new ArrayStack<E>(a.length);
        for(int i = 0; i < a.length; i++)
            s.push(a[i]);
        for(int i = 0; i < a.length; i++)
            a[i] = s.pop();
  }
```

Time complexity is: $\boldsymbol{O}(n)$
Space complexity is: $\boldsymbol{O}(n)$

Example use in Java

```
public class Tester {

  // … other methods
  public intReverse(Integer a[]) {
      Stack<Integer> s;
      s = new ArrayStack<Integer>();

      … (code to reverse array a) …
  }
```

```
public floatReverse(Float f[]) {
    Stack<Float> s;
    s = new ArrayStack<Float>();

    … (code to reverse array f) …
}
```

Better Stack Implementation

- Use linked lists instead of arrays.
- No need to define a maximum size.
- When push(), add new element/node at the tail of the list.
- pop() removes the node at the tail of the list.

What is the complexity?

Parentheses Matching

- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Is it?

Parentheses Matching Algorithm

Algorithm ParenMatch(*X*,*n*):

Input: An array *X* of *n* tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number

Output: **true** if and only if all the grouping symbols in *X* match

Let *S* be an empty stack

for *i*=0 to *n*-1 **do**

    if *X*[*i*] is an opening grouping symbol **then**

        S.push(*X*[*i*])

    else if *X*[*i*] is a closing grouping symbol **then**

Notice that item is removed here

        if *S*.isEmpty() **then**

            return false {nothing to match with}

        if *S*.**pop**() does not match the type of *X*[*i*] **then**

            return false {wrong type}

if *S*.isEmpty() **then**

    return true {every symbol matched}

else return false {some symbols were never matched}

HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

Operator precedence

* has precedence over +/–

Example: x + y * z is:

x + (y * z) rather than (x + y) * z

Associativity

operators of the same precedence group evaluated from left to right

Example: x – y + z is:

(x – y) + z rather than x – (y + z)

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Two stacks:

- opStk holds operators
- Use $ to hold a special “end of input” token with lowest precedence

Algorithm doOp()

x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push(y **op** x)

Algorithm repeatOps(refOp)

while (valStk.size() > 1 ∧
    prec(refOp) ≤
    prec(opStk.top())
  doOp()

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

Output: the value of the expression

while there’s another token z
  if isNumber(z) **then**
    valStk.push(z)
  else
    repeatOps(z);
    opStk.push(z)
repeatOps($);
return valStk.top()

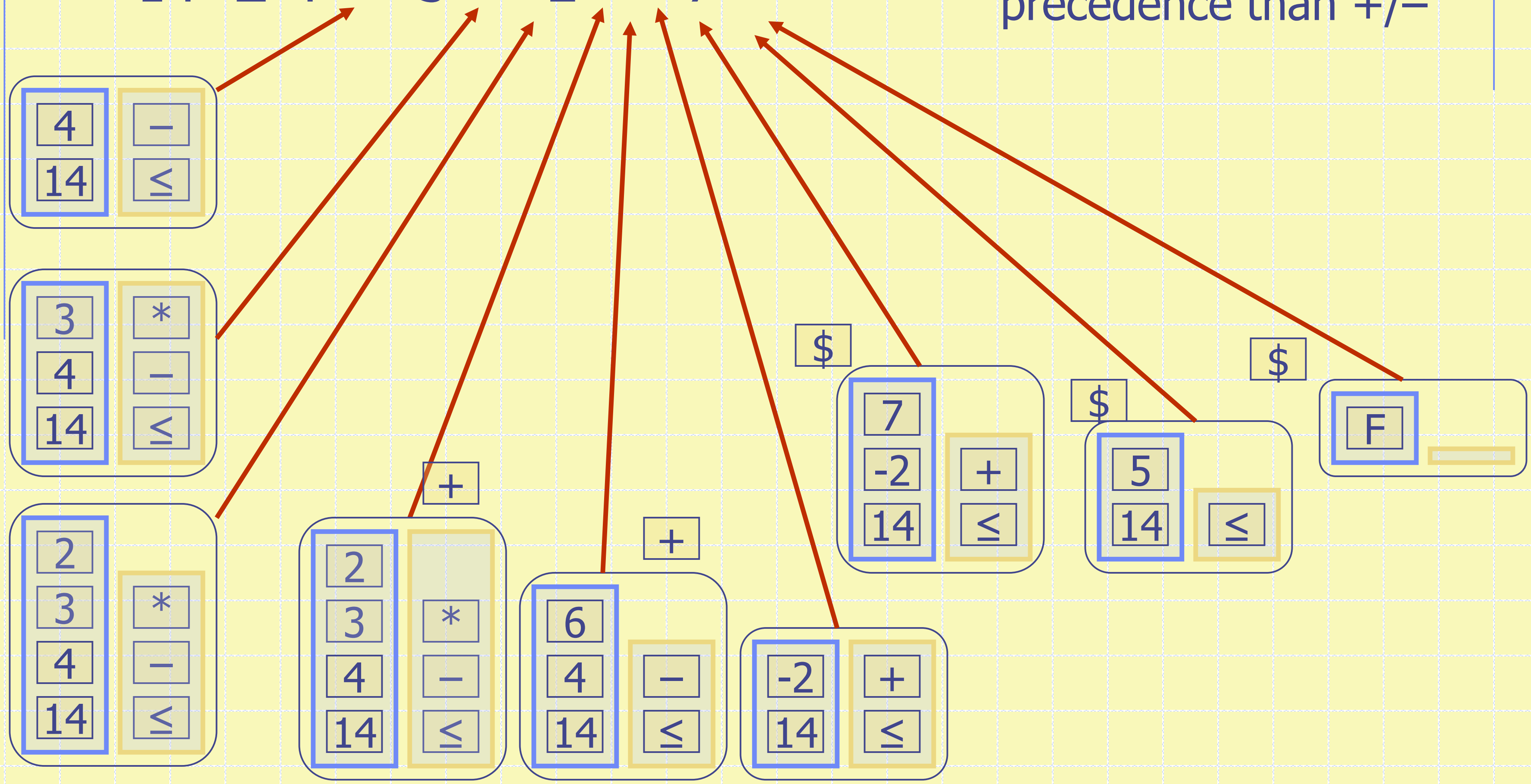
Force the execution of all remaining operators

Algorithm on an Example Expression

Slide by Matt Stallmann included with permission.

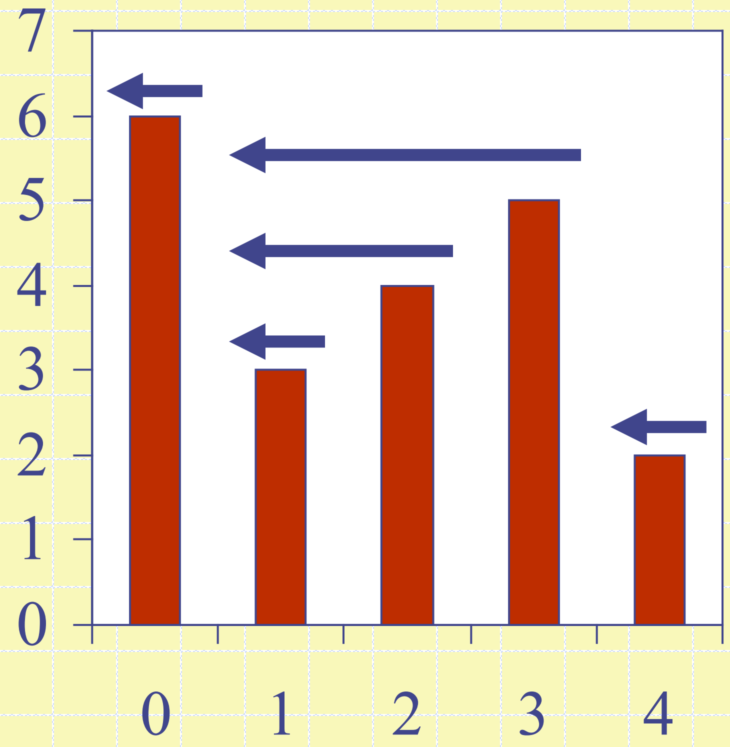
$14 \le 4 - 3 * 2 + 7$

Operator ≤ has lower precedence than +/–



Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

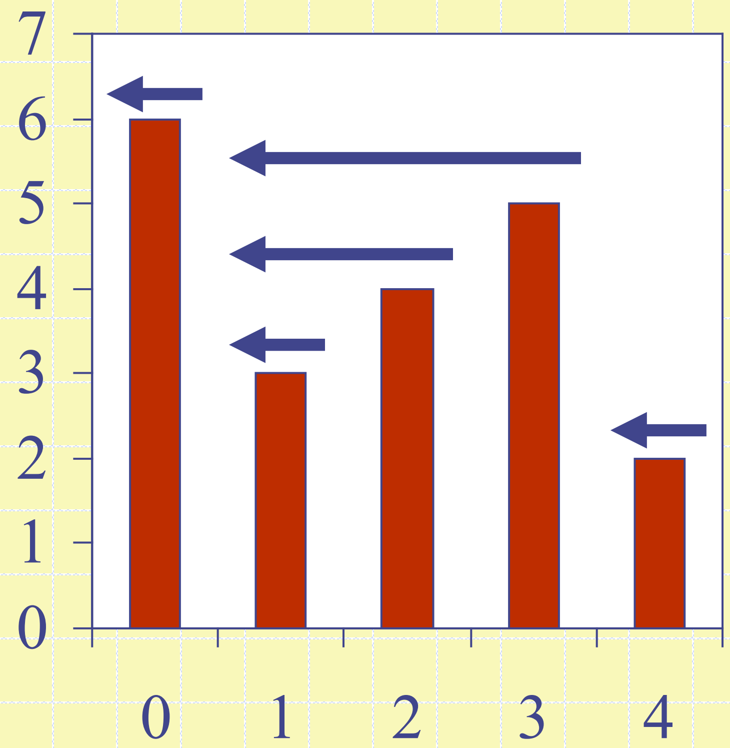
Quadratic Algorithm

Algorithm *spans1*(X, n)	
Input array X of n integers	
Output array S of spans of X	**#**
$S \leftarrow$ new array of n integers	n
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
$s \leftarrow 1$	n
while $s \leq i \wedge X[i - s] \leq X[i]$	$1 + 2 + \ldots + (n - 1)$
$s \leftarrow s + 1$	$1 + 2 + \ldots + (n - 1)$
$S[i] \leftarrow s$	n
return S	1

- Algorithm ***spans1*** runs in $O(n^2)$ time

Computing Spans with a Stack

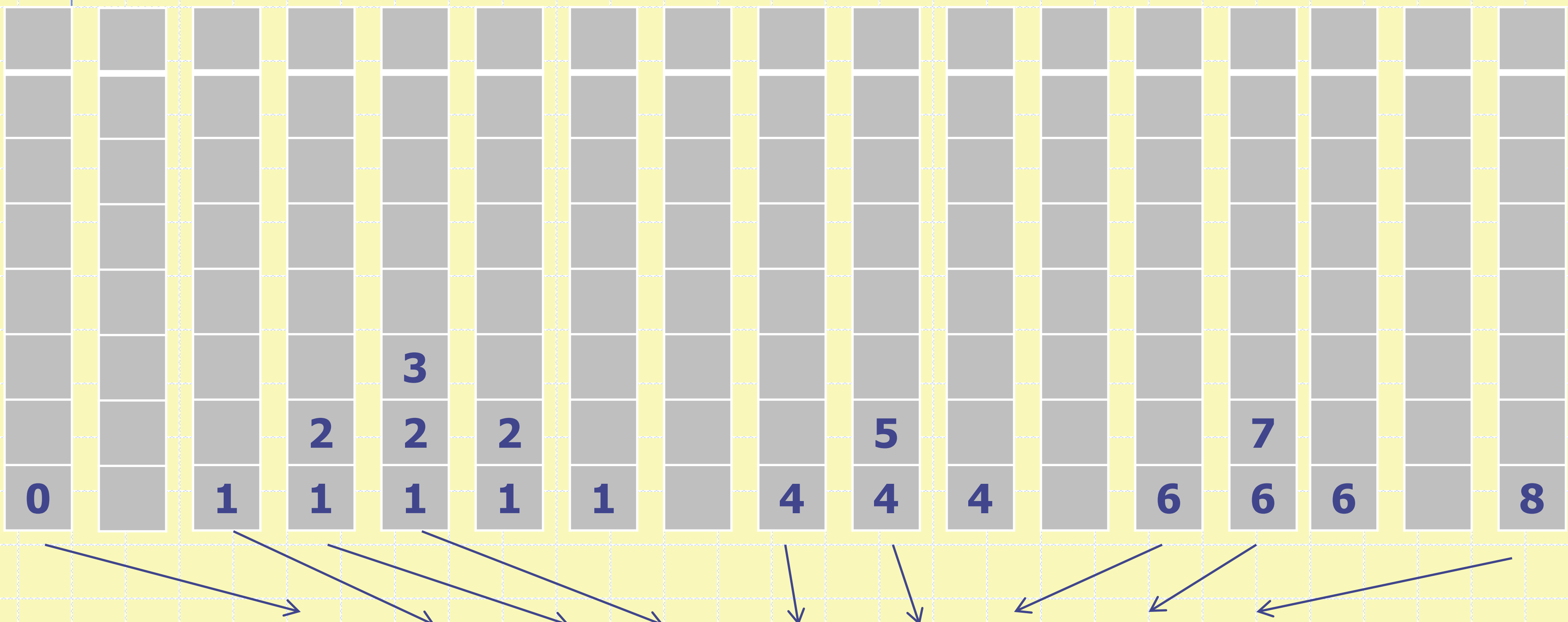
- We keep in a stack the indices of the elements <u>visible</u> when “looking back”
- We scan the array from left to right
  - Let i be the current index
  - We push the index as long as the one prior to it has a smaller value
  - We pop all elements otherwise
- Each index of the array
  - Is pushed into the stack exactly once
  - Is popped from the stack at most once
- Stack height for the pushed index represents the needed value



Computing Spans with a Stack

- Example

X index	0	1	2	3	4	5	6	7	8
Value	60	30	40	50	20	70	30	80	20



S									
	1	1	2	3	1	2	1	2	1

Linear Algorithm

- We iterate on the array n times
- Each index is then pushed and popped at most once, which totals to $n + n$
- We record the value at each index of the resulted Span array, which totals to n times
- Consequently algorithm ***spans2*** has a complexity of $O(n)$

Growable Array-based Stack

- In a push() operation, if the stack is full (no more empty locations in the array), we can throw an exception and abort/reject the operation.
- Alternatively, we can extend the array; which is actually replacing it with a larger one.
- How large should the new array be?
  - Incremental strategy: increase the size by a constant c
  - Doubling strategy: double the size

Algorithm $add(o)$
  if $t = S.length - 1$ **then**
    $A \leftarrow$ **new array of size …**
    for $i \leftarrow 0$ **to** t **do**
      $A[i] \leftarrow S[i]$
    $S \leftarrow A$
  $t \leftarrow t + 1$
  $S[t] \leftarrow o$

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n push() operations.
- We assume that we start with an empty stack represented by an array of size 1.
- We refer to the average time taken by a push() operations over the series of operations, i.e., $T(n)/n$, the ***amortized* time** of a push() operation.

Incremental Strategy Analysis

- We need to find the amortized time to perform one push() operation.
  - That is the total time to perform n push() operations / n.
- In general, we need to replace the array $k = n/c$ times for all n push() to take place.
  - For instance if $n = 100$, and $c = 4$, we need to go through 25 (100/4) replacements for all push() operations to take place.
  - Notice also that each replacement is larger than the previous one by c.

Incremental Strategy Analysis

- Consequently, the total time $T(n)$ of a series of n push() operations is proportional to:

$$n + c + 2c + 3c + 4c + \ldots + kc =$$

$$n + c(1 + 2 + 3 + \ldots + k) =$$

$$n + ck(k + 1)/2$$

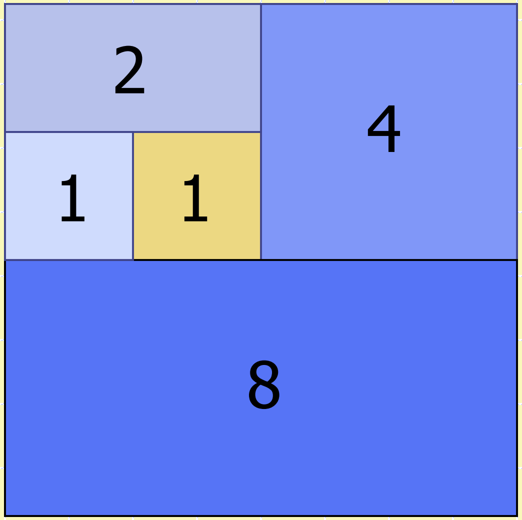
- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$. That is
- $T(n)$ is the complexity to perform n push() operations. <u>Hence, the amortized time of one single push() operation is $O(n)$.</u>

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times.
  - For instance, to perform 1000 push() operations, we need to expand the array 10 times (1 -> 2 -> 4 -> 8 -> 16 -> 32 -> 64 -> 128 -> 256 -> 512 -> 1024).
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + 1 + 2 + 4 + 8 + \ldots + 2^k = n + 2^{k+1} - 1 = n + 2n - 1 = 3n - 1.$$

geometric series



Doubling Strategy Analysis

- Consequently, $T(n)$ (which is needed to perform n push() operations) is $O(n)$.
- Hence, the amortized time of a single push() operation is $O(1)$.

geometric series