Fall 2017

HUNTER COLLEGE

STAT 701

Advanced Probability Theory I

Final Exam

Last Name: ________________________________

First Name: ________________________________

December 13, 2017

Instructions

- There are 4 questions, each question is between 5-7 points. The maximal score is 23 points. A perfect score is **20** points.
- There are two versions of Question 3 with different points. Please choice one version and only one to answer.
- Show all work. You will receive partial credit for partially completed problems.
- You may use any references.

Q1. (6pt).
(1) Assume that U_i's are independent and identically uniformly distributed with minimum 0 and maximum 1. Show that

$$\Pr\left(\cup_{i=1}^{n}\left\{U_i \leq 1-(1-\alpha)^{1/n}\right\}\right)=\alpha$$

for any $0<\alpha<1$.
(2) Assume that V_i's are identically uniformly distributed with minimum 0 and maximum 1. Show that

$$\Pr\left(\cup_{i=1}^{n}\{V_i \leq \alpha/n\}\right)\leq\alpha$$

for any $0<\alpha<1$.
(3) For any $0<\alpha<1$, is $1-(1-\alpha)^{1/n}$ greater or smaller than α/n? Prove your conclusion.

Q2. (5pt).
The moment generating function of a random variable X is

$$M_X(t) = \frac{e^{2t}}{1-t^2}.$$

Find its mean and variance.

Q3. (5pt)
Choose one and only one question from (A) and (B)

(A) There is a sequence $X_1, X_2, \cdots, X_n$, where X_i's are independent and identically distributed. We call X_m a record if

$$X_m > \max\{X_1, \cdots, X_{m-1}\}.$$

We further assume that the distribution of X_i is continuous, and X_1 is defined to be a record.

Find the expected value of the number of records in this sequence.

(B) We throw a fair dice with 6 faces until a number appears 6 times consecutively. Find the expected number of throws.

Note, you only need to solve either (A) or (B).

Q4. (7pt).
(1) Let X_i's be independent real-valued random variables with mean 0 and let $S_n = \sum_{i=1}^n X_i$. Show that

$$\Pr\left(\max_{1\le k\le n} |S_k| > \frac{3}{2}\right) \le \frac{4}{9}\mathrm{var}(S_n).$$

(2) Let X_i be a Bernoulli random variable with value 1 and -1, each with probability 0.5. Say, $\Pr(X_i = 1) = \Pr(X_i = -1) = 0.5$. Suppose that X_1 and X_2 are independent. Let $S_1 = X_1$ and $S_2 = X_1 + X_2$. Calculate

$$\Pr\left(\max_{1\le k\le 2} |S_k| > \frac{3}{2}\right)$$

and

$$\mathrm{var}(S_2).$$

Compare the value of $\Pr\left(\max_{1\le k\le 2} |S_k| > \frac{3}{2}\right)$ and $\frac{4}{9}\mathrm{var}(S_2)$.

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Please list probabilistic and statistical topics which you are interested in or which are useful to you and you would like to learned from STAT 701 and 702.

End of the final exam of Stat 701 (Instructor: Jiangtao Gou)