SPRING 2017 HUNTER COLLEGE STAT 213 Section 05 Introduction to Applied Statistics FINAL EXAM

Last Name:	
First Name:	
Graduation :	Vear·

- 1. Please do not tear paper. Please do not leave blank for any question. Show your work.
- 2. There are 10 questions, each question is 5 points. A perfect score is 50 points.
- 3. You have 120 minutes for this exam (3:00 pm 5:00 pm).
- 4. Sign your name in the front of classroom before leaving.

Formulas

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \dots + x_n$$

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + \sigma z$$

$$s_x^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}$$

$$s_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}}$$

$$\Pr(|X - \mu| \le k\sigma) \ge 1 - \frac{1}{k^2}$$
, for any distribution

 $\Pr(|X - \mu| \le \sigma) \approx 0.68$, $\Pr(|X - \mu| \le 2\sigma) \approx 0.95$, $\Pr(|X - \mu| \le 3\sigma) \approx 0.997$, for normal distribution

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

$$r = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{(n-1)s_x s_y}$$

$$\widehat{y} = a + bx$$

$$b = r \frac{s_y}{s_x}$$

$$b = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n} (x_i - \overline{x})^2}$$

$$a = \overline{y} - b\overline{x}$$

$$SSTo = \sum_{i=1}^{n} (y_i - \overline{y})^2$$
, $SSTo = \sum_{i=1}^{n} y_i^2 - n\overline{y}^2$

$$SSResid = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$
, $SSResid = \sum_{i=1}^{n} y_i^2 - a \sum_{i=1}^{n} y_i - b \sum_{i=1}^{n} x_i y_i$

$$r^2 = 1 - \frac{SSResid}{SSTo}$$

$$s_e = \sqrt{\frac{SSResid}{n-2}}$$

If events E_1, \dots, E_k are all mutually exclusive, then $\Pr(E_1 \cup \dots \cup E_k) = \Pr(E_1) + \dots + \Pr(E_k)$

$$\Pr(E \mid F) = \frac{\Pr(E \cap F)}{P(F)}$$

The event E and F are independent if and only if $Pr(E \cap F) = Pr(E) Pr(F)$

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$

$$Pr(E \cap F) = Pr(E|F) Pr(F) = Pr(F|E) Pr(E)$$

If events B_1, \dots, B_k are all mutually exclusive with $\Pr(B_1) + \dots + \Pr(B_k) = 1$, then for any event E, $\Pr(E) = \Pr(E \mid B_1) \Pr(B_1) + \dots + \Pr(E \mid B_k) \Pr(B_k)$

If events B_1, \dots, B_k are all mutually exclusive with $\Pr(B_1) + \dots + \Pr(B_k) = 1$, then for any event E, $\Pr(B_i \mid E) = \frac{\Pr(E|B_i) \Pr(B_i)}{\Pr(E|B_1) \Pr(B_1) + \dots + \Pr(E|B_k) \Pr(B_k)}$

For continuous random variable X, $\Pr(a < X \le b) = \Pr(X \le b) - \Pr(X \le a)$

Mean value of a discrete random variable $\mu_X = \sum x \cdot p(x)$

Standard deviation of a discrete random variable $\sigma_X = \sqrt{\sum (x - \mu_X)^2 \cdot p(x)}$

Y = a + bX. Mean $\mu_Y = a + b\mu_X$. Standard deviation $\sigma_Y = |b|\sigma_X$

 $Y=a_1X_1+\cdots+a_nX_n$. Mean $\mu_Y=a_1\mu_{X_1}+\cdots+a_n\mu_{X_n}$. Standard deviation (when X_i 's are independent) $\sigma_Y=\sqrt{a_1^2\sigma_{X_1}^2+\cdots+a_n^2\sigma_{X_n}^2}$

Binomial distribution
$$p(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}, \, \mu_X = np, \, \sigma_X = \sqrt{np(1-p)}$$

Geometric distribution $p(x) = (1-p)^{x-1}p$, $\mu_X = 1/p$

Continuity correction (X is a discrete variable (integer values), Y is the corresponding Normal random variable): If $\Pr(X \leq m)$ use $\Pr(Y < m + 0.5)$; If $\Pr(X < m)$ use $\Pr(Y < m - 0.5)$

Sampling distribution of \overline{X} : $\mu_{\overline{X}} = \mu$, $\sigma_{\overline{X}} = \sigma/\sqrt{n}$

Sampling distribution of \widehat{p} : $\mu_{\widehat{p}} = p$, $\sigma_{\widehat{p}} = \sqrt{p(1-p)/n}$

Critical value $z_{\alpha/2}$ is defined as: $\Pr(Z > z_{\alpha/2}) = \alpha/2$, where Z is a normally distributed random variable with mean 0 and SD 1.

 $(1-\alpha) \times 100\%$ confidence interval for p: $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$n = p(1-p) \left(\frac{z_{\alpha/2}}{B}\right)^2$$

 $(1-\alpha) \times 100\%$ confidence interval for μ : $\overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$n = \left(\frac{z_{\alpha/2}\sigma}{B}\right)^2$$

 $(1-\alpha) \times 100\%$ confidence interval for μ : $\overline{x} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}$

Finite population correction factor: $\sqrt{\frac{N-n}{N-1}}$

 $z=\frac{\overline{x}-\mu}{\sigma/\sqrt{n}}$, where μ is the hypothesized value under H_0

 $t=\frac{\overline{x}-\mu}{s/\sqrt{n}}, d.f.=n-1$, where μ is the hypothesized value under H_0

Random Number Table

 $\begin{array}{c} 34234\ 27566\ 94454\ 20349\ 69224\ 69483\ 21821\ 38248\ 62410\ 16481\ 54270\ 14344\ 60679\ 95118\\ 44916\ 95522\ 17144\ 05395\ 40643\ 08340\ 52134\ 20753\ 41452\ 52797\ 45320\ 67751\ 00459\ 28894\\ 43588\ 46388\ 64547\ 10072\ 00054\ 56665\ 60274\ 22889\ 35043\ 72024\ 87641\ 67346\ 28230\ 19021\\ 20090\ 16885\ 26498\ 97659\ 10735\ 24621\ 56406\ 07936\ 06463\ 37439\ 17953\ 23294\ 07272\ 55338\\ 11140\ 70292\ 66278\ 31434\ 09408\ 48929\ 30366\ 12613\ 39316\ 59206\ 26094\ 25430\ 00863\ 01122\\ 53461\ 69887\ 94050\ 48120\ 85909\ 45984\ 92318\ 26757\ 49997\ 27162\ 22226\ 10476\ 45725\ 39980\\ 83773\ 52393\ 73092\ 84437\ 71657\ 66721\ 54971\ 90220\ 84475\ 28268\ 70330\ 17587\ 07148\ 56945\\ 07552\ 29174\ 17424\ 52673\ 46928\ 90721\ 32783\ 80040\ 64827\ 57350\ 79781\ 12488\ 40923\ 82176 \end{array}$

STUDENT'S t PERCENTAGE POINTS

```
40.0\% \, 33.3\% \, 25.0\% \, 20.0\% \, 12.5\% \, 10.0\% \, 5.0\% \, 2.5\% \, 1.0\% \, 0.5\% \, 0.1\%
   one-tail
   two-tail
              80.0\% \, 66.7\% \, 50.0\% \, 40.0\% \, 25.0\% \, 20.0\% \, 10.0\% \quad 5.0\% \quad 2.0\% \quad 1.0\% \quad 0.2\%
             60.0\% 66.7\% 75.0\% 80.0\% 87.5\% 90.0\% 95.0\% 97.5\% 99.0\% 99.5\% 99.9\%
cum. prob
              0.325\ 0.577\ 1.000\ 1.376\ 2.414\ 3.078\ 6.31412.70631.82163.657318.31
              0.289 \ 0.500 \ 0.816 \ 1.061 \ 1.604 \ 1.886 \ 2.920 \ 4.303 \ 6.965 \ 9.925 \ 22.327
              0.277\ 0.476\ 0.765\ 0.978\ 1.423\ 1.638\ 2.353\ 3.182\ 4.541\ 5.841\ 10.215
              0.271\ 0.464\ 0.741\ 0.941\ 1.344\ 1.533\ 2.132\ 2.776\ 3.747\ 4.604\ 7.173
              0.267\ 0.457\ 0.727\ 0.920\ 1.301\ 1.476\ 2.015\ 2.571\ 3.365\ 4.032\ 5.893
              0.265\ 0.453\ 0.718\ 0.906\ 1.273\ 1.440\ 1.943\ 2.447\ 3.143\ 3.707\ 5.208
               0.263 \ 0.449 \ 0.711 \ 0.896 \ 1.254 \ 1.415 \ 1.895 \ 2.365 \ 2.998 \ 3.499 \ 4.785
              0.262\ 0.447\ 0.706\ 0.889\ 1.240\ 1.397\ 1.860\ 2.306\ 2.896\ 3.355\ 4.501
              0.261\ 0.445\ 0.703\ 0.883\ 1.230\ 1.383\ 1.833\ 2.262\ 2.821\ 3.250\ 4.297
              0.260\ 0.444\ 0.700\ 0.879\ 1.221\ 1.372\ 1.812\ 2.228\ 2.764\ 3.169\ 4.144
         10
              0.260\ 0.443\ 0.697\ 0.876\ 1.214\ 1.363\ 1.796\ 2.201\ 2.718\ 3.106\ 4.025
         11
              0.259\ 0.442\ 0.695\ 0.873\ 1.209\ 1.356\ 1.782\ 2.179\ 2.681\ 3.055\ 3.930
         12
              0.259 \ 0.441 \ 0.694 \ 0.870 \ 1.204 \ 1.350 \ 1.771 \ 2.160 \ 2.650 \ 3.012 \ 3.852
         13
              0.258\ 0.440\ 0.692\ 0.868\ 1.200\ 1.345\ 1.761\ 2.145\ 2.624\ 2.977\ 3.787
         14
         15
              0.258\ 0.439\ 0.691\ 0.866\ 1.197\ 1.341\ 1.753\ 2.131\ 2.602\ 2.947\ 3.733
              0.258\ 0.439\ 0.690\ 0.865\ 1.194\ 1.337\ 1.746\ 2.120\ 2.583\ 2.921\ 3.686
         16
              0.257 0.438 0.689 0.863 1.191 1.333 1.740 2.110 2.567 2.898 3.646
         17
              0.257 \ 0.438 \ 0.688 \ 0.862 \ 1.189 \ 1.330 \ 1.734 \ 2.101 \ 2.552 \ 2.878 \ 3.610
         19
              0.257 \ 0.438 \ 0.688 \ 0.861 \ 1.187 \ 1.328 \ 1.729 \ 2.093 \ 2.539 \ 2.861 \ 3.579
              0.257\ 0.437\ 0.687\ 0.860\ 1.185\ 1.325\ 1.725\ 2.086\ 2.528\ 2.845\ 3.552
         20
         21
              0.257\ 0.437\ 0.686\ 0.859\ 1.183\ 1.323\ 1.721\ 2.080\ 2.518\ 2.831\ 3.527
              0.256 \ 0.437 \ 0.686 \ 0.858 \ 1.182 \ 1.321 \ 1.717 \ 2.074 \ 2.508 \ 2.819 \ 3.505
              0.256\ 0.436\ 0.685\ 0.858\ 1.180\ 1.319\ 1.714\ 2.069\ 2.500\ 2.807\ 3.485
              0.256 \ 0.436 \ 0.685 \ 0.857 \ 1.179 \ 1.318 \ 1.711 \ 2.064 \ 2.492 \ 2.797 \ 3.467
              0.256\ 0.436\ 0.684\ 0.856\ 1.178\ 1.316\ 1.708\ 2.060\ 2.485\ 2.787\ 3.450
              0.256\ 0.436\ 0.684\ 0.856\ 1.177\ 1.315\ 1.706\ 2.056\ 2.479\ 2.779\ 3.435
              0.256\ 0.435\ 0.684\ 0.855\ 1.176\ 1.314\ 1.703\ 2.052\ 2.473\ 2.771\ 3.421
         27
              0.256\ 0.435\ 0.683\ 0.855\ 1.175\ 1.313\ 1.701\ 2.048\ 2.467\ 2.763\ 3.408
              0.256 \ 0.435 \ 0.683 \ 0.854 \ 1.174 \ 1.311 \ 1.699 \ 2.045 \ 2.462 \ 2.756 \ 3.396
              0.256 \ 0.435 \ 0.683 \ 0.854 \ 1.173 \ 1.310 \ 1.697 \ 2.042 \ 2.457 \ 2.750 \ 3.385
         35
              0.255\ 0.434\ 0.682\ 0.852\ 1.170\ 1.306\ 1.690\ 2.030\ 2.438\ 2.724\ 3.340
              0.255\ 0.434\ 0.681\ 0.851\ 1.167\ 1.303\ 1.684\ 2.021\ 2.423\ 2.704\ 3.307
              0.255\ 0.434\ 0.680\ 0.850\ 1.165\ 1.301\ 1.679\ 2.014\ 2.412\ 2.690\ 3.281
         45
              0.255\ 0.433\ 0.679\ 0.849\ 1.164\ 1.299\ 1.676\ 2.009\ 2.403\ 2.678\ 3.261
              0.255\ 0.433\ 0.679\ 0.848\ 1.163\ 1.297\ 1.673\ 2.004\ 2.396\ 2.668\ 3.245
              0.254 \ 0.433 \ 0.679 \ 0.848 \ 1.162 \ 1.296 \ 1.671 \ 2.000 \ 2.390 \ 2.660 \ 3.232
              0.253\ 0.431\ 0.674\ 0.842\ 1.150\ 1.282\ 1.645\ 1.960\ 2.326\ 2.576\ 3.090
```

NORMAL CUMULATIVE DISTRIBUTION FUNCTION

0.000.01 0.020.030.040.050.06 0.070.08 0.09 0.0 $0.5000\ 0.5040\ 0.5080\ 0.5120\ 0.5160\ 0.5199\ 0.5239\ 0.5279\ 0.5319\ 0.5359$ $0.5398\ 0.5438\ 0.5478\ 0.5517\ 0.5557\ 0.5596\ 0.5636\ 0.5675\ 0.5714\ 0.5753$ 0.2 0.5793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6064 0.6103 0.61410.3 $0.6179\ 0.6217\ 0.6255\ 0.6293\ 0.6331\ 0.6368\ 0.6406\ 0.6443\ 0.6480\ 0.6517$ 0.4 $0.6554 \ 0.6591 \ 0.6628 \ 0.6664 \ 0.6700 \ 0.6736 \ 0.6772 \ 0.6808 \ 0.6844 \ 0.6879$ 0.5 $0.6915\ 0.6950\ 0.6985\ 0.7019\ 0.7054\ 0.7088\ 0.7123\ 0.7157\ 0.7190\ 0.7224$ 0.6 $0.7257\ 0.7291\ 0.7324\ 0.7357\ 0.7389\ 0.7422\ 0.7454\ 0.7486\ 0.7517\ 0.7549$ 0.7 $0.7580\ 0.7611\ 0.7642\ 0.7673\ 0.7703\ 0.7734\ 0.7764\ 0.7794\ 0.7823\ 0.7852$ 0.8 $0.7881\ 0.7910\ 0.7939\ 0.7967\ 0.7995\ 0.8023\ 0.8051\ 0.8078\ 0.8106\ 0.8133$ 0.9 $0.8159\ 0.8186\ 0.8212\ 0.8238\ 0.8264\ 0.8289\ 0.8315\ 0.8340\ 0.8365\ 0.8389$ 1.0 $0.8413\ 0.8438\ 0.8461\ 0.8485\ 0.8508\ 0.8531\ 0.8554\ 0.8577\ 0.8599\ 0.8621$ 1.1 $0.8643\ 0.8665\ 0.8686\ 0.8708\ 0.8729\ 0.8749\ 0.8770\ 0.8790\ 0.8810\ 0.8830$ 1.2 $0.8849\ 0.8869\ 0.8888\ 0.8907\ 0.8925\ 0.8944\ 0.8962\ 0.8980\ 0.8997\ 0.9015$ $0.9032\ 0.9049\ 0.9066\ 0.9082\ 0.9099\ 0.9115\ 0.9131\ 0.9147\ 0.9162\ 0.9177$ 1.3 $0.9192\ 0.9207\ 0.9222\ 0.9236\ 0.9251\ 0.9265\ 0.9279\ 0.9292\ 0.9306\ 0.9319$ 1.5 $0.9332\ 0.9345\ 0.9357\ 0.9370\ 0.9382\ 0.9394\ 0.9406\ 0.9418\ 0.9429\ 0.9441$ $0.9452\ 0.9463\ 0.9474\ 0.9484\ 0.9495\ 0.9505\ 0.9515\ 0.9525\ 0.9535\ 0.9545$ 1.7 $0.9554\ 0.9564\ 0.9573\ 0.9582\ 0.9591\ 0.9599\ 0.9608\ 0.9616\ 0.9625\ 0.9633$ 1.8 $0.9641\ 0.9649\ 0.9656\ 0.9664\ 0.9671\ 0.9678\ 0.9686\ 0.9693\ 0.9699\ 0.9706$ 1.9 $0.9713\ 0.9719\ 0.9726\ 0.9732\ 0.9738\ 0.9744\ 0.9750\ 0.9756\ 0.9761\ 0.9767$ 2.0 $0.9772\ 0.9778\ 0.9783\ 0.9788\ 0.9793\ 0.9798\ 0.9803\ 0.9808\ 0.9812\ 0.9817$ 2.1 $0.9821 \ 0.9826 \ 0.9830 \ 0.9834 \ 0.9838 \ 0.9842 \ 0.9846 \ 0.9850 \ 0.9854 \ 0.9857$ $0.9861\ 0.9864\ 0.9868\ 0.9871\ 0.9875\ 0.9878\ 0.9881\ 0.9884\ 0.9887\ 0.9890$ 2.3 $0.9893\ 0.9896\ 0.9898\ 0.9901\ 0.9904\ 0.9906\ 0.9909\ 0.9911\ 0.9913\ 0.9916$ $0.9918\ 0.9920\ 0.9922\ 0.9925\ 0.9927\ 0.9929\ 0.9931\ 0.9932\ 0.9934\ 0.9936$ 2.5 $0.9938\ 0.9940\ 0.9941\ 0.9943\ 0.9945\ 0.9946\ 0.9948\ 0.9949\ 0.9951\ 0.9952$ 2.6 $0.9953\ 0.9955\ 0.9956\ 0.9957\ 0.9959\ 0.9960\ 0.9961\ 0.9962\ 0.9963\ 0.9964$ 2.7 $0.9965 \ 0.9966 \ 0.9967 \ 0.9968 \ 0.9969 \ 0.9970 \ 0.9971 \ 0.9972 \ 0.9973 \ 0.9974$ 2.8 $0.9974\ 0.9975\ 0.9976\ 0.9977\ 0.9977\ 0.9978\ 0.9979\ 0.9979\ 0.9980\ 0.9981$ 2.9 $0.9981\ 0.9982\ 0.9982\ 0.9983\ 0.9984\ 0.9984\ 0.9985\ 0.9985\ 0.9986\ 0.9986$ 3.0 $0.9987\ 0.9987\ 0.9987\ 0.9988\ 0.9988\ 0.9989\ 0.9989\ 0.9989\ 0.9990\ 0.9990$ 3.1 $0.9990 \ 0.9991 \ 0.9991 \ 0.9991 \ 0.9992 \ 0.9992 \ 0.9992 \ 0.9993 \ 0.9993$ 3.2 $0.9993 \ 0.9993 \ 0.9994 \ 0.9994 \ 0.9994 \ 0.9994 \ 0.9995 \ 0.9995 \ 0.9995$ 3.3 0.9995 0.9995 0.9995 0.9996 0.9996 0.9996 0.9996 0.9996 0.99963.4 $0.9997\ 0.9997\ 0.9997\ 0.9997\ 0.9997\ 0.9997\ 0.9997\ 0.9997\ 0.9998$ $0.9998\ 0.9998\ 0.9998\ 0.9998\ 0.9998\ 0.9998\ 0.9998\ 0.9998\ 0.9998$ 3.6 $0.9998 \ 0.9998 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999$ 3.7 $0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999$ 3.8 $0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999 \ 0.9999$ 3.9 $1.0000\ 1.0000\ 1.0000\ 1.0000\ 1.0000\ 1.0000\ 1.0000\ 1.0000\ 1.0000$

1. Plots for Quantitative Variables (Chapter 1,3,4)

Cristiano Ronaldo dos Santos Aveiro, is a Portuguese professional soccer player.

The stem-and-leaf display shows the numbers of league appearances by season by Cristiano Ronaldo from season 2002-03 to season 2016-17.

Table 1: Number of Appearances (3|5 means 35)

stem	leaf
3	1
3	5
4	03
4	7789
5	03344
5	55

Find out (a) the minimum, (b) the maximum, (c) the median, (d) the mean, and (e) the interquartile range. (f) Draw the boxplot, and (g) report whether outliers are observed. (h) Discuss the skewness of the distribution of the numbers of appearances (symmetric, positively or negatively skewed).



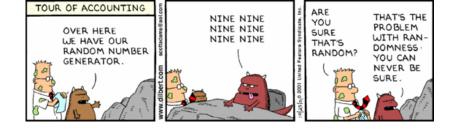
Figure 1: Cristiano Ronaldo

2. Sampling (Chapter 2)

Please randomly draw four students from a group of eighteen by using Simple Random Sampling without Replacement (SRS):

- 1 Alan
- 2 Lucy
- 3 Tom
- 4 Azar
- 5 Jayne
- 6 Nadima
- 7 Matthew
- 8 Sushi
- 9 Mohammed
- 10 Rachel
- $11~\mathrm{Ben}$
- $12~\mathrm{Emma}$
- 13 Grace
- 14 Anna
- 15 Sophie
- $16~{\rm Karen}$
- 17 Joshua
- 18 James

Briefly describe the sampling procedure you use.



3. Regression (Chapter 5)

A investigation of the properties of bricks used to line aluminum smelter pots was published in *The American Ceramic Society Bulletin* in February 2005. Six different commercial bricks were evaluated. The life span of a smelter pot depends on the porosity of the brick lining (the less porosity, the longer is the life); consequently, the researchers measured the apparent porosity of each brick specimen (percentage), as well as the mean pore diameter of each brick (micrometer). The data are given in the table below.

Brick	Apparent Porosity (%)	Mean Pore Diameter (μm)
1	18.8	12.0
2	18.3	9.7
3	16.3	7.3
4	6.9	5.3
5	17.1	10.9
6	20.4	16.8
$\frac{1}{3}$	16.3 6.9 17.1	7.3 5.3 10.9

Summary statistics for the two variables are shown in another table.

	Mean	SD
Apparent Porosity	16.30	4.82
Mean Pore Diameter	10.33	4.00

- (1) Find the correlation between porosity and mean pore diameter.
- (2) Find the least squares line relating porosity (y) to mean pore diameter (x).
- (3) Predict the apparent percentage of porosity for a brick with a mean pore diameter of 10 micrometers.

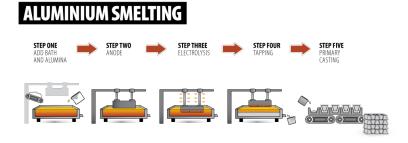


Figure 2: Aluminum Smelting

4. Bayes' Rule (Chapter 6)

Researchers want to evaluate an intelligent robotic controller that aims to capture the intent of a wheelchair user and aid in navigation. Consider the following scenario. From a certain location in a room, a wheelchair user will either (1) turn to the right and navigate through a door, (2) proceed straight to the other side of the room, or (3) turn slightly left and stop at a table. Denote these three events as D (for door), S (straight), and T (for table). Based on previous trips, $\Pr(D) = 0.5$, $\Pr(S) = 0.2$, and $\Pr(T) = 0.3$. The wheelchair is installed with a robot-controlled joystick. When the user intends to go through the door, he points the joystick straight 30% of the time; when the user intends to go straight, he points the joystick straight 40% of the time; and when the user intends to go to the table, she points the joystick straight 5% of the time. If the wheelchair user points to joystick straight, what is his most likely destination?



Figure 3: A wheelchair user in a room

5. Discrete Random Variables (Chapter 7)
Suppose an individual plays a gambling game where it is possible to lose \$1.00, break even, win \$3.00, or win \$5.00 each time she plays. The probability distribution for each outcome is provided by the following table:

Outcome	-\$1.00	\$0.00	\$3.00	\$5.00
Probability	0.30	0.40	0.20	0.10

- (1) Verify that the discrete probability distribution above is well-defined.
- (2) Find the mean and standard deviation of this discrete random variable.
- (3) Suppose that the casino decides that the game does not have an impressive enough top prize with the lower payouts, and decides to change the outcomes, as shown below

Outcome	-\$3.00	-\$1.00	\$5.00	\$9.00
Probability	0.30	0.40	0.20	0.10

Find the relation between new outcome and previous outcomes. Based on the relation, find the mean and standard deviation of the new random variable.



6. Normal Distribution (Chapter 7)

Under normal distribution, find the relation between IQR and SD.

When making a boxplot, we use

value > upper quartile +
$$1.5 \times IQR$$

or

value
$$<$$
 lower quartile $-1.5 \times IQR$

to identify outliers. When assuming normal distribution, what is the proportion of data which are identified as outliers?

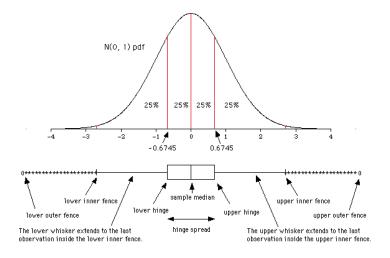


Figure 4: Boxplot and Normal Distribution

7. Normal Probability Plot (Chapter 7)

Scientists found two new types of insects in Rocky Mountain, with sample size 4 and 5. For insect A, the body lengths (centimeter) are

for insect B, the body lengths (centimeter) are

- (1) Make Normal Probability Plots for insect A and insect B. You may put two sets of data on one plot with different symbols (for example, \circ and \times).
- (2) Check the normality based on your plots.

How to make a Normal Probability Plot

- 1. Sort the data. $x_1 \leq x_2 \leq \cdots \leq x_n$
- 2. Calculate the corresponding quantile

$$z_i = \Phi^{-1} \left(\frac{i - \frac{1}{2}}{n} \right)$$

3. Make a scatter plot for $\{z_i, x_i\}_{i=1}^n$.

8. Confidence Intervals (Chapter 8, 9)

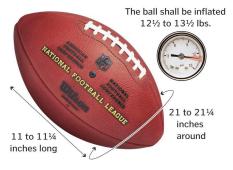
Consider the pharmaceutical company that desires an estimate of the mean increase in blood pressure of patients who take a new drug. The blood pressure increases (measured in points) for the n=6 patients in the human testing phase are recorded, with the average blood pressure increases 2.283 points and sample standard deviation 0.950. (1) Use this information to construct a 95% confidence interval for μ , the mean increase in blood pressure associated with the new drug for all patients in the population. (2) What is the margin of error?



9. Sample Size Estimation (Chapter 9)

Suppose the manufacturer of official NFL footballs uses a machine to inflate the new balls to a pressure of 13.5 pounds. When the machine is properly calibrated, the mean inflation pressure is 13.5 pounds, but uncontrollable factors cause the pressures of individual footballs to vary randomly by following a normal distribution with mean 13.5 pounds and SD 0.1 pounds. For quality control purposes, the manufacturer wishes to estimated the mean inflation pressure to within 0.025 pound of its true value (margin of error is 0.025 pound) with a 99% confidence interval. What sample size should be specified for the experiment?

OFFICIAL BALL DIMENSIONS



10. Hypothesis Test (Chapter 10)

Mirex is a chlorinated hydrocarbon that was commercialized as an insecticide and later banned because of its impact on the environment.

Researchers tested 12 farm-raised salmon for organic contaminants. They found the mean concentration of the carcinogenic insecticide mirex to be 0.0913 parts per million, with sample standard deviation s=0.0195 ppm. As a safety recommendation to recreational fishers, the Environmental Protection Agency's (EPA) recommended "screening value" for mirex is 0.08 ppm. Population distribution is assumed to be normal.

Are farmed salmon contaminated beyond the level permitted by the EPA? Use the significance level $\alpha=0.05$.





More Space