

SPRING 2016
HUNTER COLLEGE
STAT 213 Section HC1
Introduction to Applied Statistics
Final Exam

Last Name: _____

First Name: _____

Graduation Year: _____

May 25, 2016

1. Please do not leave blank for any question.
2. There are 10 questions, each question is 5 points.
3. You have 120 minutes for this exam (3:00 pm - 5:00 pm).
4. Explain briefly = Explain in one sentence or several phrases.

Formulas

$$\sum_{i=1}^n x_i = x_1 + x_2 + \cdots + x_n$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + \sigma z$$

$$s_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$$s_x = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

$$\Pr(|X - \mu| \leq k\sigma) \geq 1 - \frac{1}{k^2}, \text{ for any distribution}$$

$$\Pr(|X - \mu| \leq \sigma) \approx 0.68, \Pr(|X - \mu| \leq 2\sigma) \approx 0.95, \Pr(|X - \mu| \leq 3\sigma) \approx 0.997, \text{ for normal distribution}$$

$$r = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

$$\hat{y} = a + bx$$

$$b = r \frac{s_y}{s_x}$$

$$b = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$a = \bar{y} - b\bar{x}$$

$$SSTo = \sum_{i=1}^n (y_i - \bar{y})^2$$

$$SSTo = \sum_{i=1}^n y_i^2 - n\bar{y}^2$$

$$SSResid = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$SSResid = \sum_{i=1}^n y_i^2 - a \sum_{i=1}^n y_i - b \sum_{i=1}^n x_i y_i$$

$$r^2 = 1 - \frac{SSResid}{SSTo}$$

$$s_e = \sqrt{\frac{SSResid}{n-2}}$$

If events E_1, \dots, E_k are all mutually exclusive, then $\Pr(E_1 \cup \dots \cup E_k) = \Pr(E_1) + \dots + \Pr(E_k)$

$$\Pr(E | F) = \frac{\Pr(E \cap F)}{\Pr(F)}$$

The event E and F are independent if and only if $\Pr(E \cap F) = \Pr(E) \Pr(F)$

$$\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$$

$$\Pr(E \cap F) = \Pr(E|F) \Pr(F) = \Pr(F|E) \Pr(E)$$

If events B_1, \dots, B_k are all mutually exclusive with $\Pr(B_1) + \dots + \Pr(B_k) = 1$, then for any event E , $\Pr(E) = \Pr(E | B_1) \Pr(B_1) + \dots + \Pr(E | B_k) \Pr(B_k)$

If events B_1, \dots, B_k are all mutually exclusive with $\Pr(B_1) + \dots + \Pr(B_k) = 1$, then for any event E , $\Pr(B_i | E) = \frac{\Pr(E|B_i) \Pr(B_i)}{\Pr(E|B_1) \Pr(B_1) + \dots + \Pr(E|B_k) \Pr(B_k)}$

For continuous random variable X , $\Pr(a < X \leq b) = \Pr(X \leq b) - \Pr(X \leq a)$

Mean value of a discrete random variable $\mu_X = \sum x \cdot p(x)$

Standard deviation of a discrete random variable $\sigma_X = \sqrt{\sum (x - \mu_X)^2 \cdot p(x)}$

$Y = a + bX$. Mean $\mu_Y = a + b\mu_X$. Standard deviation $\sigma_Y = |b|\sigma_X$

$Y = a_1X_1 + \dots + a_nX_n$. Mean $\mu_Y = a_1\mu_{X_1} + \dots + a_n\mu_{X_n}$. Standard deviation (when X_i 's are independent) $\sigma_Y = \sqrt{a_1^2\sigma_{X_1}^2 + \dots + a_n^2\sigma_{X_n}^2}$

Binomial distribution $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$, $\mu_X = np$, $\sigma_X = \sqrt{np(1-p)}$

Geometric distribution $p(x) = (1-p)^{x-1}p$, $\mu_X = 1/p$

Continuity correction (X is a discrete variable (integer values), Y is the corresponding Normal random variable): If $\Pr(X \leq m)$ use $\Pr(Y < m + 0.5)$; If $\Pr(X < m)$ use $\Pr(Y < m - 0.5)$

Sampling distribution of \bar{X} : $\mu_{\bar{X}} = \mu$, $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

Sampling distribution of \hat{p} : $\mu_{\hat{p}} = p$, $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$

$(1 - \alpha) \times 100\%$ confidence interval for p : $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$n = p(1-p) \left(\frac{z_{\alpha/2}}{B} \right)^2$$

$(1 - \alpha) \times 100\%$ confidence interval for μ : $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$n = \left(\frac{z_{\alpha/2} \sigma}{B} \right)^2$$

$(1 - \alpha) \times 100\%$ confidence interval for μ : $\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$

Finite population correction factor: $\sqrt{\frac{N-n}{N-1}}$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$, where μ is the hypothesized value under H_0

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$, where μ is the hypothesized value under H_0

Power + Type II error = 1

Paired two-sample t -test: $t = \frac{\bar{x}_d - \text{hypothesized value}}{s_d / \sqrt{n}}$, $d.f. = n - 1$

Independent two-sample t -test: $t = \frac{\bar{x}_1 - \bar{x}_2 - \text{hypothesized value}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$, $d.f. = \left\lfloor \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \right\rfloor$.

Pearson's chi-squared test: $\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$

Hypothesis Test concerning slope β : $t = \frac{b - \text{hypothesized value}}{s_\beta}$, where $s_\beta = \frac{s_e}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$ and $d.f. = n - 2$.

Coefficient of multiple determination: $R^2 = 1 - \frac{SSResid}{SSTo}$

$SSResid = \sum_{i=1}^n (y_i - \hat{y}_i)^2$

$SSRegr = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$

F test for Multiple Regression: $F = \frac{SSRegr/k}{SSResid/(n-(k+1))}$ or $F = \frac{R^2/k}{(1-R^2)/(n-(k+1))}$

Total number of observations: $N = n_1 + \cdots + n_k$

Grand total: $T = \text{sum of all } N \text{ observations} = n_1 \bar{x}_1 + \cdots + n_k \bar{x}_k$

Grand mean: $\bar{\bar{x}} = T/N = \frac{n_1 \bar{x}_1 + \cdots + n_k \bar{x}_k}{n_1 + \cdots + n_k}$

$SSTr = n_1 (\bar{x}_1 - \bar{\bar{x}})^2 + \cdots + n_k (\bar{x}_k - \bar{\bar{x}})^2$, $MSTr = \frac{SSTr}{k-1}$

$SSE = (n_1 - 1) s_1^2 + \cdots + (n_k - 1) s_k^2$, $MSE = \frac{SSE}{N-k}$

$F = \frac{MSTr}{MSE}$

Tukey-Kramer MCP: For $\mu_i - \mu_j : (\bar{x}_i - \bar{x}_j) \pm q \sqrt{\frac{MSE}{2} \left(\frac{1}{n_i} + \frac{1}{n_j} \right)}$

Random Number Table

34234 27566 94454 20349 69224 69483 21821 38248 62410 16481 54270 14344
60679 95118 44916 95522 17144 05395 40643 08340 52134 20753 41452 52797
45320 67751 00459 28894 43588 46388 64547 10072 00054 56665 60274 22889
35043 72024 87641 67346 28230 19021 20090 16885 26498 97659 10735 24621
56406 07936 06463 37439 17953 23294 07272 55338 11140 70292 66278 31434
09408 48929 30366 12613 39316 59206 26094 25430 00863 01122 53461 69887
94050 48120 85909 45984 92318 26757 49997 27162 22226 10476 45725 39980
83773 52393 73092 84437 71657 66721 54971 90220 84475 28268 70330 17587
07148 56945 07552 29174 17424 52673 46928 90721 32783 80040 64827 57350
79781 12488 40923 82176 58418 76576 22101 12084 68695 72304 34919 73631
84053 99671 79376 40260 57609 58677 55473 65086 09688 22765 36651 94994
19965 18493 49468 56541 61881 45860 93925 23170 08879 78308 43464 47996
87517 42396 51200 77903 71236 38123 64018 12893 13152 65490 81917 06079

NORMAL CUMULATIVE DISTRIBUTION FUNCTION

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7703 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

STUDENT'S t PERCENTAGE POINTS

| | | | | | | | | | | | |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| one-tail | 40.0% | 33.3% | 25.0% | 20.0% | 12.5% | 10.0% | 5.0% | 2.5% | 1.0% | 0.5% | 0.1% |
| two-tail | 80.0% | 66.7% | 50.0% | 40.0% | 25.0% | 20.0% | 10.0% | 5.0% | 2.0% | 1.0% | 0.2% |
| cum. prob | 60.0% | 66.7% | 75.0% | 80.0% | 87.5% | 90.0% | 95.0% | 97.5% | 99.0% | 99.5% | 99.9% |

| | | | | | | | | | | | |
|----------|-------|-------|-------|-------|-------|-------|-------|--------|--------|--------|--------|
| 1 | 0.325 | 0.577 | 1.000 | 1.376 | 2.414 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.31 |
| 2 | 0.289 | 0.500 | 0.816 | 1.061 | 1.604 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3 | 0.277 | 0.476 | 0.765 | 0.978 | 1.423 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 |
| 4 | 0.271 | 0.464 | 0.741 | 0.941 | 1.344 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 |
| 5 | 0.267 | 0.457 | 0.727 | 0.920 | 1.301 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6 | 0.265 | 0.453 | 0.718 | 0.906 | 1.273 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 |
| 7 | 0.263 | 0.449 | 0.711 | 0.896 | 1.254 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 |
| 8 | 0.262 | 0.447 | 0.706 | 0.889 | 1.240 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 |
| 9 | 0.261 | 0.445 | 0.703 | 0.883 | 1.230 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 |
| 10 | 0.260 | 0.444 | 0.700 | 0.879 | 1.221 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 |
| 11 | 0.260 | 0.443 | 0.697 | 0.876 | 1.214 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 |
| 12 | 0.259 | 0.442 | 0.695 | 0.873 | 1.209 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 |
| 13 | 0.259 | 0.441 | 0.694 | 0.870 | 1.204 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14 | 0.258 | 0.440 | 0.692 | 0.868 | 1.200 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 |
| 15 | 0.258 | 0.439 | 0.691 | 0.866 | 1.197 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 |
| 16 | 0.258 | 0.439 | 0.690 | 0.865 | 1.194 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 |
| 17 | 0.257 | 0.438 | 0.689 | 0.863 | 1.191 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18 | 0.257 | 0.438 | 0.688 | 0.862 | 1.189 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 |
| 19 | 0.257 | 0.438 | 0.688 | 0.861 | 1.187 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 |
| 20 | 0.257 | 0.437 | 0.687 | 0.860 | 1.185 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 |
| 21 | 0.257 | 0.437 | 0.686 | 0.859 | 1.183 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 |
| 22 | 0.256 | 0.437 | 0.686 | 0.858 | 1.182 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 |
| 23 | 0.256 | 0.436 | 0.685 | 0.858 | 1.180 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 |
| 24 | 0.256 | 0.436 | 0.685 | 0.857 | 1.179 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 |
| 25 | 0.256 | 0.436 | 0.684 | 0.856 | 1.178 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 |
| 26 | 0.256 | 0.436 | 0.684 | 0.856 | 1.177 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 |
| 27 | 0.256 | 0.435 | 0.684 | 0.855 | 1.176 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 |
| 28 | 0.256 | 0.435 | 0.683 | 0.855 | 1.175 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 |
| 29 | 0.256 | 0.435 | 0.683 | 0.854 | 1.174 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 |
| 30 | 0.256 | 0.435 | 0.683 | 0.854 | 1.173 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 |
| 35 | 0.255 | 0.434 | 0.682 | 0.852 | 1.170 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.340 |
| 40 | 0.255 | 0.434 | 0.681 | 0.851 | 1.167 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 |
| 45 | 0.255 | 0.434 | 0.680 | 0.850 | 1.165 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 3.281 |
| 50 | 0.255 | 0.433 | 0.679 | 0.849 | 1.164 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 |
| 55 | 0.255 | 0.433 | 0.679 | 0.848 | 1.163 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 | 3.245 |
| 60 | 0.254 | 0.433 | 0.679 | 0.848 | 1.162 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 |
| ∞ | 0.253 | 0.431 | 0.674 | 0.842 | 1.150 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |

| | | | | | | | | | | | |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| conf. level | 20.0% | 33.3% | 50.0% | 60.0% | 75.0% | 80.0% | 90.0% | 95.0% | 98.0% | 99.0% | 99.8% |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|

CHI-SQUARED TABLE (right-tail probability)

| ν | 40.0% | 33.3% | 25.0% | 20.0% | 12.5% | 10.0% | 5.0% | 2.5% | 1.0% | 0.5% | 0.1% |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 | 0.708 | 0.936 | 1.323 | 1.642 | 2.354 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 | 10.828 |
| 2 | 1.833 | 2.197 | 2.773 | 3.219 | 4.159 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 | 13.816 |
| 3 | 2.946 | 3.405 | 4.108 | 4.642 | 5.739 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 | 16.266 |
| 4 | 4.045 | 4.579 | 5.385 | 5.989 | 7.214 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 | 18.467 |
| 5 | 5.132 | 5.730 | 6.626 | 7.289 | 8.625 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 | 20.515 |
| 6 | 6.211 | 6.867 | 7.841 | 8.558 | 9.992 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 | 22.458 |
| 7 | 7.283 | 7.992 | 9.037 | 9.803 | 11.326 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 24.322 |
| 8 | 8.351 | 9.107 | 10.219 | 11.030 | 12.636 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 26.125 |
| 9 | 9.414 | 10.215 | 11.389 | 12.242 | 13.926 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 27.877 |
| 10 | 10.473 | 11.317 | 12.549 | 13.442 | 15.198 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 29.588 |
| 11 | 11.530 | 12.414 | 13.701 | 14.631 | 16.457 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 31.264 |
| 12 | 12.584 | 13.506 | 14.845 | 15.812 | 17.703 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 32.910 |
| 13 | 13.636 | 14.595 | 15.984 | 16.985 | 18.939 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 | 34.528 |
| 14 | 14.685 | 15.680 | 17.117 | 18.151 | 20.166 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 | 36.123 |
| 15 | 15.733 | 16.761 | 18.245 | 19.311 | 21.384 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 37.697 |
| 16 | 16.780 | 17.840 | 19.369 | 20.465 | 22.595 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 | 39.252 |
| 17 | 17.824 | 18.917 | 20.489 | 21.615 | 23.799 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 | 40.790 |
| 18 | 18.868 | 19.991 | 21.605 | 22.760 | 24.997 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 | 42.312 |
| 19 | 19.910 | 21.063 | 22.718 | 23.900 | 26.189 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 | 43.820 |
| 20 | 20.951 | 22.133 | 23.828 | 25.038 | 27.376 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 | 45.315 |
| 21 | 21.991 | 23.201 | 24.935 | 26.171 | 28.559 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 | 46.797 |
| 22 | 23.031 | 24.268 | 26.039 | 27.301 | 29.737 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 | 48.268 |
| 23 | 24.069 | 25.333 | 27.141 | 28.429 | 30.911 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 | 49.728 |
| 24 | 25.106 | 26.397 | 28.241 | 29.553 | 32.081 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 | 51.179 |
| 25 | 26.143 | 27.459 | 29.339 | 30.675 | 33.247 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 | 52.620 |
| 26 | 27.179 | 28.520 | 30.435 | 31.795 | 34.410 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 | 54.052 |
| 27 | 28.214 | 29.580 | 31.528 | 32.912 | 35.570 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 | 55.476 |
| 28 | 29.249 | 30.639 | 32.620 | 34.027 | 36.727 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 | 56.892 |
| 29 | 30.283 | 31.697 | 33.711 | 35.139 | 37.881 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 | 58.301 |
| 30 | 31.316 | 32.754 | 34.800 | 36.250 | 39.033 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 | 59.703 |
| 35 | 36.475 | 38.024 | 40.223 | 41.778 | 44.753 | 46.059 | 49.802 | 53.203 | 57.342 | 60.275 | 66.619 |
| 40 | 41.622 | 43.275 | 45.616 | 47.269 | 50.424 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 | 73.402 |
| 45 | 46.761 | 48.510 | 50.985 | 52.729 | 56.052 | 57.505 | 61.656 | 65.410 | 69.957 | 73.166 | 80.077 |
| 50 | 51.892 | 53.733 | 56.334 | 58.164 | 61.647 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 | 86.661 |
| 55 | 57.016 | 58.945 | 61.665 | 63.577 | 67.211 | 68.796 | 73.311 | 77.380 | 82.292 | 85.749 | 93.168 |
| 60 | 62.135 | 64.147 | 66.981 | 68.972 | 72.751 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 | 99.607 |

F DISTRIBUTION TABLE (right-tail probability)

| $\nu_2 \backslash \nu_1$ | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 12 | 15 | 20 | 30 | 50 | ∞ |
|--------------------------|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|----------|
| | <i>q</i> | | | | | | | | | | | | | | |
| 1 | 0.100 | 49.5 | 53.6 | 55.8 | 57.2 | 58.2 | 59.1 | 59.7 | 60.5 | 61.0 | 61.5 | 62.0 | 62.6 | 63.0 | 63.3 |
| | 0.050 | 199. | 216. | 225. | 230. | 234. | 237. | 239. | 242. | 244. | 246. | 248. | 250. | 252. | 254. |
| | 0.025 | 800. | 864. | 900. | 922. | 937. | 948. | 957. | 969. | 977. | 985. | 993. | | | |
| | 0.010 | | | | | | | | | | | | | | |
| | 0.001 | | | | | | | | | | | | | | |
| 2 | 0.100 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.39 | 9.41 | 9.43 | 9.44 | 9.46 | 9.47 | 9.49 |
| | 0.050 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 |
| | 0.025 | 39.0 | 39.2 | 39.2 | 39.3 | 39.3 | 39.4 | 39.4 | 39.4 | 39.4 | 39.4 | 39.4 | 39.5 | 39.5 | 39.5 |
| | 0.010 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 100. | 100. | 100. | 100. | 100. | 100. | 100. | 99.5 |
| | 0.001 | 999. | 999. | | | | | | | | | | | | |
| 3 | 0.100 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.23 | 5.22 | 5.20 | 5.18 | 5.17 | 5.15 | 5.13 |
| | 0.050 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.79 | 8.74 | 8.70 | 8.66 | 8.62 | 8.58 | 8.53 |
| | 0.025 | 16.0 | 15.4 | 15.1 | 14.9 | 14.7 | 14.6 | 14.5 | 14.4 | 14.3 | 14.3 | 14.2 | 14.1 | 14.0 | 13.9 |
| | 0.010 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.2 | 27.1 | 26.9 | 26.7 | 26.5 | 26.4 | 26.1 |
| | 0.001 | 149. | 141. | 137. | 135. | 133. | 132. | 131. | 129. | 128. | 127. | 126. | 125. | 125. | 123. |
| 4 | 0.100 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.92 | 3.90 | 3.87 | 3.84 | 3.82 | 3.79 | 3.76 |
| | 0.050 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 5.96 | 5.91 | 5.86 | 5.80 | 5.75 | 5.70 | 5.63 |
| | 0.025 | 10.6 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 | 8.84 | 8.75 | 8.66 | 8.56 | 8.46 | 8.38 | 8.26 |
| | 0.010 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.5 | 14.4 | 14.2 | 14.0 | 13.8 | 13.7 | 13.5 |
| | 0.001 | 61.2 | 56.2 | 53.4 | 51.7 | 50.5 | 49.7 | 49.0 | 48.0 | 47.4 | 46.8 | 46.1 | 45.4 | 44.9 | 44.1 |
| 5 | 0.100 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.30 | 3.27 | 3.24 | 3.21 | 3.17 | 3.15 | 3.10 |
| | 0.050 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.74 | 4.68 | 4.62 | 4.56 | 4.50 | 4.44 | 4.36 |
| | 0.025 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 | 6.62 | 6.52 | 6.43 | 6.33 | 6.23 | 6.14 | 6.02 |
| | 0.010 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.1 | 9.89 | 9.72 | 9.55 | 9.38 | 9.24 | 9.02 |
| | 0.001 | 37.1 | 33.2 | 31.1 | 29.8 | 28.8 | 28.2 | 27.6 | 26.9 | 26.4 | 25.9 | 25.4 | 24.9 | 24.4 | 23.8 |
| 6 | 0.100 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.94 | 2.90 | 2.87 | 2.84 | 2.80 | 2.77 | 2.72 |
| | 0.050 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.06 | 4.00 | 3.94 | 3.87 | 3.81 | 3.75 | 3.67 |
| | 0.025 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 | 5.46 | 5.37 | 5.27 | 5.17 | 5.07 | 4.98 | 4.85 |
| | 0.010 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.87 | 7.72 | 7.56 | 7.40 | 7.23 | 7.09 | 6.88 |
| | 0.001 | 27.0 | 23.7 | 21.9 | 20.8 | 20.0 | 19.5 | 19.0 | 18.4 | 18.0 | 17.6 | 17.1 | 16.7 | 16.3 | 15.7 |
| 7 | 0.100 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.70 | 2.67 | 2.63 | 2.59 | 2.56 | 2.52 | 2.47 |
| | 0.050 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.64 | 3.57 | 3.51 | 3.44 | 3.38 | 3.32 | 3.23 |
| | 0.025 | 6.54 | 5.89 | 5.52 | 5.29 | 5.12 | 4.99 | 4.90 | 4.76 | 4.67 | 4.57 | 4.47 | 4.36 | 4.28 | 4.14 |
| | 0.010 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.62 | 6.47 | 6.31 | 6.16 | 5.99 | 5.86 | 5.65 |
| | 0.001 | 21.7 | 18.8 | 17.2 | 16.2 | 15.5 | 15.0 | 14.6 | 14.1 | 13.7 | 13.3 | 12.9 | 12.5 | 12.2 | 11.7 |
| 8 | 0.100 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.54 | 2.50 | 2.46 | 2.42 | 2.38 | 2.35 | 2.29 |
| | 0.050 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.35 | 3.28 | 3.22 | 3.15 | 3.08 | 3.02 | 2.93 |
| | 0.025 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.29 | 4.20 | 4.10 | 4.00 | 3.89 | 3.81 | 3.67 |
| | 0.010 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.81 | 5.67 | 5.52 | 5.36 | 5.20 | 5.07 | 4.86 |
| | 0.001 | 18.5 | 15.8 | 14.4 | 13.5 | 12.9 | 12.4 | 12.0 | 11.5 | 11.2 | 10.8 | 10.5 | 10.1 | 9.80 | 9.33 |

1. Sampling (Chapter 2)

Please randomly draw two students from a group of eighteen by using Simple Random Sampling without Replacement (SRS):

- 1 Alan
- 2 Lucy
- 3 Tom
- 4 Azar
- 5 Jayne
- 6 Nadima
- 7 Matthew
- 8 Sushi
- 9 Mohammed
- 10 Rachel
- 11 Ben
- 12 Emma
- 13 Grace
- 14 Anna
- 15 Sophie
- 16 Karen
- 17 Joshua
- 18 James

Briefly describe the sampling procedure you use.



2. Plots for Quantitative Variables (Chapter 1,3,4)

The stem-and-leaf display shows the numbers of goals by Lionel A. Messi at FC Barcelona during 12 seasons of La Liga (the top professional association soccer division of the Spanish soccer league system) games from season 2004-05 to season 2015-16.

Table 1: Number of Goals (5|0 means 50)

| stem | leaf |
|------|------|
| 0 | 16 |
| 1 | 04 |
| 2 | 368 |
| 3 | 14 |
| 4 | 36 |
| 5 | 0 |

- Find (a) the minimum
(b) the maximum
(c) the median
(d) the mean
(e) the interquartile range
and
(f) draw the boxplot
(g) report whether outliers are observed.



3. Correlations and Simple Linear Regression (Chapter 5,13,14)
 Total Fat versus Calories for 5 items on the Subway menu are shown in Table 2. (data source: <https://www.subway.com/nutrition/nutritionlist.aspx>)

Table 2: Nutrition Facts

| | Fat (g) | Calories |
|---------------------|---------|----------|
| 6" Black Forest Ham | 4.5 | 290 |
| 6" Roast Beef | 5.0 | 320 |
| 6" Turkey Breast | 3.5 | 280 |
| 6" Veggie Delite | 2.5 | 230 |
| 6" Chicken Teriyaki | 4.5 | 370 |

Table 3: Product of the deviations

| | X_i | Y_i | $X_i - \bar{X}$ | $Y_i - \bar{Y}$ | $(X_i - \bar{X})(Y_i - \bar{Y})$ |
|------------------|-------|-------|-----------------|-----------------|----------------------------------|
| Black Forest Ham | 4.5 | 290 | 0.5 | -8 | -4.0 |
| Roast Beef | 5.0 | 320 | 1.0 | 22 | 22.0 |
| Turkey Breast | 3.5 | 280 | -0.5 | -18 | 9.0 |
| Veggie Delite | 2.5 | 230 | -1.5 | -68 | 102.0 |
| Chicken Teriyaki | 4.5 | 370 | 0.5 | 72 | 36.0 |
| Sum | 20 | 1490 | 0.0 | 0 | 165.0 |

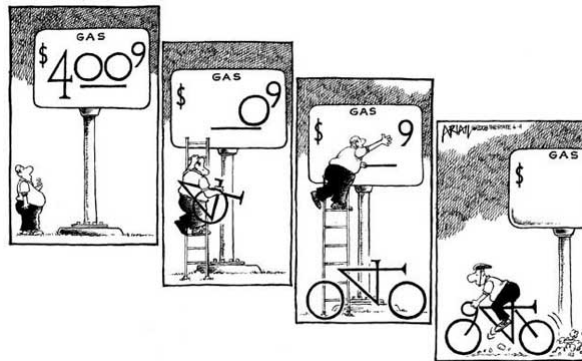
In Table 3, the sum of product of the deviations is given, where X_i 's denote total fat and Y_i 's denote calories.

We know that the standard deviation of Total Fat content is 1.000 ($s_X = 1.000$), and the standard deviation of Calories is 51.672 ($s_Y = 51.672$).

- Find the correlation between Total Fat content and Calories.
- Find the ratio of the residual sum of squares SS_{Resid} to the total sum of squares $SSTo$.
- Find the linear regression equation for predicting calories from total fat.
- Test if the slope $\beta = 0$. Use the significance level $\alpha = 0.05$.

4. Bayes' Rule (Chapter 6)

At a certain gas station 50% of the customers request regular gas, 30% request plus gas, and 20% request premium gas. Of those customers requesting regular gas, 25% fill up their tanks. Of those customers requesting plus gas, 45% fill up their tanks, while of those requesting premium, 40% fill up their tanks. If the next customer fills up the tank, what is the probability that premium gas is requested?



5. Discrete Random Variables (Chapter 7)

Suppose an individual plays a gambling game where it is possible to lose \$1.00, break even, win \$3.00, or win \$5.00 each time she plays. The probability distribution for each outcome is provided by the following table:

| | | | | |
|-------------|---------|--------|--------|--------|
| Outcome | −\$1.00 | \$0.00 | \$3.00 | \$5.00 |
| Probability | 0.60 | 0.25 | 0.10 | 0.05 |

- (1) Verify that the discrete probability distribution above is well-defined.
- (2) Find the mean and standard deviation of this discrete random variable.
- (3) Suppose that the casino decides that the game does not have an impressive enough top prize with the lower payouts, and decides to change the outcomes, as shown below

| | | | | |
|-------------|---------|---------|--------|---------|
| Outcome | −\$3.00 | −\$0.50 | \$7.00 | \$12.00 |
| Probability | 0.60 | 0.25 | 0.10 | 0.05 |

Find the linear relation between new outcome and previous outcomes. Based on the relation, find the mean and standard deviation of the new random variable.



6. Sample Size (Chapter 8,9)

A Company claims its program will allow your computer to download movies quickly. We'll test the free evaluation copy by downloading a movie several times, hoping to estimate the mean download time with a margin of error of only 30 seconds. We think the standard deviation of download times is about 2.5 minutes. How many trial download must we run if we want 95% confidence in our estimate with a margin of error of 30 seconds?



7. Hypothesis Test (Chapter 10,11)

Our bodies have a natural electrical field that is known to help wounds heal. Does changing the field strength slow healing? A series of experiments with newts investigated this question. the data below are the healing rates of cuts (micrometers per hour) in a matched pairs experiment. The pairs are the two hind limbs of the same newt, with the body's natural field in one limb (control) and half the natural value in the other limb (experimental). Is there good evidence that changing the electrical field from its natural level slows healing? Choose the appropriate two-sample t -test, and use significance level 5% to answer this question.

| Newt | 1 | 2 | 3 | 4 | 5 |
|--------------|----|----|----|----|----|
| Control | 25 | 36 | 31 | 45 | 57 |
| Experimental | 24 | 33 | 27 | 42 | 26 |



8. Hypothesis Test (Chapter 10,11)

Do college students who have volunteered for community service work differ from those who have not? A study obtained data from 57 students who had done service work and 17 student who had not. One of the response variables was a measure of attachment to friends (roughly, secure relationships), measured by the the Inventory of Parent and Peer Attachment. Here are the results:

| Group | Condition | n | \bar{x} | s |
|-------|------------|-----|-----------|-------|
| 1 | Service | 57 | 105.32 | 10.68 |
| 2 | No service | 17 | 96.82 | 14.26 |

By using significance level 5%, choose the appropriate two-sample t -test to show whether college students who have volunteered for community service work differ from those who have not.

9. Categorical Data Analysis (Chapter 12)

A poker-dealing machine is supposed to deal cards at random, as if from an infinite deck. In a test, you counted 200 cards, and observed the following:

Spades: 63

Hearts: 39

Diamonds: 43

Clubs: 55

Could it be that the suits are equally likely? Use significance level $\alpha = 0.05$.

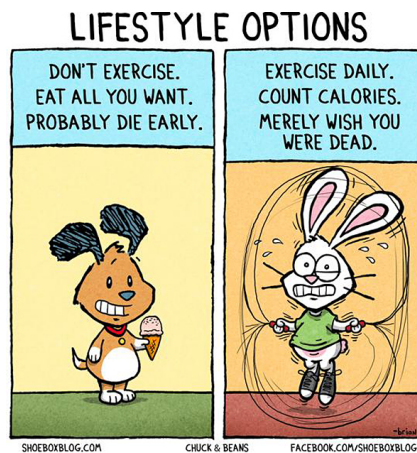


10. Analysis of Variance (ANOVA) (Chapter 15)

What conditions help overweight people exercise regularly? Subjects were randomly assigned to three treatments: a single two-hour long exercise 1 day per week, two one-hour medium exercise 2 days per week, and 20-minute short exercise 6 days per week. The study report contains the weight loss (in lb) after six months of treatment

| Group | Number of Subjects | Weight loss |
|----------------------|--------------------|-------------|
| Single long exercise | 3 | 1, 2, 3 |
| Two medium exercises | 2 | 4, 6 |
| Six short exercises | 2 | 5, 7 |

Calculate (a) the overall mean response $\bar{\bar{x}}$, (b) the Mean square for treatment MSTR, (c) the Mean square for error MSE, (d) the F -statistic, (e) test if there is no difference among three exercise plans by using significance level $\alpha = 5\%$, and (f) summarize your calculation by using an ANOVA table.



More space

More space

End of the final exam of Stat 213 Sec HC1 (Instructor: Jiangtao Gou)