FALL 2016

HUNTER COLLEGE

STAT 213 Section 06/HC1

Introduction to Applied Statistics

Final Exam

Last Name: ______________________________

First Name: ______________________________

Graduation Year: ________________________

December 21, 2016

Final Exam of Stat 213 Sec 06/HC1 (Fall 2016)

1. Please do not leave blank for any question.
2. There are 10 questions, each question is 5 points.
3. You have 120 minutes for this exam (3:00 pm - 5:00 pm).
4. Explain briefly = Explain in one sentence or several phrases.

Formulas

$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$

$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

$z = \frac{x-\mu}{\sigma}$

$x = \mu + \sigma z$

$s_x^2 = \frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n-1}$

$s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n-1}}$

$\Pr\left(|X-\mu| \leq k\sigma\right) \geq 1 - \frac{1}{k^2}$, for any distribution

$\Pr\left(|X-\mu| \leq \sigma\right) \approx 0.68$, $\Pr\left(|X-\mu| \leq 2\sigma\right) \approx 0.95$, $\Pr\left(|X-\mu| \leq 3\sigma\right) \approx 0.997$, for normal distribution

$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$

$r = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{(n-1)s_x s_y}$

$\widehat{y} = a + bx$

$b = r\frac{s_y}{s_x}$

$b = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}$

$a = \overline{y} - b\overline{x}$

$SSTo = \sum_{i=1}^{n}(y_i - \overline{y})^2$

$SSTo = \sum_{i=1}^{n} y_i^2 - n\overline{y}^2$

$SSResid = \sum_{i=1}^{n}\left(y_i - \widehat{y}_i\right)^2$

$SSResid = \sum_{i=1}^{n} y_i^2 - a\sum_{i=1}^{n} y_i - b\sum_{i=1}^{n} x_i y_i$

$r^2 = 1 - \frac{SSResid}{SSTo}$

$s_e = \sqrt{\frac{SSResid}{n-2}}$

If events $E_1, \cdots, E_k$ are all mutually exclusive, then $\Pr(E1 \cup \cdots \cup E_k) = \Pr(E_1) + \cdots + \Pr(E_k)$

$\Pr(E \mid F) = \frac{\Pr(E \cap F)}{P(F)}$

The event E and F are independent if and only if $\Pr(E \cap F) = \Pr(E)\Pr(F)$

$\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$

$\Pr(E \cap F) = \Pr(E|F)\Pr(F) = \Pr(F|E)\Pr(E)$

If events $B_1, \cdots, B_k$ are all mutually exclusive with $\Pr(B_1) + \cdots + \Pr(B_k) = 1$, then for any event E, $\Pr(E) = \Pr(E \mid B_1)\Pr(B_1) + \cdots + \Pr(E \mid B_k)\Pr(B_k)$

If events $B_1, \cdots, B_k$ are all mutually exclusive with $\Pr(B_1) + \cdots + \Pr(B_k) = 1$, then for any event E, $\Pr(B_i \mid E) = \frac{\Pr(E|B_i)\Pr(B_i)}{\Pr(E|B_1)\Pr(B_1)+\cdots+\Pr(E|B_k)\Pr(B_k)}$

For continuous random variable X, $\Pr(a < X \leq b) = \Pr(X \leq b) - \Pr(X \leq a)$

Mean value of a discrete random variable $\mu_X = \sum x \cdot p(x)$

Standard deviation of a discrete random variable $\sigma_X = \sqrt{\sum (x - \mu_X)^2 \cdot p(x)}$

$Y = a + bX$. Mean $\mu_Y = a + b\mu_X$. Standard deviation $\sigma_Y = |b|\sigma_X$

$Y = a_1X_1 + \cdots + a_nX_n$. Mean $\mu_Y = a_1\mu_{X_1} + \cdots + a_n\mu_{X_n}$. Standard deviation (when X_i's are independent) $\sigma_Y = \sqrt{a_1^2\sigma_{X_1}^2 + \cdots + a_n^2\sigma_{X_n}^2}$

Binomial distribution $p(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$, $\mu_X = np$, $\sigma_X = \sqrt{np(1-p)}$

Geometric distribution $p(x) = (1-p)^{x-1}p$, $\mu_X = 1/p$

Continuity correction (X is a discrete variable (integer values), Y is the corresponding Normal random variable): If $\Pr(X \leq m)$ use $\Pr(Y < m + 0.5)$; If $\Pr(X < m)$ use $\Pr(Y < m - 0.5)$

Sampling distribution of $\overline{X}$: $\mu_{\overline{X}} = \mu$, $\sigma_{\overline{X}} = \sigma/\sqrt{n}$

Sampling distribution of $\widehat{p}$: $\mu_{\widehat{p}} = p$, $\sigma_{\widehat{p}} = \sqrt{p(1-p)/n}$

$(1-\alpha) \times 100\%$ confidence interval for p: $\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$

$n = p(1-p)\left(\frac{z_{\alpha/2}}{B}\right)^2$

$(1-\alpha) \times 100\%$ confidence interval for μ: $\overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$n = \left(\frac{z_{\alpha/2}\sigma}{B}\right)^2$

$(1-\alpha) \times 100\%$ confidence interval for μ: $\overline{x} \pm t_{\alpha/2, n-1}\frac{s}{\sqrt{n}}$

Finite population correction factor: $\sqrt{\frac{N-n}{N-1}}$

$z = \frac{\overline{x}-\mu}{\sigma/\sqrt{n}}$, where μ is the hypothesized value under H_0

$t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$, where μ is the hypothesized value under H_0

Power + Type II error = 1

Paired two-sample t-test: $t = \frac{\overline{x}_d - \text{hypothesized value}}{s_d/\sqrt{n}}$, $d.f. = n - 1$

Independent two-sample t-test: $t = \frac{\overline{x}_1 - \overline{x}_2 - \text{hypothesized value}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$, $d.f. = \left\lfloor \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \right\rfloor$.

Pearson's chi-squared test: $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$

Hypothesis Test concerning slope β: $t = \frac{b - \text{hypothesized value}}{s_\beta}$, where $s_\beta = \frac{s_e}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$ and $d.f. = n - 2$.

Coefficient of multiple determination: $R^2 = 1 - \frac{SSResid}{SSTo}$
SSResid $= \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$
SSRegr $= \sum_{i=1}^{n}(\hat{y}_i - \overline{y})^2$
F test for Multiple Regression: $F = \frac{\text{SSRegr}/k}{\text{SSResid}/(n-(k+1))}$ or $F = \frac{R^2/k}{(1-R^2)/(n-(k+1))}$

Tukey-Kramer MCP: For $\mu_i - \mu_j$: $(\overline{x}_i - \overline{x}_j) \pm q\sqrt{\frac{\text{MSE}}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$

Random Number Table

34234	27566	94454	20349	69224	69483	21821	38248	62410	16481	54270	14344	60679	95118	44916	95522
17144	05395	40643	08340	52134	20753	41452	52797	45320	67751	00459	28894	43588	46388	64547	10072
00054	56665	60274	22889	35043	72024	87641	67346	28230	19021	20090	16885	26498	97659	10735	24621
56406	07936	06463	37439	17953	23294	07272	55338	11140	70292	66278	31434	09408	48929	30366	12613
39316	59206	26094	25430	00863	01122	53461	69887	94050	48120	85909	45984	92318	26757	49997	27162
22226	10476	45725	39980	83773	52393	73092	84437	71657	66721	54971	90220	84475	28268	70330	17587
07148	56945	07552	29174	17424	52673	46928	90721	32783	80040	64827	57350	79781	12488	40923	82176
58418	76576	22101	12084	68695	72304	34919	73631	84053	99671	79376	40260	57609	58677	55473	65086
09688	22765	36651	94994	19965	18493	49468	56541	61881	45860	93925	23170	08879	78308	43464	47996
87517	42396	51200	77903	71236	38123	64018	12893	13152	65490	81917	06079	60150	97939	58013	04348
38787	88585	39192	60813	49064	84312	52009	95803	64422	85121	96466	88989	11420	44128	72563	87258
90057	08216	53741	43723	00334	03943	66559	78713	15693	31310	11016	71899	62691	63759	60554	70167

NORMAL CUMULATIVE DISTRIBUTION FUNCTION

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

STUDENT'S t PERCENTAGE POINTS

one-tail	40.0%	33.3%	25.0%	20.0%	12.5%	10.0%	5.0%	2.5%	1.0%	0.5%	0.1%
two-tail	80.0%	66.7%	50.0%	40.0%	25.0%	20.0%	10.0%	5.0%	2.0%	1.0%	0.2%
cum. prob	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.325	0.577	1.000	1.376	2.414	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.500	0.816	1.061	1.604	1.886	2.920	4.303	6.965	9.925	22.327
3	0.277	0.476	0.765	0.978	1.423	1.638	2.353	3.182	4.541	5.841	10.215
4	0.271	0.464	0.741	0.941	1.344	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.457	0.727	0.920	1.301	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.453	0.718	0.906	1.273	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.449	0.711	0.896	1.254	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.447	0.706	0.889	1.240	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.445	0.703	0.883	1.230	1.383	1.833	2.262	2.821	3.250	4.297
10	0.260	0.444	0.700	0.879	1.221	1.372	1.812	2.228	2.764	3.169	4.144
11	0.260	0.443	0.697	0.876	1.214	1.363	1.796	2.201	2.718	3.106	4.025
12	0.259	0.442	0.695	0.873	1.209	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.441	0.694	0.870	1.204	1.350	1.771	2.160	2.650	3.012	3.852
14	0.258	0.440	0.692	0.868	1.200	1.345	1.761	2.145	2.624	2.977	3.787
15	0.258	0.439	0.691	0.866	1.197	1.341	1.753	2.131	2.602	2.947	3.733
16	0.258	0.439	0.690	0.865	1.194	1.337	1.746	2.120	2.583	2.921	3.686
17	0.257	0.438	0.689	0.863	1.191	1.333	1.740	2.110	2.567	2.898	3.646
18	0.257	0.438	0.688	0.862	1.189	1.330	1.734	2.101	2.552	2.878	3.610
19	0.257	0.438	0.688	0.861	1.187	1.328	1.729	2.093	2.539	2.861	3.579
20	0.257	0.437	0.687	0.860	1.185	1.325	1.725	2.086	2.528	2.845	3.552
21	0.257	0.437	0.686	0.859	1.183	1.323	1.721	2.080	2.518	2.831	3.527
22	0.256	0.437	0.686	0.858	1.182	1.321	1.717	2.074	2.508	2.819	3.505
23	0.256	0.436	0.685	0.858	1.180	1.319	1.714	2.069	2.500	2.807	3.485
24	0.256	0.436	0.685	0.857	1.179	1.318	1.711	2.064	2.492	2.797	3.467
25	0.256	0.436	0.684	0.856	1.178	1.316	1.708	2.060	2.485	2.787	3.450
26	0.256	0.436	0.684	0.856	1.177	1.315	1.706	2.056	2.479	2.779	3.435
27	0.256	0.435	0.684	0.855	1.176	1.314	1.703	2.052	2.473	2.771	3.421
28	0.256	0.435	0.683	0.855	1.175	1.313	1.701	2.048	2.467	2.763	3.408
29	0.256	0.435	0.683	0.854	1.174	1.311	1.699	2.045	2.462	2.756	3.396
30	0.256	0.435	0.683	0.854	1.173	1.310	1.697	2.042	2.457	2.750	3.385
35	0.255	0.434	0.682	0.852	1.170	1.306	1.690	2.030	2.438	2.724	3.340
40	0.255	0.434	0.681	0.851	1.167	1.303	1.684	2.021	2.423	2.704	3.307
45	0.255	0.434	0.680	0.850	1.165	1.301	1.679	2.014	2.412	2.690	3.281
50	0.255	0.433	0.679	0.849	1.164	1.299	1.676	2.009	2.403	2.678	3.261
55	0.255	0.433	0.679	0.848	1.163	1.297	1.673	2.004	2.396	2.668	3.245
60	0.254	0.433	0.679	0.848	1.162	1.296	1.671	2.000	2.390	2.660	3.232
∞	0.253	0.431	0.674	0.842	1.150	1.282	1.645	1.960	2.326	2.576	3.090
conf. level	20.0%	33.3%	50.0%	60.0%	75.0%	80.0%	90.0%	95.0%	98.0%	99.0%	99.8%

CHI-SQUARED TABLE (right-tail probability)

ν	40.0%	33.3%	25.0%	20.0%	12.5%	10.0%	5.0%	2.5%	1.0%	0.5%	0.1%
1	0.708	0.936	1.323	1.642	2.354	2.706	3.841	5.024	6.635	7.879	10.828
2	1.833	2.197	2.773	3.219	4.159	4.605	5.991	7.378	9.210	10.597	13.816
3	2.946	3.405	4.108	4.642	5.739	6.251	7.815	9.348	11.345	12.838	16.266
4	4.045	4.579	5.385	5.989	7.214	7.779	9.488	11.143	13.277	14.860	18.467
5	5.132	5.730	6.626	7.289	8.625	9.236	11.070	12.833	15.086	16.750	20.515
6	6.211	6.867	7.841	8.558	9.992	10.645	12.592	14.449	16.812	18.548	22.458
7	7.283	7.992	9.037	9.803	11.326	12.017	14.067	16.013	18.475	20.278	24.322
8	8.351	9.107	10.219	11.030	12.636	13.362	15.507	17.535	20.090	21.955	26.125
9	9.414	10.215	11.389	12.242	13.926	14.684	16.919	19.023	21.666	23.589	27.877
10	10.473	11.317	12.549	13.442	15.198	15.987	18.307	20.483	23.209	25.188	29.588
11	11.530	12.414	13.701	14.631	16.457	17.275	19.675	21.920	24.725	26.757	31.264
12	12.584	13.506	14.845	15.812	17.703	18.549	21.026	23.337	26.217	28.300	32.910
13	13.636	14.595	15.984	16.985	18.939	19.812	22.362	24.736	27.688	29.819	34.528
14	14.685	15.680	17.117	18.151	20.166	21.064	23.685	26.119	29.141	31.319	36.123
15	15.733	16.761	18.245	19.311	21.384	22.307	24.996	27.488	30.578	32.801	37.697
16	16.780	17.840	19.369	20.465	22.595	23.542	26.296	28.845	32.000	34.267	39.252
17	17.824	18.917	20.489	21.615	23.799	24.769	27.587	30.191	33.409	35.718	40.790
18	18.868	19.991	21.605	22.760	24.997	25.989	28.869	31.526	34.805	37.156	42.312
19	19.910	21.063	22.718	23.900	26.189	27.204	30.144	32.852	36.191	38.582	43.820
20	20.951	22.133	23.828	25.038	27.376	28.412	31.410	34.170	37.566	39.997	45.315
21	21.991	23.201	24.935	26.171	28.559	29.615	32.671	35.479	38.932	41.401	46.797
22	23.031	24.268	26.039	27.301	29.737	30.813	33.924	36.781	40.289	42.796	48.268
23	24.069	25.333	27.141	28.429	30.911	32.007	35.172	38.076	41.638	44.181	49.728
24	25.106	26.397	28.241	29.553	32.081	33.196	36.415	39.364	42.980	45.559	51.179
25	26.143	27.459	29.339	30.675	33.247	34.382	37.652	40.646	44.314	46.928	52.620
26	27.179	28.520	30.435	31.795	34.410	35.563	38.885	41.923	45.642	48.290	54.052
27	28.214	29.580	31.528	32.912	35.570	36.741	40.113	43.195	46.963	49.645	55.476
28	29.249	30.639	32.620	34.027	36.727	37.916	41.337	44.461	48.278	50.993	56.892
29	30.283	31.697	33.711	35.139	37.881	39.087	42.557	45.722	49.588	52.336	58.301
30	31.316	32.754	34.800	36.250	39.033	40.256	43.773	46.979	50.892	53.672	59.703
35	36.475	38.024	40.223	41.778	44.753	46.059	49.802	53.203	57.342	60.275	66.619
40	41.622	43.275	45.616	47.269	50.424	51.805	55.758	59.342	63.691	66.766	73.402
45	46.761	48.510	50.985	52.729	56.052	57.505	61.656	65.410	69.957	73.166	80.077
50	51.892	53.733	56.334	58.164	61.647	63.167	67.505	71.420	76.154	79.490	86.661
55	57.016	58.945	61.665	63.577	67.211	68.796	73.311	77.380	82.292	85.749	93.168
60	62.135	64.147	66.981	68.972	72.751	74.397	79.082	83.298	88.379	91.952	99.607

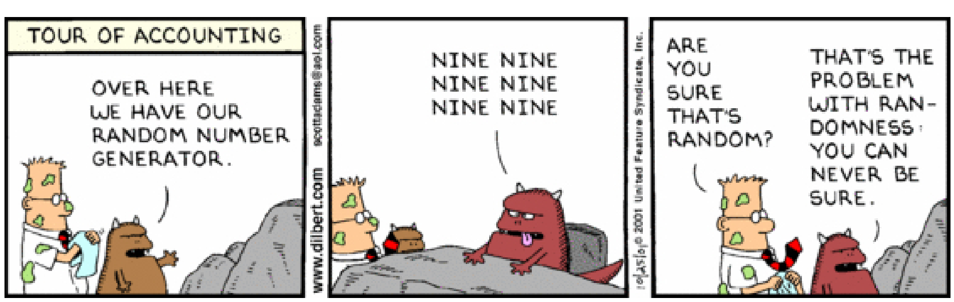
F DISTRIBUTION TABLE (right-tail probability)

$\nu_2 \backslash \nu_1$	q	2	3	4	5	6	7	8	10	12	15	20	30	50	∞
1	0.100	49.5	53.6	55.8	57.2	58.2	59.1	59.7	60.5	61.0	61.5	62.0	62.6	63.0	63.3
	0.050	199.	216.	225.	230.	234.	237.	239.	242.	244.	246.	248.	250.	252.	254.
	0.025	800.	864.	900.	922.	937.	948.	957.	969.	977.	985.	993.			
	0.010														
	0.001														
2	0.100	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.39	9.41	9.43	9.44	9.46	9.47	9.49
	0.050	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
	0.025	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5
	0.010	99.0	99.2	99.2	99.3	99.3	99.4	100.	100.	100.	100.	100.	100.	100.	99.5
	0.001	999.	999.												
3	0.100	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.23	5.22	5.20	5.18	5.17	5.15	5.13
	0.050	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.70	8.66	8.62	8.58	8.53
	0.025	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.4	14.3	14.3	14.2	14.1	14.0	13.9
	0.010	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.2	27.1	26.9	26.7	26.5	26.4	26.1
	0.001	149.	141.	137.	135.	133.	132.	131.	129.	128.	127.	126.	125.	125.	123.
4	0.100	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.92	3.90	3.87	3.84	3.82	3.79	3.76
	0.050	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.86	5.80	5.75	5.70	5.63
	0.025	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.84	8.75	8.66	8.56	8.46	8.38	8.26
	0.010	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.5	14.4	14.2	14.0	13.8	13.7	13.5
	0.001	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.0	47.4	46.8	46.1	45.4	44.9	44.1
5	0.100	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.30	3.27	3.24	3.21	3.17	3.15	3.10
	0.050	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.62	4.56	4.50	4.44	4.36
	0.025	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.62	6.52	6.43	6.33	6.23	6.14	6.02
	0.010	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.1	9.89	9.72	9.55	9.38	9.24	9.02
	0.001	37.1	33.2	31.1	29.8	28.8	28.2	27.6	26.9	26.4	25.9	25.4	24.9	24.4	23.8
6	0.100	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.94	2.90	2.87	2.84	2.80	2.77	2.72
	0.050	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.94	3.87	3.81	3.75	3.67
	0.025	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.46	5.37	5.27	5.17	5.07	4.98	4.85
	0.010	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.87	7.72	7.56	7.40	7.23	7.09	6.88
	0.001	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.4	18.0	17.6	17.1	16.7	16.3	15.7
7	0.100	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.70	2.67	2.63	2.59	2.56	2.52	2.47
	0.050	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.51	3.44	3.38	3.32	3.23
	0.025	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.76	4.67	4.57	4.47	4.36	4.28	4.14
	0.010	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.62	6.47	6.31	6.16	5.99	5.86	5.65
	0.001	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.1	13.7	13.3	12.9	12.5	12.2	11.7
8	0.100	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.54	2.50	2.46	2.42	2.38	2.35	2.29
	0.050	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.22	3.15	3.08	3.02	2.93
	0.025	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.29	4.20	4.10	4.00	3.89	3.81	3.67
	0.010	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.81	5.67	5.52	5.36	5.20	5.07	4.86
	0.001	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.5	11.2	10.8	10.5	10.1	9.80	9.33

1. Sampling (Chapter 2)
   Please randomly draw two students from a group of eighteen by using Simple Random Sampling without Replacement (SRS):

   1 Alan
   2 Lucy
   3 Tom
   4 Azar
   5 Jayne
   6 Nadima
   7 Matthew
   8 Sushi
   9 Mohammed
   10 Rachel
   11 Ben
   12 Emma
   13 Grace
   14 Anna
   15 Sophie
   16 Karen
   17 Joshua
   18 James

   Briefly describe the sampling procedure you use.

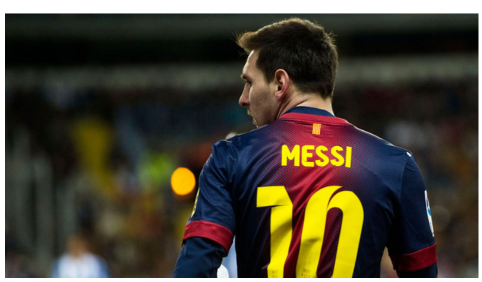


2. Plots for Quantitative Variables (Chapter 1,3,4)
The stem-and-leaf display shows the numbers of goals by Lionel A. Messi at FC Barcelona during 13 seasons of La Liga, Copa del Rey, Champions League, and other games from season 2003-04 to season 2015-16.

Table 1: Number of Goals (5|8 means 58)

stem	leaf
0	578
1	67
2	
3	8
4	117
5	38
6	0
7	3

Find (a) the minimum,
(b) the maximum,
(c) the median,
(d) the mean,
(e) the interquartile range,
and
(f) draw the boxplot, and briefly describe the steps of drawing a boxplot,
(g) report whether outliers are observed, and briefly describe the outlier detection techniques you use.



3. Correlations and Simple Linear Regression (Chapter 5,13,14)
Total Fat versus Calories for 5 items on the Subway menu are shown in Table 2. (data source: https://www.subway.com/nutrition/nutritionlist.aspx)

Table 2: Nutrition Facts

	Fat (g)	Calories
6" Black Forest Ham	4.5	290
6" Roast Beef	5.0	320
6" Turkey Breast	3.5	280
6" Veggie Delite	2.5	230
6" Chicken Teriyaki	4.5	370
6" Classic Tuna	25.0	480

Table 3: Product of the deviations

	X_i	Y_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)$
Black Forest Ham	4.5	290	-3.0	-38.3	115.0
Roast Beef	5.0	320	-2.5	-8.3	20.8
Turkey Breast	3.5	280	-4.0	-48.3	193.3
Veggie Delite	2.5	230	-5.0	-98.3	491.7
Chicken Teriyaki	4.5	370	-3.0	41.7	-125.0
Classic Tuna	25	480	17.5	151.7	2654.2
Sum	45.0	1970	0.0	0	3350.0

In Table 3, the sum of product of the deviations is given, where X_i's denote total fat and Y_i's denote calories.

We know that the standard deviation of Total Fat content is 8.6197 ($s_X = 8.6197$), and the standard deviation of Calories is 87.5024 ($s_Y = 87.5024$).

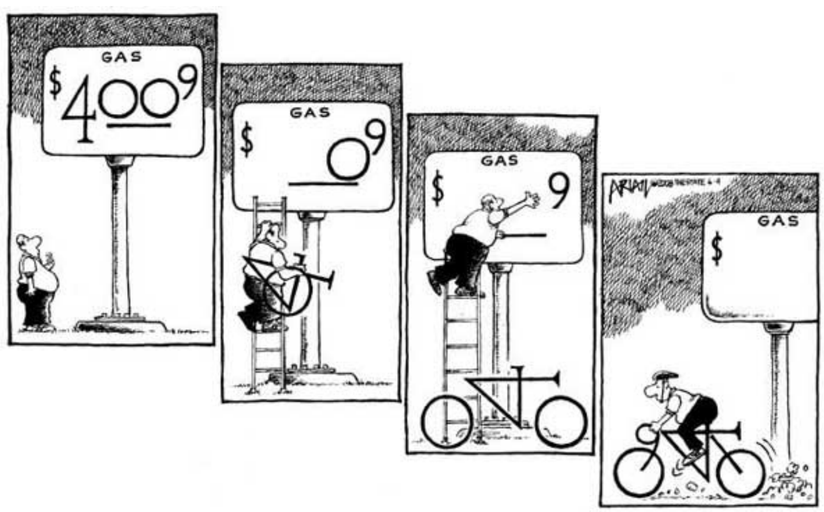
(a) Find the correlation between Total Fat content and Calories.

(b) Find the ratio of the residual sum of squares $SSResid$ to the total sum of squares $SSTo.$.

(c) Find the linear regression equation for predicting calories from total fat.

(d) Test if the slope $\beta = 0$. Use the significance level $\alpha = 0.05$.

4. Bayes' Rule (Chapter 6)
At a certain gas station 60% of the customers request regular gas, 15% request plus gas, and 25% request premium gas. Of those customers requesting regular gas, 25% fill up their tanks. Of those customers requesting plus gas, 45% fill up their tanks, while of those requesting premium, 40% fill up their tanks. If the next customer fills up the tank, what is the probability that premium gas is requested?



5. Discrete Random Variables (Chapter 7)
Suppose an individual plays a gambling game where it is possible to lose \$1.00, break even, win \$3.00, or win \$5.00 each time she plays. The probability distribution for each outcome is provided by the following table:

Outcome	$-\$1.00$	\$0.00	\$3.00	\$5.00
Probability	0.60	0.25	0.10	0.05

(1) Verify that the discrete probability distribution above is well-defined.
(2) Find the mean and standard deviation of this discrete random variable.
(3) Suppose that the casino decides that the game does not have an impressive enough top prize with the lower payouts, and decides to change the outcomes, as shown below

Outcome	$-\$3.00$	$-\$0.50$	\$7.00	\$12.00
Probability	0.60	0.25	0.10	0.05

Find the linear relation between new outcome and previous outcomes. Based on the relation, find the mean and standard deviation of the new random variable.

6. Normal Distribution (Chapter 7)

   (You may need a Normal Table for this question.)

   In Fuel Economy Guide (Model Year 2013), Environmental Protection Agency (EPA) fuel economy estimates for automobile models tested predicted a mean of 23.8 mpg (miles per gallon) and a standard deviation of 6.2 mpg. Assume that a Normal model can be applied.

   (data source: www.fueleconomy.gov)

   An auto dealer introduced you a fuel-efficient car. He told you that this car's gas mileage is higher than 98% of vehicles.

   Find the gas mileage of this car.

Figure 1: Gas Mileage

7. Sample Size (Chapter 8,9)
A Company claims its program will allow your computer to download movies quickly. We'll test the free evaluation copy by downloading a movie several times, hoping to estimate the mean download time with a margin of error of only 30 seconds. We think the standard deviation of download times is about 2.5 minutes. How many trial download must we run if we want 95% confidence in our estimate with a margin of error of 30 seconds?

8. Hypothesis Test (Chapter 10,11)
Our bodies have a natural electrical field that is known to help wounds heal. Does changing the field strength slow healing? A series of experiments with newts investigated this question. the data below are the healing rates of cuts (micrometers per hour) in a matched pairs experiment. The pairs are the two hind limbs of the same newt, with the body's natural field in one limb (control) and half the natural value in the other limb (experimental). Is there good evidence that changing the electrical field from its natural level slows healing? Choose the appropriate two-sample t-test, and use significance level 5% to answer this question.

Newt	1	2	3	4	5
Control	25	36	31	45	57
Experimental	24	33	27	42	26

9. Categorical Data Analysis (Chapter 12)
   A poker-dealing machine is supposed to deal cards at random, as if from an infinite deck. In a test, you counted 200 cards, and observed the following:
   Spades: 63
   Hearts: 39
   Diamonds: 43
   Clubs: 55
   Could it be that the suits are equally likely? Use significance level $\alpha = 0.05$.

10. F-test (Chapter 14,15)

A multiple regression model is estimated to study the factors which contribute to the price of energy bars

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + e$$

where y is the price (dollar), x_1 is the calorie content, x_2 is the protein content (gram), and x_3 is the fat content (gram).

Based on a sample of 10 energy bars, the regression result is

$$\hat{y} = 0.298 + 0.0016x_1 + 0.0582x_2 + 0.0533x_3$$

Given the regression sum of squares equals 3.5126 and the residual sum of squares equals 1.2531.
(1) A energy bar's nutrition facts are displayed at bottom of this page. Use the regression model to predict its price.
(2) Calculate the coefficient of multiple determination R^2.
(3) Test if $\beta_1 = \beta_2 = \beta_3 = 0$. Use significance level $\alpha = 0.05$. (Hint: Use F-test)

Nutrition Facts	**Amount/Serving**	**%DV***	**Amount/Serving**	**%DV***	**Amount/Serving**	**%DV***
Serv. Size 1 Bar (68g)	**Total Fat** 3.5g	**5%**	**Cholesterol** 0mg	**0%**	Insoluble Fiber 3g	
Calories 240	Saturated Fat 0.5g	**3%**	**Sodium** 120mg	**5%**	Sugars 24g	
Calories from Fat 30	Trans Fat 0g		**Potassium** 250mg	**7%**	Other Carb. 16g	
*Percent Daily Values (DV) are based on a 2,000 calorie diet	Polyunsaturated Fat 1g		**Total Carb.** 45g	**15%**	**Protein** 9g	**18%**
	Monounsaturated Fat 1.5g		Dietary Fiber 5g	**20%**		
R18	Vit. A 10% • Vit. C 10% • Calcium 20% • Iron 10% • Vit. D 15% • Vit. E 10% • Thiamin (B1) 10% • Riboflavin (B2) 10% • Niacin 10% Vitamin B6 10% • Vitamin B12 10% • Phosphorus 25% • Magnesium 20%					

More space

More space

End of the final exam of Stat 213 Sec 06/HC1 (Instructor: Jiangtao Gou)