Organization Structure with Information Synthesis

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1 Synthesis model

This is the full mathematical specification of the model. It is *not* graph theoretic. The model is in the spirit of CalvArmengol, Antoni, Joan Mart, and Andrea Prat. "Communication and influence." Theoretical Economics 10.2 (2015): 649-690, but is far broader (more information about that model is in the commented out version of this tex). In this write-up, I sometimes make assumptions for notational convenience but note that such assumptions can be relaxed.

The model contains N agents indexed by $i = 1...N$. There are also K random variables representing a different part of the state of the world or the environment. For simplicity, assume that each element of the environment is independent so that

$$\theta_k \sim f_k \tag{1}$$

where θ_k is the state of random variable k and f_k is a probability density/distribution function. In general $K \neq N$. For simplicity, assume that each of θ_k is one-dimensional. Allowing θ to be multi-dimensional as well as allowing each θ to be correlated would be a trivial extension at the expense of more notation.

Each agent i takes an action. Each agent also sends and receives messages. We assume that agent i broadcasts. That is, whatever agent i says can be heard by all other agents, if they choose to do so (think mass emails).

First, I will discuss how agents receive messages and observe the environment, then talk about how they choose their action and finally discuss how they send messages.

1.1 Receiving Observations

In this section, we will discuss the observations an agent receives. Let E_i be a $1 \times G_i$ row vector. Later we will discuss the elements of E_i but intuitively, E_i represents observations from the environment and whatever any of the other agents say. Then, the observations of agent i is given by

$$\mathbf{O_i} = \gamma_i(\mathbf{E_i} + \mathbf{W_i} \odot \mathcal{N}_i^e(0, \Sigma_i^e)) \tag{2}$$

where W_i is a $1 \times G_i$ row vector of the form $(\frac{1}{w_1}, \frac{1}{w_1} ... \frac{1}{w_{G_i}})$ and $\odot$ represents element-wise multiplication. γ_i is a function from $\mathbb{R}^{G_i} \to \mathbb{R}^{G_i}$. For now, we can just think of γ_i as the identity function but later, we might want it to be non-linear. The term $\mathcal{N}_i^e(0, \Sigma_i^e)$ represents a $1 \times G_i$ vector of normal random variables with standard deviation Σ_i^e.

Intuitively, agent i's observations, O_i, are simply observations of the environment or the physical world *and* messages sent by other players, both which are corrupted by noise.

1.2 Choosing Actions

Given what agent i observes, O_i, agent i's action is given by

$$\mathbf{A_i} = \alpha_i(O_i X_i) \tag{3}$$

where X_i is a $G_i \times D_i$ matrix. This means that A_i is a $1 \times D_i$ row vector. Intuitively, D_i represents the number of different actions that agent i can take. That is to say, if D_i is 2, then agent i's action space is two dimensional and thus agent i can take 2 actions. α is a function but for now think of it as the identity.

1.3 Sending Messages

Given agent i's observation O_i, what agent i says is given by

$$M_i = \beta(O_i \Omega_i + \mathcal{N}_i^m(0, \Sigma_i^m)) \tag{4}$$

where Ω_i is a $G_i \times F_i$ matrix and $\mathcal{N}_i^m(0, \Sigma_i^m)$ is a $1 \times F_i$ vector of normal random variables. That means that M_i is a $1 \times F_i$ vector. Intuitively, F_i represents the number of distinct things that agent i can say. Intuitively, agent i can determine what it says by setting elements of Ω_i and the magnitude of those elements determine the signal to noise ratio. β is a function but for now, think of it as the identity.

1.4 Composition of E_i

So what exactly is E_i? Simple, E_i is just the concatenation of the environment and what all other agents $j < i$ say. So, for agent 1, E_i is a $1 \times K$ vector. For agent 2, E_i is a $1 \times K + F_1$ vector. For agent 3, E_i is a $1 \times K + F_1 + F_2$ vector. In general, for agent i, E_i is a $1 \times K + \sum_{j<i} F_i$ vector.

2 Optimization Problem

As shorthand, let $\Omega = [\Omega_1, \Omega_2 ... \Omega_N]$, $X = [X_1, X_2 ... X_N]$, $W = [W_1, W_2 ... W_n]$ and $A = [A_1, A_2 ... A_N]$.

Finally, we can write the welfare function as:

$$F = U(\theta, A) - ||\Omega||^d - ||W||^d \tag{5}$$

where $||\mathbf{q}||^d$ represents the L^d norm of matrix $\mathbf{q}$ and U is some function of the environment and the states of the nodes. The important element to note about the welfare function is that it does *not* explicitly depend on X, which represents the agent's internal computation on how to combine inputs to outputs. Of course, X implicitly enter the utility function through U (since A is a function of X), but there is no cost associated with the weights. The optimization problem then becomes

$$\max_{\Omega, X, W} F \tag{6}$$

2.1 Signal-to Noise Ratio

The main element of this model is that there is a **clear direct relationship** between signal-to-noise ratio and communication cost. The higher the signal-to-noise ratio, the higher the penalization in the welfare function. **JG COMMENT: Question to Milo: Do you see why we need to penalize for the magnitude of Ω?. If not, agent i can just blow up his message so that the noise is irrelevant and all agents listening to him can just shrink it down.**

3 Real-World Interpretation of Variables

Item	Type	Interpretation
N	parameter	Number of Agents in an Organization
K	parameter	Number of random variables in the environment
θ_k	Random Variable	Environment External to the Organization
f_k	parameter	Distribution of environment variable k
E_i	Variable	All of the things (environment and other people) that agent i could possible listen t.o
W_i	Variable	A matrix that represents how agent i allocates its attention to everything it could possibly listen to
O_i	Variable	What agent i "observes" after it decides where to allocate its attention
A_i	Variable	The action that agent i takes
X_i	Variable	A variable that represents how agent i combines what it observes O_i to determine what it does.
M_i	Variable	What agent i says
Ω_i	Variable	How agent i transforms its outputs to what it says.