

Efficient Distributed Stochastic Dual Coordinate Ascent

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Problem Overview

Implementation

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Problem Overview - The Primal Problem

Many machine learning problems can be formulated as the Regularized Finite Sum Minimization (RFSM) problem.

$$\min_{w \in \mathbb{R}^d} P(w) \tag{1}$$

where

$$P(w) = \frac{1}{n} \sum_{i=1}^n \phi(w^\top x_i, y_i) + \lambda g(w)$$

$$w, x_i \in \mathbb{R}^d, \text{ for } i = 1, \dots, n$$

$$y_i \in \mathbb{R}, \text{ for } i = 1, \dots, n$$

$\phi(z, y)$ is convex in z

$g(w)$ is convex in w

Problem Overview - The Dual Problem

We consider the case when $g(w) = \frac{1}{2}\|w\|_2^2$, then the dual problem is given by

$$\max_{\alpha \in \mathbb{R}^n} D(\alpha) \quad (2)$$

where

$$D(\alpha) = \frac{1}{n} \sum_{i=1}^n -\phi_i^*(-\alpha_i) - \frac{\lambda}{2} \left\| \frac{1}{\lambda n} \sum_{i=1}^n \alpha_i x_i \right\|^2$$

$$x_i \in \mathbb{R}^d, \text{ for } i = 1, \dots, n$$

$$\alpha \in \mathbb{R}^n$$

$$\phi_i^*(u) = \max_z (zu - \phi_i(z))$$

The two key papers influencing our work are:

- Stochastic Dual Coordinate Ascent (SDCA) [Shalev-Shwartz and Zhang, 2013]
- Distributed SDCA [Yang, 2013, Yang et al., 2013]

What is SDCA?

- **SDCA** - randomly pick a coordinate axis of $\alpha \in \mathbb{R}^n$, find update that best improves the objective
- **Distributed SDCA** - randomly pick k coordinate axes of $\alpha \in \mathbb{R}^n$, simultaneously find updates that best improve the objective (independently)

SDCA Algorithm

SDCA procedure:

- Let $w^{(0)} = w(\alpha^{(0)})$
- **Iterate:** for $t = 1, 2, \dots, T$
 - Randomly pick i
 - **Find $\Delta\alpha_i$ to maximize**
$$-\phi_i^*(-(\alpha^{(t-1)} + \Delta\alpha_i)) - \frac{\lambda n}{2} \|w^{(t-1)} + (\lambda n)^{-1} \Delta\alpha_i x_i\|^2$$
 - $\alpha^{(t)} \leftarrow \alpha^{(t-1)} + \Delta\alpha_i e_i$
 - $w^{(t)} \leftarrow w^{(t-1)} + (\lambda n)^{-1} \Delta\alpha_i x_i$
- **Output (Random option):**
Let $\bar{\alpha} = \alpha^{(t)}$ and $\bar{w} = w^{(t)}$ for some random
 $t \in T_0 + 1, \dots, T$
Return \bar{w}

Remark: The red steps spend the largest proportion of computing resources.

What if we ran SDCA on the GPU?

Two approaches:

- Naive approach: simply parallelize all tensor operations (e.g., dot product, matrix-vector multiplication, etc.)

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- Naive approach: simply parallelize all tensor operations (e.g., dot product, matrix-vector multiplication, etc.)
- Better: mimic the distributed approach by [Yang, 2013] on a GPU

Implementation

The key area of concern:

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- Memory
 - Allocation
 - Communication (via PCIE bus rather than network)
 - How can we reduce cognitive load of writing this code?

Dealing with Memory Allocation

Naive approach:

```
void MemSync::PushToGpu(const vector &x) {  
    double *dx;  
    int n_bytes = sizeof(double) * x.size();  
  
    // Allocate GPU memory  
    cudaMalloc((double**) &dx, n_bytes);  
  
    // Copy data to GPU  
    cudaMemcpy(dx, &x[0], n_bytes,  
               cudaMemcpyHostToDevice);  
}
```

On a small dataset(200 points in \mathbb{R}^3) we hit over 200k allocations, which comprised nearly **95%** of the GPU compute time (≈ 13 seconds).

Can we do better?

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Yes!

Dealing with Memory Allocation

Better approach:

```
class MemSync {  
    // class code  
    static double *dx_;  
};  
  
void MemSync::PushToGpu(const vector &x) {  
    int n_bytes = sizeof(double) * x.size();  
  
    // Copy data to GPU  
    cudaMemcpy(MemSync::dx_, &x[0],  
               n_bytes, cudaMemcpyHostToDevice);  
}
```

The use of static class pointers reduced the 200k allocations down to only **3 memory allocations**, which comprised only **0.03%** compute time ($\approx 180\mu s$).

What about the cost of communication?

Copying data to and from the GPU is the next most expensive operation.

- About 50% of the compute time, or 768 ms (using the same toy dataset, after we have fixed the memory allocation issue)

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- About 50% of the compute time, or 768 ms (using the same toy dataset, after we have fixed the memory allocation issue)
- Mostly unnecessary!

Consider the following algorithm using the GPU:

1: $\Delta\omega_i \leftarrow f(\mathbf{x}, \omega)$

2: $\omega_i \leftarrow \omega_i + \Delta\omega_i$

To handle this efficiently we should:

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To handle this efficiently we should:

- Reuse ω in step 2 since we already moved it to the GPU for step 1
- Perform step 2 on the GPU since the data is already there. No need to pull it off and then move it back to the GPU

The sad reality is this is quite complicated.

- Lots of book-keeping
- Are there edge cases?
- Need to watch out for memory leaks. Remember, **no** GC!

How can we handle this complexity?

Wrappers

We use wrappers (also known as decorators) to add additional functionality to our code. For example,

```
double VectorDotProduct(const vector &x,  
                        const vector &y) {  
    #ifdef GPU  
        return VectorDotProduct_gpu(x, y);  
    #else  
        return VectorDotProduct_cpu(x, y);  
    #endif  
}
```

To handle the flow of data from GPU to CPU, as well as book-keeping, we can use something like this:

```
class Data {  
    enum DataLocation { Gpu, Cpu };  
  
    // Pointer to GPU memory  
    std::unique_ptr<double> dx_  
    // Local reference (RAM)  
    Eigen::VectorXd x_  
    // DataLocation::Gpu or DataLocation::Cpu  
    DataLocation location_  
};
```

What about results?

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We can reformulate the distributed version as matrix-vector multiplication. This is just an extension of the naive approach!

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Our memory management class is poorly written

- X is a matrix now, not a vector
- However, we have only allocated memory for vectors

Results - Challenges

We are still finalizing results. There is a lot of non-trivial structural code behind this.

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- Algorithm performance tracking

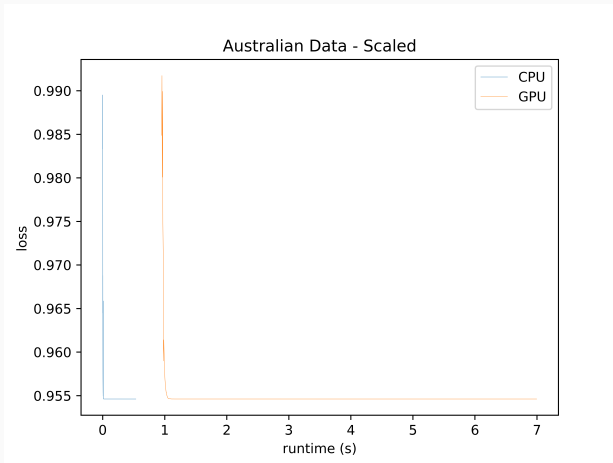
Results - Challenges

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- Loading data
- CUDA can be challenging - communication is slowing us down
- Algorithm
- Algorithm performance tracking
- Expect distributed SDCA to be the fastest, followed by CUDA accelerated SDCA, followed by SDCA

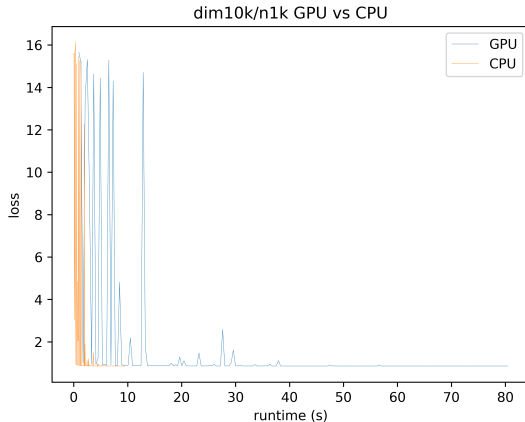
Results - Australian Dataset (\mathbb{R}^{14} , 690 points)

Naive approach



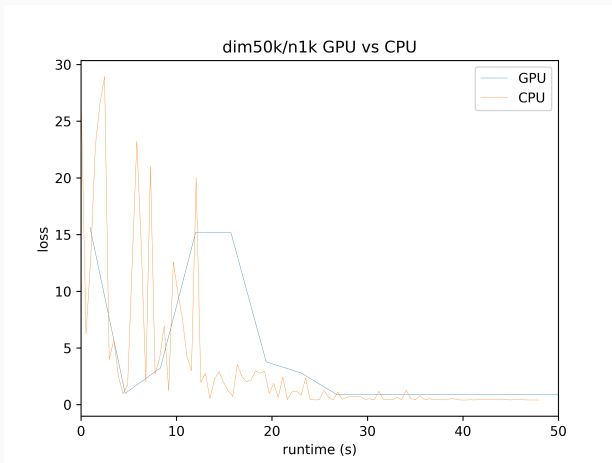
Results - Generated Dataset (\mathbb{R}^{10k} , 1000 points)

Naive approach



Results - Generated Dataset (\mathbb{R}^{50k} , 1000 points)

Naive approach





Shalev-Shwartz, S. and Zhang, T. (2013).

Stochastic dual coordinate ascent methods for regularized loss minimization.

Journal of Machine Learning Research, 14(Feb):567–599.



Yang, T. (2013).

Trading computation for communication: Distributed stochastic dual coordinate ascent.

In *Advances in Neural Information Processing Systems*, pages 629–637.



Yang, T., Zhu, S., Jin, R., and Lin, Y. (2013).

Analysis of distributed stochastic dual coordinate ascent.

arXiv preprint arXiv:1312.1031.