

## P9. Special Pythag Triple.

Set:  $a, b, c \in \mathbb{N}$ ,  $a < b < c$ ,  $a + b + c = 1000$   
up:  $a^2 + b^2 = c^2$

To Find: The Product of  $abc$ .

Recall: ~~Simon's Spe~~ Simon's Favorite Factoring Trick.

Suppose you have an equation in the form of

$1xy + ax + by = c$ , where  $a, b, c$  are int constants.

Then you can factor LHS by

$$\Rightarrow (x+b)(y+a) - ba = c$$

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$$\Rightarrow (x+b)(y+a) = c + ba$$

Solving:

Solve System of Equations.

$$a + b + c = 1000 \quad (a.)$$

$$a^2 + b^2 = c^2 \quad (b.)$$



$$\Rightarrow \cancel{a+b+c=1000=c} \quad c = a+b-1000 \quad (c.)$$

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Plug (c.) into (b.)

$$\Rightarrow a^2 + b^2 = (a+b-1000)^2$$

$$\Rightarrow a^2 + \overset{+ab}{\cancel{ab}} - 1000a + \overset{+ab}{\cancel{ab}} + b^2 - 1000b - 1000a - 1000b + M$$

a milli  
↓

$$\Rightarrow a^2 + b^2 = a^2 + b^2 + M + 2ab - 2000a - 2000b$$

$$\Rightarrow 0 = M + 2ab - 2000a - 2000b$$

I see Simon!

$$\Rightarrow [-2ab + 2000a + 2000b = M] - \frac{1}{2}$$

$$\Rightarrow ab - 1000a - 1000b = -\frac{1}{2}M \quad \text{Simon!}$$

$$\Rightarrow (a-1000)(b-1000) = -\frac{1}{2}M + (-1000)^2$$

$$\Rightarrow [(a-1000)(b-1000) = \frac{1}{2}M] \quad \text{Multiply by } -1 \text{ twice}$$

$$\Rightarrow (1000-a)(1000-b) = \frac{1}{2}M$$

$$\frac{1}{2}M \approx 490,000$$

So... a & b must make our terms,

a & b made in the range of [600-800] ish

(they must sum to a little less than  $2 \cdot 1000 = 1400$ ).

then just search w a program.