Week 1

Arrays

- Accessable by *indexing*
- Continuous area of storage (in most programming langauges)
  - Provides constant-time access to an indexed memory location
- Used at the core of many data structures
- Need to indicate number of elements the array has when initiasing it. **e.g. in Java**

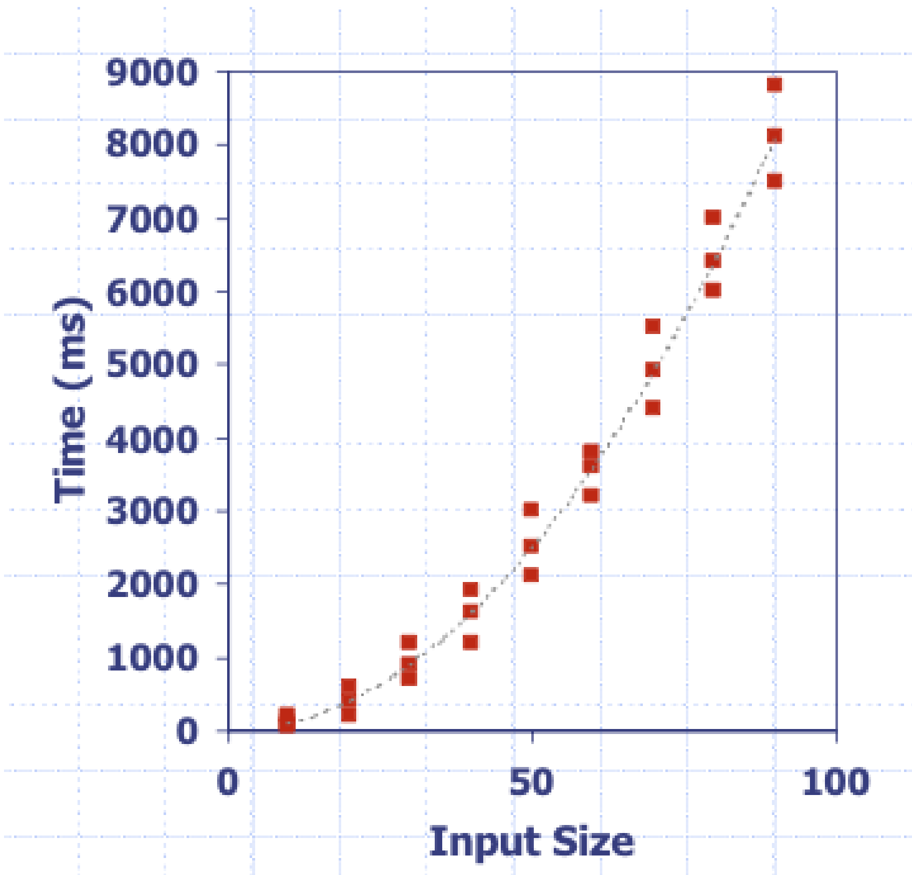
```java
int [] array = new int[4];
```

Algorithm Analysis - Intro

How does an algorithm's runtime and memory usage increase with the size of the input. - Most algorithms transform input data into output data - Running time of an algorithm typically grows with input size - Averagecase time is often difficult to determineq - We focus on the worstcase running time - easier to analyse - crucial for games, finance, robotics, eScience, .. ### Space Usage - Algorithms require space to store intermediate results - Space usage can vary dramatically depending on - algorithm implementation - iteration or recursion - Average-case space usage is often difficult to determine - We also focus on the worst-case space usage - easier to analyse - *crucial* for embedded systems - Data structures need space to hold data elements - We will focus primarily on runtime analysis

Experimental Studies

- Write program implementing the algorithm
- Run program with inputs of varing size and composition
- Use a method lies `System.currentTimeMillis()` to measure actaul running time
- Plot the results



Limitations of Experiments - Need to implement the algorithm - May be difficult - Results may not be indicative of the running time on other inputs not included in the experiment - Comparing algorithms requires the same hardware and software envionments.

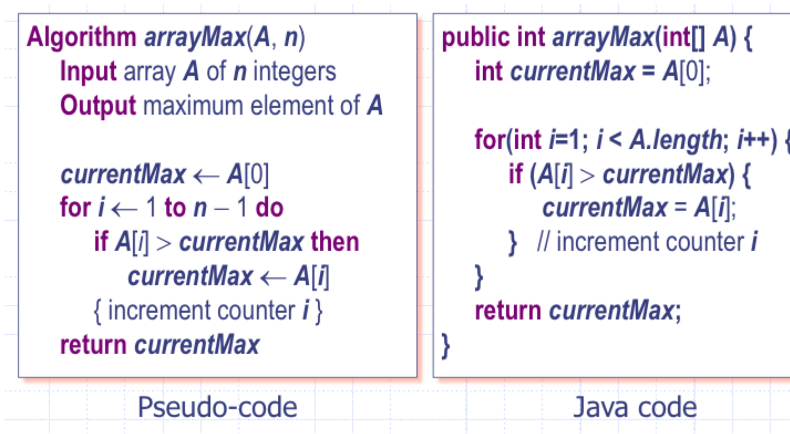
Theoretical Analysis

- Use a high-level description of the algorithm
  - instead of an implementation
- Characterises running time as a function of input size, n
- Tkes into account akk possible inputs
  - at least those that are "bad"
- Evaluation is independent of the hardware and software envionment ### Theoretical Analysis Steps

1. Express algorithm as pseudo-code
2. Count primitive operations
3. Describe algorithm as `f(n)`

- function of n

4. Preform asymptotic analysis
    - express in **asymptotic notation** ## Pseudo-Code



- High level description of an algorithm

Counting Promative Operations

Assignment `int num = 10; // 1 operation` 1. Assigning a value to a variable `int num = A[10]; // 2 operations` 1. Indexing into an array 2. Assigning a value to a variable

Loops

```
while(i < 10) { // 1 operations per iteration + 1
    // operations
}
do {
    // operations
} while(i < 10) // 1 operation per iteration

for(int i = 0; i < n; i++) {        // 1 + (n + 1)
    ... let counted operations = X  // n * X
}                                   // n
```

```
int i = 0   // executed once
i < m       // conditional is evaluated n + 1 times
i++         // increment is evaluated n times
{/*body*/}  // X instructsions within loop are evaluated n times
```

Describe as f(n) - Function of n

- By inspecting the psuedo-code, we can determine the **maximum** number of primatives operations executed by an algotithm as a function of the input size.

```
Algorithm arrayMax(A, n)              # Operations
    currentMax <- A[0]                # 2
    for i <- 1 to n - 1 do            # 1 + 2 * n or 2 + n
        if A[i] > currentMax then     # 2 * (n - 1)
            currentMax <- A[i]        # 2 * (n - 1)
        { increment counter i }       # n - 1
    return currentMax                 # 1
Total:                                # 7 * n [+ lower order terms]
```

- `arrayMax` execute seven and primitive operations in the worst case
  - a = time taken by the fastest primative operation
  - b = time taken by the slowest primative operation
- Let $T(n)$ be the worst time case of `arrayMax`
  - $a\dot{7}n \leq T(n) \leq b\dot{7}n$
- Run time $T(n)$, is bounded by two linear functions
-

Growth Rate

- Seven functions that often appear in algoryth analysis

Name	Math	Found
Constant	≈ 1	
Logarithmic	$\approx \log_2 n$	Searching a sorted list
Linear	$\approx n$	Searching an unsorted list
N-Log-N	$\approx n \log_2 n$	
Quadratic	$\approx n^2$	Nested loops
Cubic	$\approx n^3$	Nested nested loops
Exponential	$\approx 2^n$	Loop where the number of operations doubles each iteration

Effect of growth rate

```
n <- 64
k <- 1
for i <- 0 to n-1 do
    for j <- 0 to k-1 do
        pick()
        { increment counter j }
    k <- k * 2
    { increment counter i }
```

- Suppose pick takes 10-9s to executeq
- This harmless looking loop would take 585 years to runn

– assuming 82 109instructions per secq
– Would still take 7 years if pick takes a single instruction(unrealistic)

Comparision of Two Algorithms

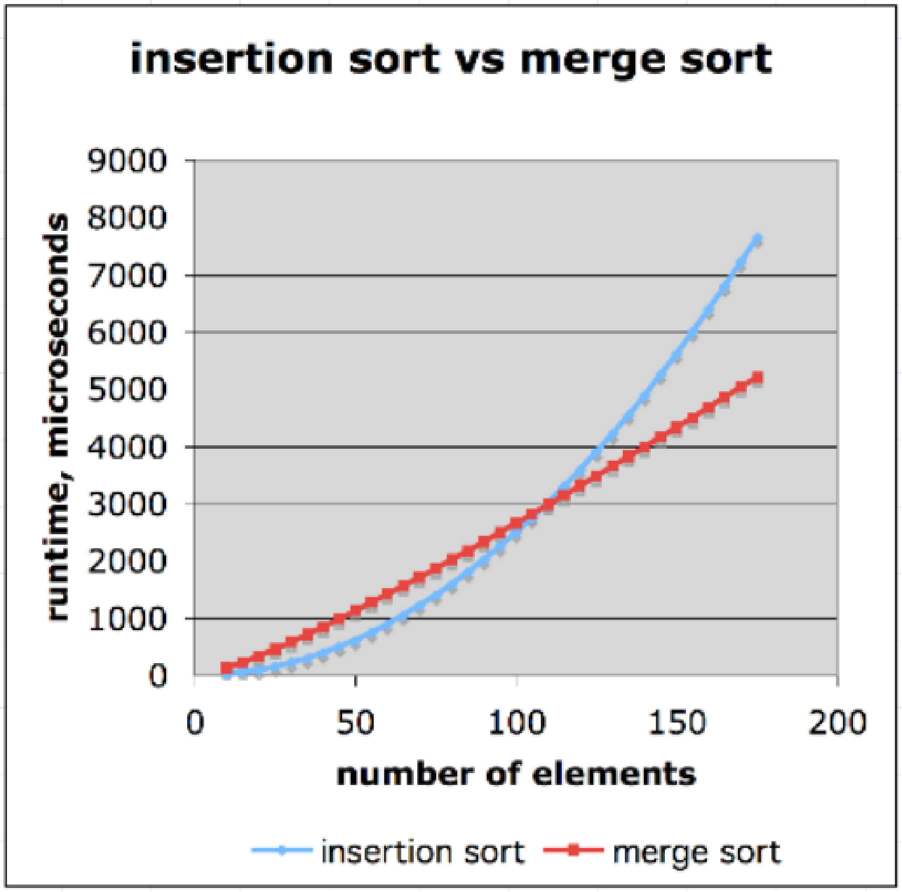


Figure 1: graph comparing insertation sprt and merge sort

Name	Complexity
Insertation sort	$\frac{n^2}{4}$
Merge sort	$2\log_2(n)$

If it takes merge sort 0.5s to sort a list of a million items, it would take insertation sort 40m

Analyisis Process

- To perform an asymptotic analysis of the worst-case running time of an algorithm
  - find the worst-case number of primitive operations executed as a function of the input size f(n)
    - since constant factors and lower-order terms do not affect the growth rate for large n they are usually disregarded when counting primitive operations
  - express this function with big-O notation

What Difference do Lower-Order Terms Make?

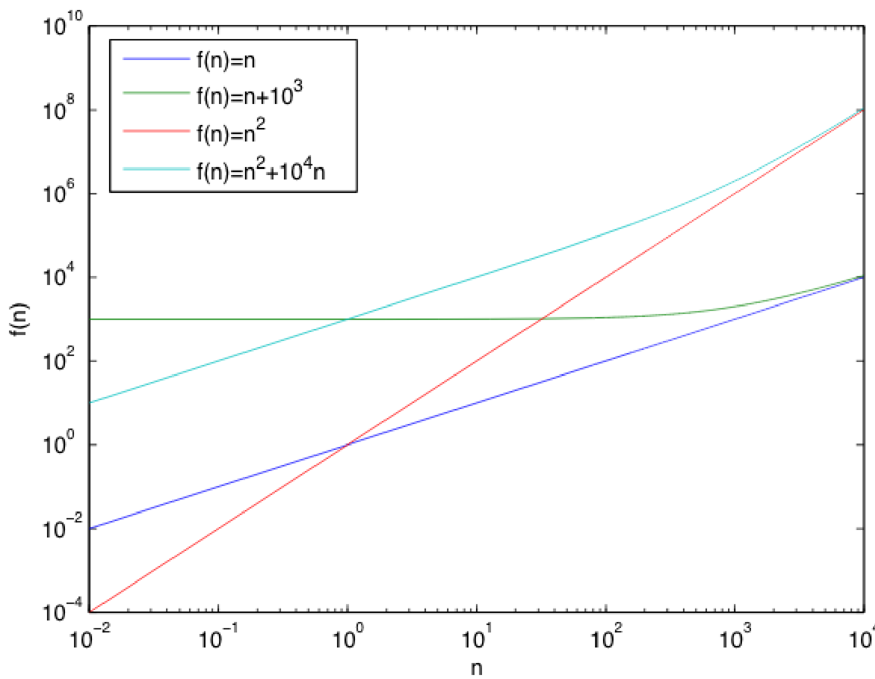


Figure 2: Graph showing show functions with the lower order terms included converge on the ones without them

What Difference Does a Constant Factor Make?

Big-O Notation

- Big-O notation describes an upper bound on a function
- $f(n)$ is $O(g(n))$ if $f(n)$ is asymptotically less than or equal to $g(n)$

Given functions $f(n)$ and $g(n)$, we say that

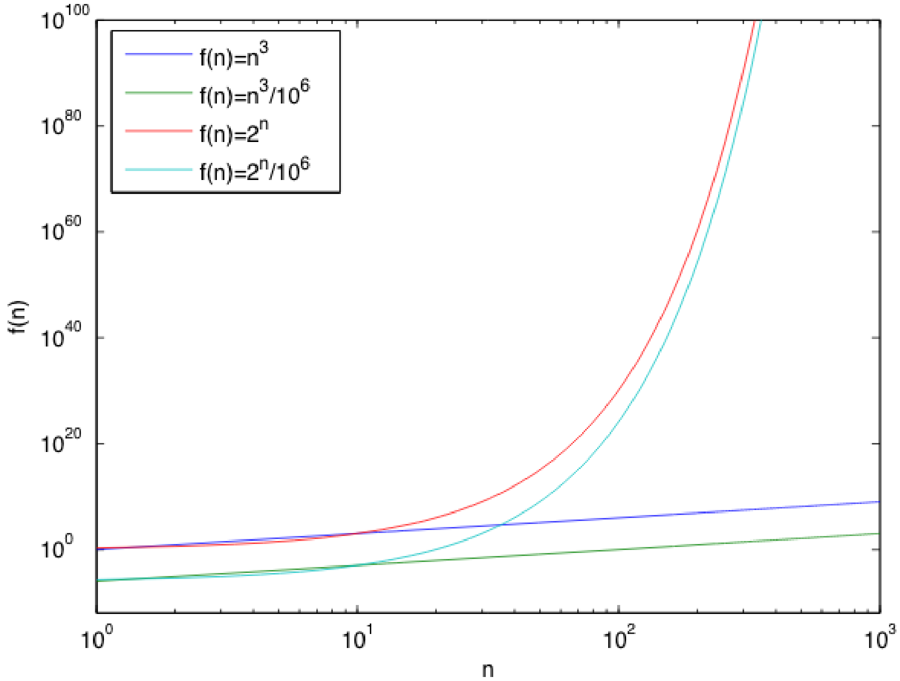


Figure 3: Graph showing converence of constant factor

$$f(n) \text{ is } O(g(n))$$

if there are positive constants c and n_0 such that

$$f(n) \leq c\dot{g}(n) \text{ for } n \geq n_0$$

Big-O and Growth Rate

	$f(n)$is$O(g(n))$	$g(n)$is$O(f(n))$
$g(n)$ grows more	yes	**no**

Big-O Rules

- **Rule 1** - If $f(n)$ is a polynomial of degree d, then $f(n)$ is $O(n^d)$
  - drop lower-order terms
  - drop constant factors (coefficients)

$$\text{e.g. } 3n^4 + 7n^3 + 5 \text{ is } O(n^4)$$

- **Rule 2** - Use the smallest possible class of functions (the "tighest" possible bound)
- "$2n$ is $O(n)$" instead of "$2n$ is $O(n^2)$"

___ Rule 3___ Use the simplest expression of the class - "$3n + 5 \text{is} O(n)$" instead of "$3n + 5 \text{is} O(3n)$"

Big-O Examples

$7n - 2 \text{is} O(n)$ - need c $>$ 0 and n_0 $>=$ 1 such that $7n - 2 <= c \times n$ for $N >= N_0$ true for $c = 7$ and $n_0 = 1$ $3n^3 + 20n^2 + 5 \text{is} O(n^3)$ - need c $>$ 0 and n_0 $>=$ 1 such that $3n^3 + 20n^2 + 5 <= c \times n^3$ for N $>=$ N_0$ true for $c = 4$ and $n_0 = 21$ $3\log(n) + 5 \text{is} O(log(n))$ - need c $>$ 0 and n_0 $>=$ 1 such that $3\log(n) + 5 <= c \times \log(n)$ for $n >= n_0 true for $c = 7$ and $n_0 = 1$