Figure 1: Cylindrical coordinate

As shown in Fig.1, we first attach a Cartesian coordinate x_c, y_c, z_c to the cylinder, these three vectors represent the location and orientation of the cylinder in the world frame. The axis of the cylinder is along the z_c axis. For any point in the cylindrical coordinate, its coordinates are

$$p_c = \begin{bmatrix} r \\ \theta \\ h \end{bmatrix} \tag{1}$$

where r is the radius of that point, θ is the angle between the point and the x_c axis, and h is the height.

We then attach a tangential coordinate frame x_t, y_n, z_t at the point p_c, this tangential coordinate will be used to indicate the orientation of a body at point x_c. x_t is aligned with $\mathbf{r} \times z_c$, where $\mathbf{r}$ is the vector along the radius. y_n is the normal vector of the circle, it is aligned with $\mathbf{r}$, z_t is aligned with z_c.