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Parameter Estimation

#1 $$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-y)^2}{2\sigma^2}}$$

$x_1, x_2, x_3, \dots x_n$ = sample of size n

$$L(x_1, x_2, \dots x_n) = f(x_1) \cdot f(x_2) \dots f(x_n)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-y)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-y)^2}{2\sigma^2}}\right) \dots$$

taking ln on both sides

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i - y)^2}{2\sigma^2}\right) \quad - (1)$$

take partial derivative w.r.t μ of the above equation.

$$\frac{\partial}{\partial \mu} \ln(L) = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i - \mu)}{2\sigma^2}\right) = 0$$

$$= \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

Hence, $\theta_1 = \bar{x}$ is therefore sample mean

Taking derivative w.r.t. σ^2 (of eqn (1))

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i - \mu)^2}{2\sigma^2} = 0$$

$$-n + \sum_{i=1}^{n} -\frac{(x_i - \mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \left(\frac{x_i - \mu)^2}{\sigma^2}\right.$$

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$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} (x_i - \mu)^2\right)$$

Hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$

#2 Binomial Distribution - ${}^{n}C_{x_i} \ \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i} \ \theta^{x_i} (1-\theta)^{n-x_i}$$

Log on both sides

$$\text{Log } L = \sum_{i=1}^{n} \left(\log ({}^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\text{Log } L = \sum_{i=1}^{n} \text{Log}({}^{n}C_{x_i}) + \log \theta \sum_{i=1}^{n} x_i + \log (1-\theta) \sum_{i=1}^{n} (n - x_i)$$

differentiate w.r.t θ

$$\frac{d \text{ Log}(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n - x_i) = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta} \sum x_i \neq \theta$$

$$\theta = \frac{\sum x_i}{n^2}$$