

#Paired t-test prototype  
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#A paired t-test compares two sets of measurements,  
#which are exactly matched for each individual of a population

**Hypotheses:**

$H_0: \text{mean}(x_1) = \text{mean}(x_2)$

$H_1: \text{mean}(x_1) \neq \text{mean}(x_2)$

#Read in data: I'm using "iris," a data set built into r.  
iris

#The following test will compare Sepal.Length between "versicolor" and "virginica"  
#Assign subsets of two equal-length vectors

```
versicolor <- iris$Sepal.Length[iris$Species == "versicolor"]  
virginica <- iris$Sepal.Length[iris$Species == "virginica"]
```

#A paired t-test is a parametric test. Must check the assumptions:

**Assumptions:**

#Observed values  $X_{1,1}, X_{1,2}, X_{1,3}, \dots, X_{1,n}$  are a random sample exactly matched with

#Observed values  $X_{2,1}, X_{2,2}, X_{2,3}, \dots, X_{2,n}$  across individuals 1,2,3, ... ,n.

#Let  $d_i = X_{2,i} - X_{1,i}$  for each individual  $i$  are a random sample from  $\sim N(\mu, \sigma)$ .

#Variance of the population is unknown.

#Note: paired t-test is reasonably robust for deviations from normal distribution.

#verify that data vectors are the same length

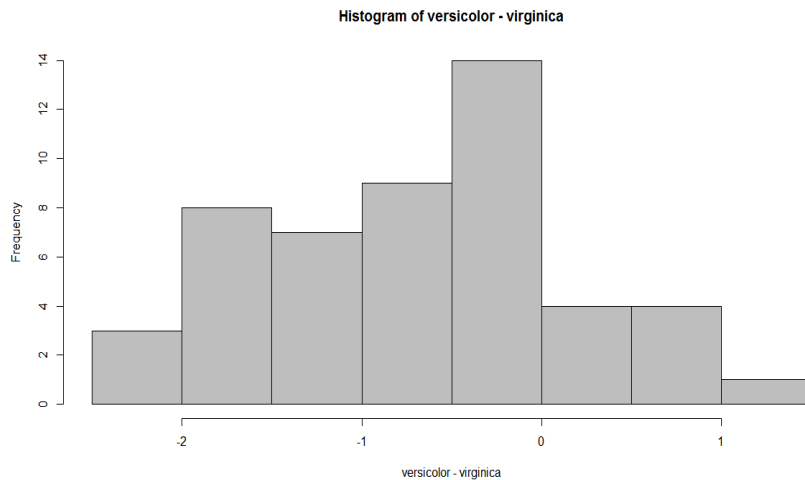
```
length(versicolor)
```

```
[1] 50
```

```
length(virginica)
```

```
[1] 50
```

#Visual confirmation that differences between pairs are normally distributed  
hist(versicolor - virginica, col="grey")



#Below are the manual calculations for paired t-test

#Assign vectors versicolor and virginica to variables "x1" and "x2" respectively.

```
x1 <- versicolor
```

```
x2 <- virginica
```

#assign variable "n" to sample size

```
n <- length(versicolor)
```

#assign mean and standard deviation variables

```
x1bar <- mean(versicolor)
```

```
x2bar <- mean(virginica)
```

```
s1 <- sd(versicolor)
```

```
s2 <- sd(virginica)
```

#dbar is the mean difference between paired datum

```
d <- x1-x2
```

```
dbar <- mean(d)
```

```
s_d <- sqrt(var(d))
```

#Manual calculation of test statistic, "t"

```
t <- dbar/(s_d/sqrt(n))
```

```
t
```

```
[1] -5.275345
```

#Probability (P) value (two sided case)

```
degf <- n-1
```

```
P <- 2 * pt(t, degf)
```

```
P
```

```
[1] 2.989652e-06
```

#Confidence intervals

```
alpha <- 0.05
```

```
#T distribution
```

```
c1 <- qt(alpha/2,degf)
```

```
c1 <- -c1
```

```
#Confidence Intervals
```

```
ci_l <- dbar-c1*(s_d/sqrt(n))
```

```
ci_u <- dbar+c1*(s_d/sqrt(n))
```

```
CI <- c(ci_l, ci_u)
```

```
CI
```

```
[1] -0.900371 -0.403629
```

```
#Run the test using the built in function
```

```
t.test(versicolor,virginica,paired=TRUE, conf.level=0.95)
```

Output:

Paired t-test

**data: versicolor and virginica**

**t = -5.2753, df = 49, p-value = 2.99e-06**

**alternative hypothesis: true difference in means is not equal to 0**

**95 percent confidence interval:**

**-0.900371 -0.403629**

**sample estimates:**

**mean of the differences**

**-0.652**