```r
#Two sample t-test (for populations with UNEQUAL variance) prototype
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#A test for two sets of measurments from samples failing F-test for equal variances.

#Assumptions:
# Observed values x1.1...x1.n are a random sample from a normal distribution.
# Observed values x2.1...x2.n are a random sample from a normal distribution.
# Variances are unequal and unknown
# Both sample are independent.
## Note this test is reasonably robust for deviation from normal distribution.

# Hypotheses:
#1) Null: Mu1 is equal to Mu2.
#2) Alternative: Mu1 is not equal to Mu2 (two sided case)
#		Mu1 < Mu2 (one sided case lower tail)
#		Mu1 > Mu2 (one sided case upper tail)

#Paperwork

#read in data
iris

#assign variables
x1 <- iris$Sepal.Length[iris$Species=="setosa"]
x1

x2 <- iris$Sepal.Length[iris$Species=="versicolor"]
x2

#assign number of observations
n1 <- length(x1)
n1

n2 <- length(x2)
n2

#assign means
x1bar <- mean(x1)
x1bar
x2bar <- mean(x2)
x2bar

#assign standard deviations
s1 <- sqrt(var(x1))
s1
s2 <- sqrt(var(x2))
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s2

#####Test Statistic#####
t <- (x1bar-x2bar)/sqrt((s1^2/n1)+(s2^2/n2)) #t is the normalized mean (x1bar-x2bar)
t
```
[1] -10.52099

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#Satterthwaite's Method Degrees of Freedom:
df_s <- (s1^2/n1+s2^2/n2)^2/(((s1^2/n1)^2/(n1-1))+(s2^2/n2)^2/(n2-1))
df_s
```
[1] 86.538

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#Sampling distribution: if assumptions hold and Null Hypothesis is true, then t~t(df_s)

#Critical Values of the Test  Note: two-tailed CVs are mirror values because of normal distribution
alpha <- 0.05 #probability of type 1 error

c1 <- qt(alpha/2, df_s) #this is the two-sided lower critical value
c1

c2 <- qt(1-alpha/2, df_s)#this is the two-sided higher critical value
c2

abs_c <- abs(c1) #If using two-sided test, use absolute value of c1.

c3 <- qt(alpha, df_s) #one sided case lower CV
c3

c4 <- qt(1-alpha, df_s) #one sided case upper CV
c4

#Decision Rules:
#If abs(t) is > abs_c, then reject Null, otherwise accept Null
#If t < c3, then reject Null ** one sided case lower tail
#If t > c4, then reject Null ** one sided case upper tail

#Probability (P) Value

p1 <- 2*(pt(t, df_s)) #two sided case, if t < or equal to 0
p1
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[1] 3.746743e-17

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p2 <- 2*(1-pt(t, df_s)) #two sided case, if t > 0
p2

p3 <- pt(t, df_s) #one sided case lower tail
p3
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p4 <- 1-pt(t, df_s) #one sided case upper tail
p4

#Confidence Intervals for the Difference in Mean:

#two sided case = (ci_a, ci_b)
ci_a <- x1bar-x2bar-abs_c*sqrt((s1^2/n1)+(s2^2/n2))
ci_a
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[1] -1.105707

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ci_b <- x1bar-x2bar+abs_c*sqrt((s1^2/n1)+(s2^2/n2))
ci_b
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[1] -0.7542926

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ci_l <- x1bar-x2bar-c3*sqrt((s1^2/n1)+(s2^2/n2)) #one sided lower tail
ci_l

ci_u <- x1bar-x2bar-c4*sqrt((s1^2/n1)+(s2^2/n2)) #one sided upper tail
ci_u

# Now, test the built in R function
t.test(x1,x2,alternative = "two.sided",var.equal = FALSE, conf.level = 0.95)
```

Results of Hypothesis Test

Null Hypothesis: difference in means = 0

Alternative Hypothesis: True difference in means is not equal to 0

Test Name: Welch Two Sample t-test

Estimated Parameter(s): mean of x = 5.006
mean of y = 5.936

Data: x1 and x2

Test Statistic: t = -10.52099

Test Statistic Parameter: df = 86.538

P-value: 3.746743e-17

95% Confidence Interval: LCL = -1.1057074
UCL = -0.7542926