

```
#Two sample t-test (for populations with EQUAL variance) prototype
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#2/22/2016
```

```
#A test for two sets of measurements in which the variances are approximately equal.
```

```
#Assumptions:
```

```
# Observed values  $x_{1.1} \dots x_{1.n}$  are a random sample from a normal distribution.
```

```
# Observed values  $x_{2.1} \dots x_{2.n}$  are a random sample from a normal distribution.
```

```
# Variances are approximately equal but unknown
```

```
# Both sample are independent.
```

```
## Note this test is reasonably robust for deviation from normal distribution in the two-tailed case when sample sizes are similar.
```

```
# Hypotheses:
```

```
#1) Null:  $\mu_1$  is equal to  $\mu_2$ .
```

```
#2) Alternative:  $\mu_1$  is not equal to  $\mu_2$ 
```

```
#Paperwork
```

```
#read in data
```

```
iris
```

```
#assign variables
```

```
x1 <- iris$Sepal.Length[iris$Species=="setosa"]
```

```
x1
```

```
x2 <- iris$Sepal.Length[iris$Species=="versicolor"]
```

```
x2
```

```
#assign number of observations
```

```
n1 <- length(x1)
```

```
n1
```

```
n2 <- length(x2)
```

```
n2
```

```
#assign means
```

```
 $\bar{x}_1$  <- mean(x1)
```

```
 $\bar{x}_1$ 
```

```
 $\bar{x}_2$  <- mean(x2)
```

```
 $\bar{x}_2$ 
```

```
#assign standard deviations
```

```
 $s_1$  <- sqrt(var(x1))
```

```
 $s_1$ 
```

```
 $s_2$  <- sqrt(var(x2))
```

```
 $s_2$ 
```

#Pooled Sample Variance

#Both sample variances are pooled and adjusted for differences in sample size).

```
sp <- ((n1-1)*s1^2+(n2-1)*s2^2)/(n1+n2-2)
```

```
sp
```

```
[1] 0.1953408
```

#####Test Statistic#####

```
t <- (x1bar-x2bar)/sqrt(sp*(1/n1+1/n2))
```

```
t
```

```
[1] -10.52099
```

#Critical Values of the Test (probability of type 1 error) Note: two-tailed CVs are mirror values because of normal distribution

```
alpha <- 0.05
```

```
c1 <- qt(alpha/2, n1+n2-2) #this is the two-sided lower critical value
```

```
c1
```

```
[1] -1.984467
```

```
c2 <- qt(1-alpha/2, n1+n2-2)#this is the two-sided higher critical value
```

```
c2
```

```
[1] 1.984467
```

```
abs_c <- abs(c1) #If using two-sided test, use absolute value of c1.
```

```
c3 <- qt(alpha, n1+n2-2) #one sided case lower CV
```

```
c3
```

```
c4 <- qt(1-alpha, n1+n2-2) #one sided case upper CV
```

```
c4
```

#Decision Rules:

#If  $abs(t) > abs\_c$ , then reject Null, otherwise accept Null

#If  $t < c3$ , then reject Null \*\* one sided case lower tail

#If  $t > c4$ , then reject Null \*\* one sided case upper tail

#Probability (P) Value

```
p1 <- 2*pt(t, n1+n2-2) #two sided case, if t < or equal to 0
```

```
p1
```

```
[1] 8.985235e-18
```

```
p2 <- 2*(1-pt(t, n1+n2-2)) #two sided case, if t > 0
```

```
p2
```

```
p3 <- pt(t, n1+n2-2) #one sided case lower tail
```

```
p3
```

```
p4 <- 1-pt(t, n1+n2-2) #one sided case upper tail
```

p4

#Confidence Intervals for the Difference in Mean:

#two sided case = (ci\_a, ci\_b)

ci\_a <- x1bar-x2bar-abs\_c\*sqrt(sp\*(1/n1+1/n2))

ci\_a

**[1] -1.105417**

ci\_b <- x1bar-x2bar+abs\_c\*sqrt(sp\*(1/n1+1/n2))

ci\_b

**[1] -0.7545835**

ci\_l <- x1bar-x2bar-c3\*sqrt(sp\*(1/n1+1/n2)) #one sided lower tail

ci\_l

ci\_u <- x1bar-x2bar-c4\*sqrt(sp\*(1/n1+1/n2)) #one sided upper tail

ci\_u

# Now, the built in R function

t.test(x1,x2,alternative = "two.sided",var.equal = TRUE, conf.level = 0.95)

### Results of Hypothesis Test

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**Null Hypothesis:            difference in means = 0**

**Alternative Hypothesis:    True difference in means is not equal to 0**

**Test Name:                Two Sample t-test**

**Estimated Parameter(s):    mean of x = 5.006**

**mean of y = 5.936**

**Data:                      x1 and x2**

**Test Statistic:            t = -10.52099**

**Test Statistic Parameter:   df = 98**

**P-value:                    8.985235e-18**

**95% Confidence Interval:    LCL = -1.1054165**

**UCL = -0.7545835**