AP Calculus Cheat Sheet*

Derivative rules

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(cf) = cf'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(f+g) = f'+g'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(fg) = f'g+fg'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f}{g}\right) = \frac{f'g-fg'}{g^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} x^n = nx^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f(g(x)) = f'(g(x))\, g'(x)$$

Derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x} a^x = a^x \ln a$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln x = \frac{1}{x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \log_a x = \frac{1}{x\ln a}$$

$$\tfrac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$$

$$\tfrac{\mathrm{d}}{\mathrm{d}x} \cos x = -\sin x$$

$$\tfrac{\mathrm{d}}{\mathrm{d}x} \sec x = \sec x \tan x$$

$$\tfrac{\mathrm{d}}{\mathrm{d}x} \tan x = \sec^2 x$$

$$\tfrac{\mathrm{d}}{\mathrm{d}x} \csc x = -\csc x \cot x$$

$$\tfrac{\mathrm{d}}{\mathrm{d}x} \cot x = -\csc^2 x$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \cot^{-1} x = -\frac{1}{1+x^2}$$

L'Hôpital's rule

If f/g is an indeterminate form, then $\displaystyle\lim_{x\to a}\frac{f}{g} = \lim_{x\to a}\frac{f'}{g'}$.

Summation rules

$$\sum_{i=m}^{n} ca = c\sum_{i=m}^{n} a$$

$$\sum_{i=m}^{n} (a+b) = \sum_{i=m}^{n} a + \sum_{i=m}^{n} b$$

$$\sum_{i=1}^{n} c = nc$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

Integral rules

$$\int_a^b f(x)\,\mathrm{d}x = F(b)-F(a)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_a^x f(t)\,\mathrm{d}t\right] = f(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_g^h f(t)\,\mathrm{d}t\right] = f(h)h' - f(g)g'$$

$$\int cf\,\mathrm{d}x = c\int f\,\mathrm{d}x$$

$$\int (f+g)\,\mathrm{d}x = \int f\,\mathrm{d}x + \int g\,\mathrm{d}x$$

$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C$$

$$\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u$$

Integrals

(See also the derivatives to the left)

$$\textstyle\int \ln x\,\mathrm{d}x = x\ln x - x + C$$

$$\textstyle\int \tan x\,\mathrm{d}x = \ln|\sec x| + C$$

$$\textstyle\int \sec x\,\mathrm{d}x = \ln|\sec x + \tan x| + C$$

$$\textstyle\int \csc x\,\mathrm{d}x = -\ln|\csc x + \cot x| + C$$

$$\textstyle\int \cot x\,\mathrm{d}x = \ln|\sin x| + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,\mathrm{d}x = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2+x^2}\,\mathrm{d}x = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Intermediate value theorem

If $f(x)$ is continuous on $[a,b]$ and n is a number between $f(a)$ and $f(b)$, then there exists $c\in(a,b)$ where $f(c)=n$.

Mean value theorem

If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there exists $c\in(a,b)$ where $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

Polar functions

$$x = r(\theta)\cos\theta$$

$$y = r(\theta)\sin\theta$$

$$\text{area} = \frac{1}{2}\int_{\theta_1}^{\theta_2} (r(\theta))^2\,\mathrm{d}\theta$$

Sequences and series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r},\ |r|<1$$

$|S-S_n| \le |a_{n+1}|$ (alternating series)

Taylor series

(For a Maclaurin series, use $c=0$)

$$t_n = \frac{1}{n!} f^{(n)}(c)\,(x-c)^n$$

$$|R_n(x)| \le \frac{\max\left|f^{(n+1)}(z)\right|\,|x-c|^{n+1}}{(n+1)!}$$

Logarithm laws

$$\ln x + \ln y = \ln(xy)$$

$$\ln x - \ln y = \ln\frac{x}{y}$$

$$y\ln x = \ln x^y$$

Trig identities

$$\sin^2 x + \cos^2 x = 1$$

$$1+\tan^2 x = \sec^2 x$$

$$1+\cot^2 x = \csc^2 x$$

$$\sin(2x) = 2\sin x\cos x$$

$$\begin{aligned}\cos(2x) &= \cos^2 x - \sin^2 x\\ &= 2\cos^2 x - 1\\ &= 1-2\sin^2 x\end{aligned}$$

$$\sin(x\pm y) = \sin x\cos y \pm \cos x \sin y$$

$$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$$