

# AP Calculus Cheat Sheet\*

## Derivative rules

$$\frac{d}{dx}f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}(cf) = cf'$$

$$\frac{d}{dx}(f+g) = f' + g'$$

$$\frac{d}{dx}(fg) = f'g + fg'$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

## Derivatives

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln a$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$$

## L'Hôpital's rule

If  $f/g$  is an indeterminate form, then

$$\lim_{x \rightarrow a} \frac{f}{g} = \lim_{x \rightarrow a} \frac{f'}{g'}$$

## Summation rules

$$\sum_{i=m}^n ca = c \sum_{i=m}^n a$$

$$\sum_{i=m}^n (a+b) = \sum_{i=m}^n a + \sum_{i=m}^n b$$

$$\sum_{i=1}^n c = nc$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left[ \frac{n(n+1)}{2} \right]^2$$

$$\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

## Integral rules

$$\int_a^b f(x) dx = F(b) - F(a)$$

$$\frac{d}{dx} \left[ \int_a^x f(t) dt \right] = f(x)$$

$$\frac{d}{dx} \left[ \int_g^h f(t) dt \right] = f(h)h' - f(g)g'$$

$$\int cf dx = c \int f dx$$

$$\int (f+g) dx = \int f dx + \int g dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int u dv = uv - \int v du$$

## Integrals

(See also the derivatives to the left)

$$\int \ln x dx = x \ln x - x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = -\ln|\csc x + \cot x| + C$$

$$\int \cot x dx = \ln|\sin x| + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

## Intermediate value theorem

If  $f(x)$  is continuous on  $[a, b]$  and  $n$  is a number between  $f(a)$  and  $f(b)$ , then there exists  $c \in (a, b)$  where  $f(c) = n$ .

## Mean value theorem

If  $f(x)$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$ , then there exists  $c \in (a, b)$  where  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

## Polar functions

$$x = r(\theta) \cos \theta$$

$$y = r(\theta) \sin \theta$$

$$\text{area} = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r(\theta))^2 d\theta$$

## Sequences and series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad |r| < 1$$

$$|S - S_n| \leq |a_{n+1}| \quad (\text{alternating series})$$

## Taylor series

(For a Maclaurin series, use  $c = 0$ )

$$t_n = \frac{1}{n!} f^{(n)}(c) (x - c)^n$$

$$|R_n(x)| \leq \frac{\max |f^{(n+1)}(z)| |x - c|^{n+1}}{(n+1)!}$$

## Logarithm laws

$$\ln x + \ln y = \ln(xy)$$

$$\ln x - \ln y = \ln \frac{x}{y}$$

$$y \ln x = \ln x^y$$

## Trig identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \end{aligned}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

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