

Borůvka's Algorithm

Algorithms That Changed the World

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Background

- Developed by Otakar Borůvka in 1926 [1]
- For optimising the electrical network for Moravia
- First minimum spanning tree (MST) algorithm



Principles

- Graph is defined as $G = (V, E)$, connected, undirected, and weighted
- A forest is defined, with each tree containing just one vertex
- The minimum edges connecting each tree to any other is included in the MST
- Any ties must be resolved to prevent cycles
- Trees that are now connected are merged
- This continues until there is one tree in the forest \rightarrow the MST
- Nested loop with halving nature means $O(|E|\log|V|)$

Example

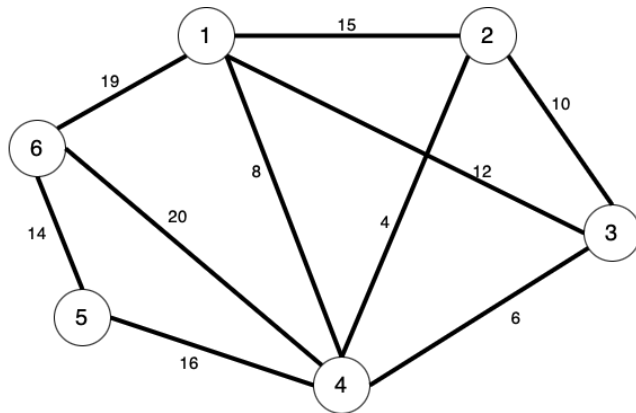


Figure: A graph

Example

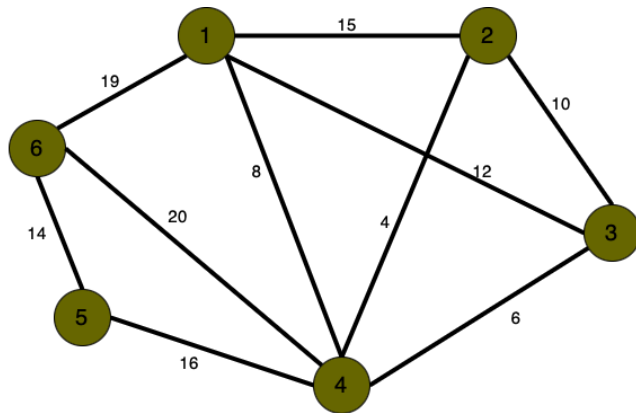


Figure: Initialisation of the Forest

Example

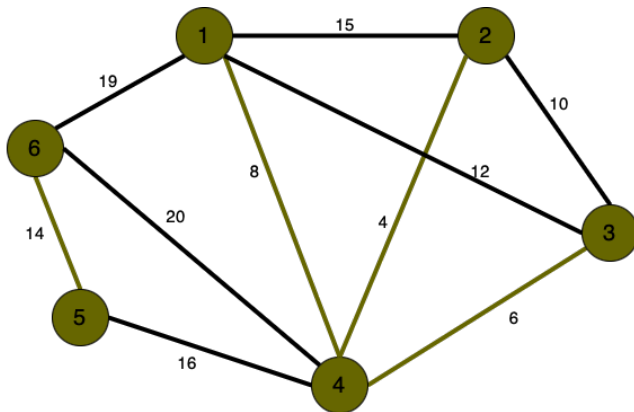


Figure: After first iteration

Example

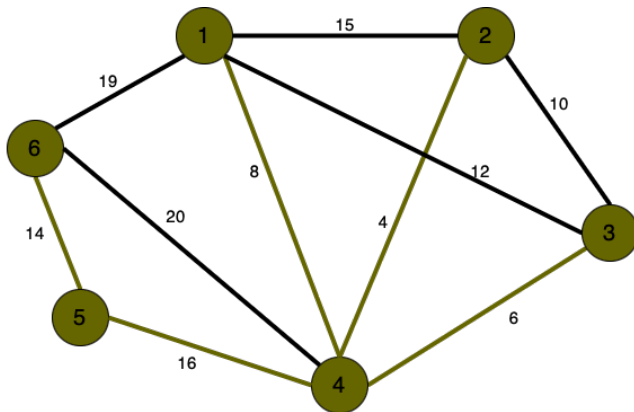


Figure: Final Step

Algorithm Borůvka's Algorithm

Require: $G = (V, E)$ such that G is undirected, weighted, and connected, and $V = \{V_1, V_2, \dots, V_n\}$

Ensure: MST is the minimum spanning tree of G

$F \leftarrow \{(V_1, \emptyset), (V_2, \emptyset), \dots, (V_n, \emptyset)\}$

$MST \leftarrow (V, \emptyset)$

while $|F| > 1$ **do**

$E' \leftarrow \emptyset$

for component T in F **do**

 Find the shortest edge e connecting T to another component T'

$E' \leftarrow E' \cup \{e\}$

end for

 Add $\forall e \in E'$ to MST

 Merge components in F using E'

end while

return MST

The Code

Importance

- The first MST algorithm
- Used for many electrical networks, water supply systems, and transport networks
- Led to development of other algorithms
 - ▶ Parallelised versions [2]
 - ▶ Stochastic version with complexity $O(|E|)$ [3]

References

- [1] Otakar Borůvka. “O jistém problému minimálním”. In: *Práce Mor. Přírodově d. spol. v Brně (Acta Societ. Scient. Natur. Moravicae)* (1926).
- [2] Sun Chung and Anne Condon. “Parallel implementation of Borůvka’s minimum spanning tree algorithm”. In: *Proceedings of International Conference on Parallel Processing*. IEEE. 1996, pp. 302–308.
- [3] David R Karger, Philip N Klein, and Robert E Tarjan. “A randomized linear-time algorithm to find minimum spanning trees”. In: *Journal of the ACM (JACM)* 42.2 (1995), pp. 321–328.

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