Entropy-Guided Gain Scheduling for Low-Bit X-Band Marine Radar HDR Imaging

J. C. Vaught, A. Advisor

***Abstract*—Low-cost navigation radars digitise returns with only 4–8 effective bits; manual gain settings either drown weak echoes in noise or saturate strong scatterers. We introduce an *entropy-guided gain scheduler* that (i) learns four operational regimes—noise-floor, target, information-rich and saturation—from a 20 000-sweep calibration, (ii) selects three gains that maximise Shannon entropy while respecting analogue-gain slew limits, and (iii) fuses the multi-gain stack with exposure-weighted HDR to recover $\geq 12\,$dB of lost dynamic range. Lake-based field trials show a *10 dB* probability-of-detection lift on 0.1 m^2 buoys at P$_{fa} = 10^{-6}$ versus the radar's factory fixed-gain mode, validating the approach under sea-state-2 clutter.**

***Index Terms*—high dynamic range, Shannon entropy, adaptive gain control, marine radar, small-target detection, information theory**

I. Introduction

Small, low-RCS maritime objects are routinely masked by sea-clutter spikes when X-band radars operate at constant gain [?]. Automatic Gain Control (AGC) has existed for decades [?], yet commercial units still require manual trimming and cannot exploit multi-gain HDR stacking. Information-theoretic control promises principled adaptation: early work on waveform design [?] and multi-function radar management [?] demonstrated that maximising (relative) entropy yields near-optimal sensing policies. We extend that idea to analogue gain and show empirically that entropy peaks uniquely identify an "information-rich" zone (30–93% gain) [?].

Our contributions are:

1) Formal derivation of the four entropy zones for low-bit radar ADCs.
2) A greedy entropy-max scheduler with sub-modular ½-optimality bound [?].
3) A real-time Raspberry-Pi implementation (3 µs cell-time) and open NetCDF dataset (A-MARV-Pilot).
4) HDR exposure-fusion pipeline delivering 10 dB Pd gain on lake trials.

II. Related Work

A. Entropy in Radar Processing

Rényi-entropy selective integration in passive 5G radar [?]; information-theoretic waveform scheduling [?].

B. Sensor-Management Theory

Greedy entropy policies with sub-modular bounds [?]; foundational text by Hero [?].

C. HDR / Exposure Fusion

Multi-exposure fusion survey [?]; UltraFusion deep HDR [?]; neural exposure fusion for detection [?].

D. Small-Target Marine Detection

Entropy-feature CFAR for sea clutter [?]; GEV-CFAR for non-stationary clutter [?]; adaptive sea-clutter suppression via gain-tuning [?].

E. Dynamic-Range Recovery & ADC Limits

Modulo-ADC HDR SAR [?]; saturation correction in SAR raw data [?]; bit-depth enhancement survey [?].

Despite these advances, *no prior work couples entropy-based gain zoning with multi-gain HDR fusion on marine radars*.

III. Entropy-Zone Theory

For a discrete return X_G digitised at analogue gain G, Shannon entropy is

$$H(G) = -\sum_k p_k(G)\log_2 p_k(G).$$

Differentiating shows $\frac{dH}{dG} > 0$ until the noise-to-quantisation transition, then < 0 once ADC codes saturate, guaranteeing a single peak in the information-rich zone (proof in Appendix A). Figure **??** illustrates this four-zone structure [?].

IV. Data & Calibration

A. 20 k-Sweep Full-Factorial Design

Five range settings $\times$ 100 gains $\times$ 40 repeats captured at two South Carolina lakes; radar processing features (STC, RezBoost) disabled to isolate gain effects [?].

B. Entropy & SNR Profiles

Figure **??** shows entropy rising steeply at $G \approx 30\%$–93%, while SNR flattens after 40%, confirming the information-rich window [?].

V. Entropy-Guided Gain Scheduler

A sliding 200 ms window builds coarse histograms (1024 bins). The scheduler selects

$$G^* = \arg\max_G \hat{H}(G)$$

and fires three staggered sweeps at $G^*-\Delta$, G^*, $G^*+\Delta$ with $\Delta \approx 25\%$. Sub-modular analysis ensures $\geq 50\%$ of optimal information gain [?]. Latency measurements on the Pi show 3 µs per cell, well under the 40 µs inter-pulse budget.

VI. HDR Fusion Pipeline

Successive gains are range-registered using phase correlation on the 256 strongest FFT bins [?]. Weights are proportional to local entropy, akin to MEF [?]. BM3D (= 200/255) post-filters residual speckle [?].

VII. Experimental Results

A. Detection Benchmarks

Scenario: 0.1 m^2 buoy at 0.5–1.0 NM; sea-state 2. **Baseline:** manufacturer "Auto gain" fixed mode with CA-CFAR. **Metric:** Pd vs Pfa, 1000 Monte-Carlo insertions (Swerling-I).

TABLE I
Detection Performance Comparison

Method	Pd @ Pfa= 10^{-6}	SCR Gain
Fixed gain	0.41	0 dB
Entropy-guided 1-gain	0.62	+4 dB
Entropy-guided 3-gain (HDR)	**0.88**	**+10 dB**

ROC curves (omitted for brevity) show consistent 6–12 dB AUC improvements.

B. Ablation

Removing entropy weighting drops Pd by 12%; two-gain schedules recover only +6 dB SCR, underscoring the three-gain sweet spot.

VIII. Discussion

Entropy maximisation captures both clipping loss and noise-floor masking, outperforming SNR-only control [?]. Ghosting remains at high Δ-gain; future work will integrate neural exposure fusion [?].

IX. Conclusion

We demonstrated the first entropy-zoned gain scheduler for low-bit marine radar, recovering 12 dB dynamic range and +10 dB Pd. The public A-MARV-Pilot dataset and code will be released upon acceptance.

Appendix A
Proof of a Single Entropy Peak

A. Problem Setting and Notation

Consider the analogue radar voltage

$$V = G\,(S+N), \qquad G>0,$$

where S (target+clutter) and N (receiver noise) are independent non-negative r.v.'s with finite variance. The receiver saturates at $V_{\max}$ and is followed by an B-level uniform quantiser $Q(\cdot)$ producing the code

$$C(G) = Q\big(\min\{V, V_{\max}\}\big) \in \{0,1,\dots,B{-}1, B_{\text{sat}}\}.$$

Denote $p_k(G) = \Pr\{C(G)=k\}$ and the discrete Shannon entropy

$$H(G) \;=\; -\sum_k p_k(G)\,\log p_k(G). \qquad (1)$$

B. Behaviour Before Clipping

For gains $0<G<G_c$ (no ADC clipping), $C(G)$ is a pure scale transform of $S+N$. Differential entropy obeys $h_{cont}(G) = h_0 + \log G$ [?]. Quantisation adds a bias $O(\Delta)$ that is independent of G when all codes are populated [?]. Hence $\frac{dH}{dG} = \frac{1}{G} + O(\Delta) > 0$, so $H(G)$ is strictly increasing for $0<G<G_c$.

C. Behaviour After Clipping

Let $p_{\text{sat}}(G) = \Pr\{C(G)=B_{\text{sat}}\} > 0$ for $G>G_c$. Writing $H(G)$ as the sum of clipped and unclipped parts and differentiating gives

$$\frac{dH}{dG} = -(\log p_{\text{sat}}+1)\,\frac{dp_{\text{sat}}}{dG} + (1-p_{\text{sat}})\frac{dH_{\text{noclip}}}{dG}.$$

Because p_{sat} is increasing and $\log p_{\text{sat}} < 0$, the first term is negative. Once $p_{\text{sat}}e^{-1}$ it dominates the $O((1-p_{\text{sat}})/G)$ second term, making $dHdG<0$.

D. Existence and Uniqueness of the Peak

Since $H(G)$ rises for $0<G<G_c$ and falls for $G>\hat{G}$, $dHdG$ has exactly one zero at

$$G^\star = \arg\max_{G>0} H(G),$$

which is a strict (global) maximum because

$$\frac{d^2H}{dG^2} = -\frac{1}{G^{\star 2}} - \frac{dp_{\text{sat}}}{dG}\,\frac{1}{p_{\text{sat}}(1-p_{\text{sat}})} < 0.$$

E. Regularity of the Clipping Probability

Because $S+N$ has a continuous pdf, $p_{\text{sat}}(G) = \int_{V_{\max}/G}^{\infty} f_{S+N}(x)\,dx$ is smooth and strictly increasing for $G>G_c$ [?].

F. Practical Implications

Figure 2 of Vaught's calibration memo highlights the *information-rich* window 30–93% where $H(G)$ is within 3dB of its peak [?]. The single-peak theorem therefore justifies selecting gains inside that window for entropy-guided HDR scheduling.

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