Supplementary: Modelling the emergence of open-ended technological evolution

James Winters[1,*] **Mathieu Charbonneau**[2]

[1]Centre for Culture and Evolution, Brunel University of London

[2]Africa Institute for Research in Economics and Social Sciences, Université Mohammed VI Polytechnique

[*]Corresponding author

July 30, 2025

Abstract

Supplementary material for Winters & Charbonneau (under review). For supplementary with additional code and model runs, see: https://github.com/j-winters/bitw0r1d/blob/main/supplementary/bitw0r1d_overview.ipynb.

1 Extended model description

Bitw0r1d is a general framework for simulating the cultural evolution of technology. Adopting a macroscopic approach, which aims to explain long-term patterns of change using a minimal set of endogenous macro-level properties (Enquist, Ghirlanda and Eriksson, 2011; Mesoudi, 2011), we model the dynamics of technological evolution in terms of two causally interacting processes: one that changes the structure of technological systems ($\mathbb{T}$) and another that changes the structure of the search space ($\mathbb{S}$) (see Figure 1).

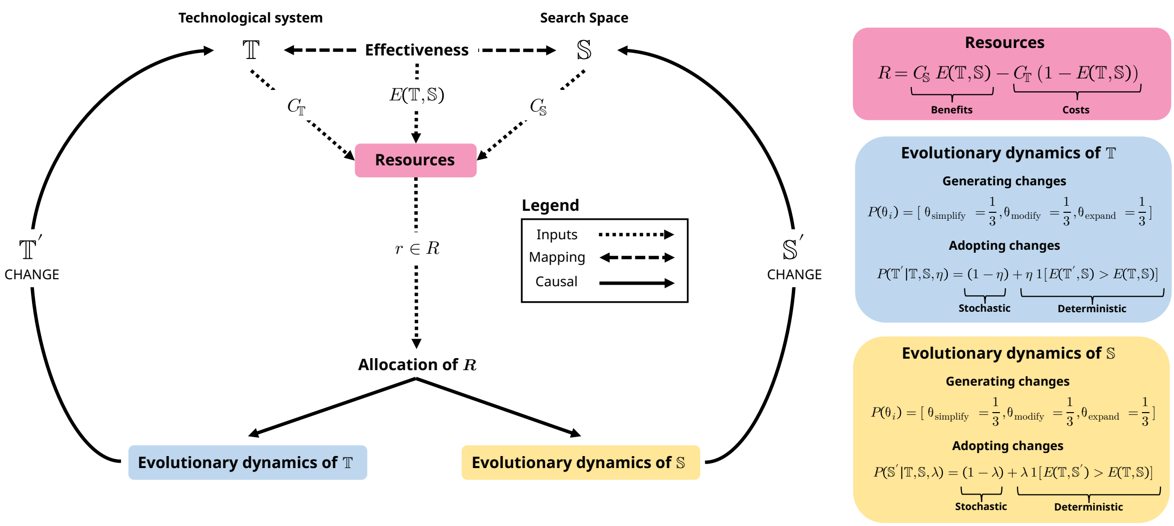

Figure 1: A diagram outlining the general dynamics of bitw0r1d. At each generation, a society needs to find effective mappings between $\mathbb{T}$ and $\mathbb{S}$. The effectiveness of this mapping as well as the complexity of $\mathbb{T}$ ($C_{\mathbb{T}}\}$) and $\mathbb{S}$ ($C_{\mathbb{S}}$) determine the amount of resources (R). These resources are then allocated to two processes: one which changes the technological systems of a society and another which changes the underlying search space. We assume both processes follow a two-step cultural evolutionary dynamic of generating and adopting changes. Our model varies the selection pressures on $\mathbb{T}$ and $\mathbb{S}$ by manipulating the extent to which the adoption of changes are stochastic or deterministic: η controls the selection pressures on $\mathbb{T}$ and λ controls the selection pressures on $\mathbb{S}$.

1.1 Technological systems ($\mathbb{T}$) and search spaces ($\mathbb{S}$)

At the start of a simulation, a society is initialised with a technological system ($\mathbb{T}$) and a search space ($\mathbb{S}$). Both $\mathbb{T}$ and $\mathbb{S}$ are operationalised as bitstrings of N-length. The use of bitstrings is a common feature of many models (Kauffman and Levin, 1987; Banzhaf *et al.*, 2016; Corominas-Murtra, Seoane and Solé, 2018; Winters, 2019). Search spaces represent the aggregate collection of needs, problems and goals facing a society and determine what constitutes an effective system of technologies, whereas whereas technological systems represent the totality of skills, techniques, artifacts and knowledge used to exploit these search spaces.

Modelling $\mathbb{T}$ and $\mathbb{S}$ as bitstrings is useful in this respect because we can represent both technological systems and search spaces as structured entities. Structure refers to how the properties of $\mathbb{T}$ and the properties of $\mathbb{S}$ are organised. In bitw0r1d, this means bitstrings can both vary in the distribution of 0s and 1s (e.g., 0011 versus 1010) and in the length of the string (e.g., 0110 versus 01101001). Taken together, these two properties allow us to measure the distance between $\mathbb{T}$ and $\mathbb{S}$, which we use to approximate effectiveness, $E(\mathbb{T},\mathbb{S})$, and the string length, which serves as a proxy for complexity, $C_{\mathbb{T}}$ and $C_{\mathbb{S}}$.

1.1.1 Effectiveness $E(\mathbb{T}, \mathbb{S})$

Effectiveness directly corresponds to the distance between $\mathbb{T}$ and $\mathbb{S}$. A technology only contributes to effectiveness to the extent it is both integrated within a wider system of technologies (Enquist, Ghirlanda and Eriksson, 2011; Buskell, Enquist and Jansson, 2019) and is useful in addressing the needs, problems and goals of a society (Arthur and Polak, 2006; Coccia and Watts, 2020; Charbonneau, Strachan and Winters, 2023). A car, for instance, is useful insomuch as it fulfils general needs related to transportation, solves specific problems associated with safety and is supported by a system of roads, mechanics and other related infrastructure. A lack of infrastructure (e.g., no road or access to fuel), minimal safety (e.g., engines or tires susceptible to exploding) and no long distance transportation needs (e.g., living on a small island) would greatly diminish the demand for something like a car.

Formally, we measure effectiveness using the Levenshtein distance (Levenshtein, 1975):

$$\mathrm{LD}[\mathbb{T},\mathbb{S}](i,j) = \begin{cases} \max(i,j) & \text{if} \min(i,j) = 0, \\ \min \begin{cases} \mathrm{LD}[\mathbb{T},\mathbb{S}](i-1,j)+1 \\ \mathrm{LD}[\mathbb{T},\mathbb{S}](i,j-1)+1 \\ \mathrm{LD}[\mathbb{T},\mathbb{S}](i-1,j-1)+1_{(\mathbb{T}_i \neq \mathbb{S}_j)} \end{cases} & \text{otherwise} \end{cases} \tag{1.1}$$

where $\mathrm{LD}[\mathbb{S},\mathbb{T}](i,j)$ is the distance between the ith element of $\mathbb{T}$ and the jth element of $\mathbb{S}$. This tells us the minimum number of single-element edits (insertions, deletions or substitutions) required to transform one bitstring into another. Fewer edits approximate a more effective outcome. As $\mathbb{T}$ and $\mathbb{S}$ can differ in lengths, we normalise the Levenshtein distance as follows to get our measure of effectiveness:

$$E(\mathbb{T},\mathbb{S}) = 1 - \frac{\mathrm{LD}[\mathbb{T},\mathbb{S}]}{\max(C_{\mathbb{T}}, C_{\mathbb{S}})} \tag{1.2}$$

where $C_{\mathbb{T}}$ and $C_{\mathbb{S}}$ refer to the string length of $\mathbb{T}$ and $\mathbb{S}$ respectively. We invert the normalised $\mathrm{LD}[\mathbb{T},\mathbb{S}]$ so that $E(\mathbb{T},\mathbb{S}) = 1.0$ represents a maximally effective outcome and $E(\mathbb{T},\mathbb{S}) = 0.0$ represents a maximally non-effective one. Using this metric allows us to formulate the task facing societies as a mapping problem: a society needs to minimise the distance between $\mathbb{T}$ and $\mathbb{S}$ to improve effectiveness. Importantly, this provides a granular way of approaching effectiveness, as different mappings of $\mathbb{T}$ and $\mathbb{S}$ can be more or less distant from one another. For instance, imagine we have $\mathbb{S}$ of 1010 and two possible technological systems, a $\mathbb{T}$ of 0100 and a $\mathbb{T}$ of 0000 — in both cases $\mathbb{T}$ is not maximally effective, in that neither perfectly map onto $\mathbb{S}$, but it is also true that 0100 is closer to optimal than 0000.

1.1.2 Complexity $C_{\mathbb{T}}$ and $C_{\mathbb{S}}$

Defining complexity is infamously a notoriously complex problem (Adami, 2002). For our purposes here, complexity refers to the the string length of technological systems (denoted as $C_{\mathbb{T}}$) and search spaces (denoted as $C_{\mathbb{S}}$). Restricting our definition of complexity to string length allows us to formulate it in terms of computational principles, i.e., the time required (on average) to find a maximally effective outcome given an initial state and an underlying process. Figure 2 provides a demonstration of this basic point by modelling an optimization process where $\mathbb{S}$ is held as a fixed entity, but $\mathbb{T}$ is allowed to vary and follows an incremental two-step process of generating and then adopting a change (see Section 1.3).

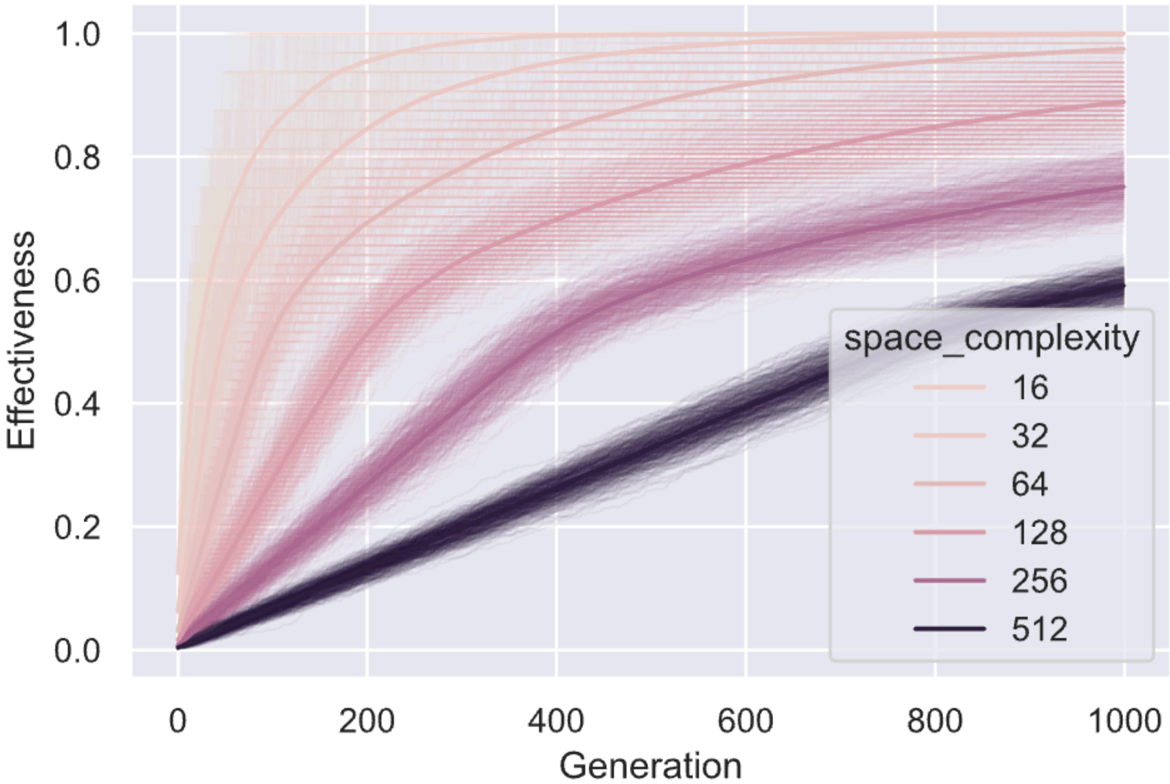

Figure 2: The level of effectiveness $E(\mathbb{T}, \mathbb{S})$ reached for $K_{\text{sim}} = 500$ simulation runs where $\mathbb{T}$ is initialised at $C_{\mathbb{T}} = 2$ and optimized for $\mathbb{S}$ with different levels of complexity, i.e., $C_{\mathbb{S}} \in [16, 32, 64, 128, 256, 512]$.

For these simulation runs, we assume the adoption of changes follows an entirely deterministic hill climbing process, i.e., a change is only adopted if it improves $E(\mathbb{T}, \mathbb{S})$. The number of generations is $G = 1000$, the initial $C_{\mathbb{T}} = 2$ and we manipulate the complexity of the search space ($C_{\mathbb{S}}$). We see that when $C_{\mathbb{T}} = 2$, it takes far longer for the dynamics to reach an optimal outcome as $C_{\mathbb{S}}$ increases. In fact, for $C_{\mathbb{S}} \geq 128$ the dynamics fail to find an optimal outcome on average within 1000 generations. See https://github.com/j-winters/bitw0r1d/blob/main/supplementary/bitw0r1d_overview.ipynb for simulation runs where $C_{\mathbb{S}}$ is initialised with the same complexity as $C_{\mathbb{T}}$.

1.2 Resources (R)

Evolutionary dynamics in our model are both constrained by and responsible for the production of resources. We conceptualise this as a two-step process of first producing and then allocating resources. The production of resources (R) is governed by the interaction between $C_{\mathbb{T}}$, $C_{\mathbb{S}}$ and E (which denotes a simplified notation for effectiveness):

$$R(C_{\mathbb{T}}, C_{\mathbb{S}}, E) = \underbrace{C_{\mathbb{S}}E}_{\text{benefits}} - \underbrace{C_{\mathbb{T}}(1-E)}_{\text{costs}} \tag{1.3}$$

where $C_{\mathbb{S}}E$ represents the benefits of effectively exploiting a search space and $C_{\mathbb{T}}(1-E)$ captures any costs arising from ineffective technological systems. We make two main assumptions here. First, gains in resources are bounded by the complexity of the search space ($C_{\mathbb{T}}$), with $E(\mathbb{T}, \mathbb{S})$ determining how close a society is to realising this resource potential. This assumes more complex search spaces are associated with an increased resource potential. Second, the costs for ineffectiveness, $1 - E(\mathbb{T}, \mathbb{S})$, scale with the complexity of technological systems ($C_{\mathbb{T}}$). As technological systems grow in complexity, a society will incur increased costs if it is maintaining skills, techniques and artifacts that do not effectively contribute to the production of resources.

The amount of resources determine the number of iterations ($i \in I$) at each generation, where a single iteration consists of making changes to $\mathbb{T}$ or making changes to $\mathbb{S}$. We convert resources to an integer as follows:

$$I(R) := \begin{cases} \lfloor R \rceil & \text{if } R \geq 1.0 \\ 1 & \text{if } R_{\text{endow}} > 0.0 \text{ and } R < 1.0 \\ 0 & \text{otherwise.} \end{cases} \tag{1.4}$$

Here, $\lfloor R \rceil$ corresponds to Python's rounding function, and R_{endow} refers to an initial resource endowment that is assigned at the start of a simulation. Introducing an endowment provides a warm up period and mitigates the initial conditions from too strongly influencing the outcomes. Societies only draw upon this endowment when $R \leq 0.0$. If this endowment is exhausted, $R_{\text{endow}} \leq 0$, the dynamics will terminate at the next generation. The reduction of the endowment corresponds to the net loss in resources. For instance, if the resources at generation g is $R(g) = -10$, then the endowment is correspondingly decreased for the next generation $R_{\text{endowment}} = 90$ and the society at $g + 1$ is provided with a single resource, $R = 1$. A special case exists where $R = 0$: we treat this as costly and subtract -1 from the endowment.

Once the resources are converted into iterations, we allocate these to two mutually exclusive processes: one which changes the technological systems of a society and another that changes the underlying search space. As a simplifying assumption, resource allocation remains unbiased, i.e., $P_{\text{allocate}} \sim \text{Bernoulli}(0.5)$. This assumes there is no inherent preference for changing $\mathbb{T}$ or $\mathbb{S}$. As such, the rate of change for technological systems and search spaces is approximately equivalent at each generation.

1.3 Cultural evolutionary dynamics (η and λ)

Cultural evolutionary dynamics are modelled as a two-step process of first generating and then adopting changes to both technological systems ($\mathbb{T}$) and search spaces ($\mathbb{S}$). For the generative component, we assume there are three possible options to change $\mathbb{T}$ or $\mathbb{S}$: to remove a randomly chosen bit (e.g., *remove*: 1011 → 101), to flip a randomly chosen bit to its Boolean complement (e.g, *modify*: 1011 → 1111) or to introduce a new randomly chosen bit at a randomly assigned position (e.g., *expand*: 1011 → 10111). The choice of option always remains unbiased, i.e., $P(\theta_i) = (\theta_{\text{simplify}}, \theta_{\text{modify}}, \theta_{\text{expand}}) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Once a change is introduced, its adoption is contingent on whether the dynamics are stochastic or deterministic. For technological systems, the adoption of a change ($\mathbb{T} \rightarrow \mathbb{T}'$) at a given iteration is underpinned by the following:

$$P(\mathbb{T}'|\mathbb{T},\mathbb{S},\eta) = \underbrace{(1-\eta)}_{\text{stochastic}} + \underbrace{\eta \cdot 1[E(\mathbb{T}',\mathbb{S}) > E(\mathbb{T},\mathbb{S})]}_{\text{deterministic}} \tag{1.5}$$

where $\mathbb{T}$ is the current technological system, $\mathbb{T}'$ represents a proposed change to this system, and $\mathbb{S}$ is the search space. η is a parameter $\in [0,1]$ that allows us to manipulate the probability a change is adopted stochastically or deterministically.

If $\eta = 0.0$, then the evolution of the system is purely stochastic and reduces to $P(\mathbb{T}' \mid \mathbb{T},\mathbb{S},\eta) = (1-\eta) = 1.0$, i.e., a change is adopted irrespective of whether or not it increases effectiveness. Conversely, for $\eta = 1.0$, the evolutionary dynamics are purely deterministic and the outcome depends on the indicator function $1[\cdot]$: here, if $\mathbb{T}'$ is more effective for exploiting a search space, as denoted by $E(\mathbb{T}',\mathbb{S}) > E(\mathbb{T},\mathbb{S})$, then $\mathbb{T}'$ is adopted as the new state of the system at the next iteration. Otherwise, if $E(\mathbb{T}',\mathbb{S}) < E(\mathbb{T},\mathbb{S})$ a society remains with the existing state of $\mathbb{T}$.

Intermediate values, $\eta = (0.0, 1.0)$, incorporate some mixture of deterministic and stochastic factors into the dynamics of adopting changes. As such, the structure of technological systems will to some extent reflect both randomly adopted changes as well as changes selected on the basis of improving effectiveness. Modelling the dynamics in this way assumes the adoption of changes is endogenous and incremental. A technological system culturally adapts insomuch as the adoption of changes helps address the needs, problems and goals of a society. Parallel dynamics hold for the evolution of search spaces:

$$P(\mathbb{S}'|\mathbb{T},\mathbb{S},\eta) = \underbrace{(1-\lambda)}_{\text{stochastic}} + \underbrace{\lambda \cdot 1[E(\mathbb{T},\mathbb{S}') > E(\mathbb{T},\mathbb{S})]}_{\text{deterministic}} \tag{1.6}$$

except λ now controls the stochastic and deterministic forces acting upon the adoption of changes to a search space ($\mathbb{S}$). Values of $\lambda \rightarrow 0.0$ increasingly adopt random changes to the search space, whereas values of $\lambda \rightarrow 1.0$ increasingly evaluate and adopt changes to $\mathbb{S}$ that improve effectiveness. This means that search spaces culturally evolve by changing the needs, problems and goals of a society. Some of these changes are random and others are selected when $\lambda \in (0.0, 1.0]$. A search space culturally adapts by adopting changes that improve effectiveness, i.e., the needs, problems and goals are restructured to more effectively map onto the existing technological capabilities of a society.

We can think of different parameter values for $\eta \in [0, 1]$ and $\lambda \in [0, 1]$ as determining the extent to which technological systems and search spaces are coupled to one another and capable of co-evolution. Special cases hold at the extremes of the parameter space: purely stochastic dynamics ($\eta = \lambda = 0.0$) mean that technological systems and search spaces are solely driven by random changes and evolve independently of one another, while purely deterministic dynamics ($\eta = \lambda = 1.0$) are entirely driven by selection for effectiveness and constitute a perfect co-evolutionary relationship (see Section 2.2 for more details).

1.4 Simulation runs

For the reported simulation runs in the main paper, societies are initialised with randomly sampled technological systems and search spaces of $C_{\mathbb{T}} = C_{\mathbb{S}} = 2$ (minimum level of complexity is $C = 1$). Parameter combinations of $\eta \in (0, 1)$ and $\lambda \in (0, 1)$ consisted of $K_{\text{sim}} = 1000$ simulations. A simulation halted under one of three conditions: (i) the maximum number of generations was reached ($\max_g = 10,000$), (ii) the upper-bound of technological complexity was either matched or exceeded ($\max_{\{C_{\mathbb{S}}\}} \geq 10,000$), or (iii) societies hit an absorbing barrier having exhausted their resources ($R \leq 0$). For (iii), we assume that absorbing barriers only come into effect if societies have drawn down an initial resource endowment (R_{endow}).

2 Extended results

2.1 Summaries of terminal mean values

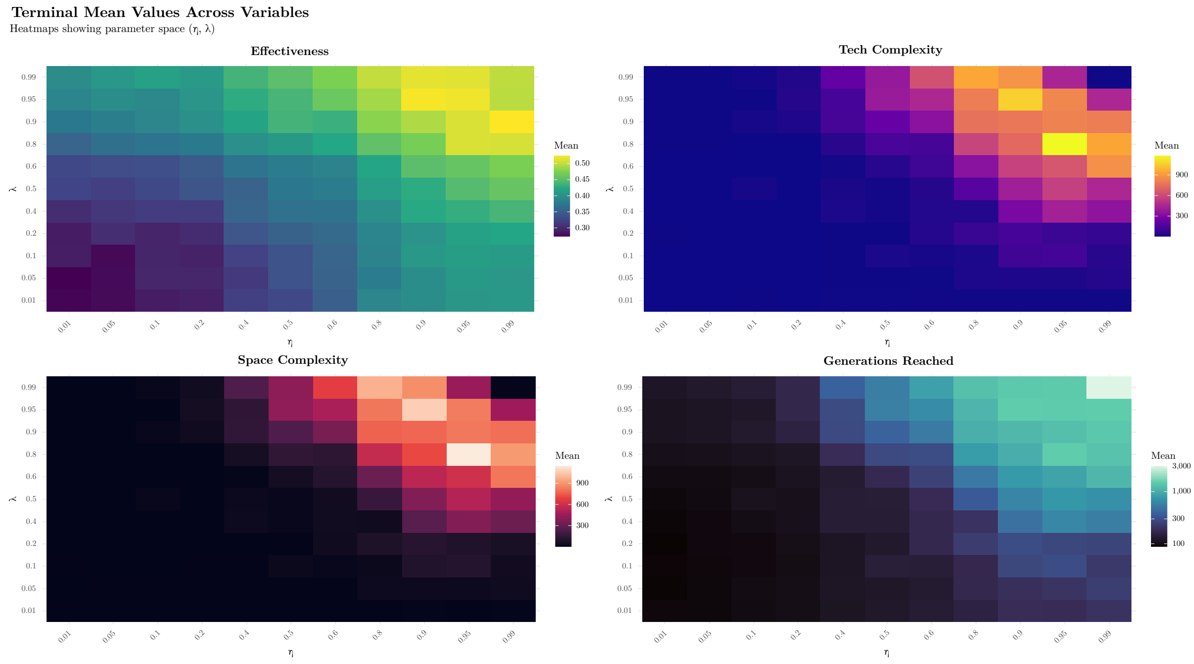

Figure 3: Heatmaps showing mean values at simulation termination for $E(\mathbb{T}, \mathbb{S})$, $C_{\mathbb{T}}$, $C_{\mathbb{S}}$, and generations reached. The patterns reveal that open-ended growth requires moderate-to-strong selection pressures on both technological systems and search spaces.

Figure 3 presents a series of heatmaps showing the mean values for $E(\mathbb{T}, \mathbb{S})$, $C_{\mathbb{T}}$, $C_{\mathbb{S}}$, and number of generations reached for parameters combinations of η and λ. See https://github.com/j-winters/bitw0r1d/blob/main/data/summaries.csv for summary statistics of other moments, i.e., variance, skewness and kurtosis. Each mean was calculated at the point of termination for a given simulation run (either due to an absorbing barrier, reaching the full 10,000 generations or exceeding the upper-limit for complexity of $\mathbb{T}$, i.e., $C_{\mathbb{T}} \geq 10,000$).

- $E(\mathbb{T}, \mathbb{S})$: Mean terminal effectiveness shows a clear gradient from low values in the bottom-left corner where both parameters are highly stochastic ($\eta \approx \lambda \approx 0.01$) to moderately higher values in regions where at least one parameter exhibits a strong selection pressure (e.g., $\eta = 0.99$). The highest effectiveness occurs when both parameters are strongly deterministic.
- $C_{\mathbb{T}}$: Mean complexity of technological systems shows a concentrated region of high values centered around $\eta = 0.8 - 0.9$ and $\lambda = 0.8 - 0.9$. Outside this region complexity drops dramatically to very low values. This pattern indicates that moderate-to-strong selection pressures on both technological systems and search spaces are necessary for sustained growth in complexity, but at the extreme growth is inhibited ($\eta = \lambda = 0.99$).
- $C_{\mathbb{S}}$: Search space complexity follows a similar pattern to $C_{\mathbb{T}}$, with a concentrated hotspot in the upper-right region of the parameter space.

- Generations Reached: The mean number of generations before termination shows that the probability of survival is maximised as both parameters approach the extreme of $\eta = \lambda = 0.99$ (with a mean of ≈ 3060 generations). A key point here is that regions with the longest survival are not strictly synonymous with $C_{\mathbb{T}}$ and $C_{\mathbb{S}}$, although the two are highly correlated with one another.

2.2 Extremes of the parameter space

Special cases hold at the extremes of the parameter space: purely stochastic dynamics ($\eta = \lambda = 0.0$) mean that technological systems and search spaces are solely driven by random changes and evolve independently of one another, while purely deterministic dynamics ($\eta = \lambda = 1.0$) are entirely driven by selection for effectiveness and constitute a perfect co-evolutionary relationship. See Figure 4 for simulation runs showing $C_{\mathbb{T}}$ with different parameter combinations of η and λ at these extremes.

Figure 4: Simulation runs for $C_{\mathbb{T}}$ across different parameter combinations of η and λ ($K_{\text{sim}} = 1000$ per combination). *Top left*: $\eta = \lambda = 0.0$, *Top right*: $\eta = \lambda = 1.0$, *Bottom left*: $\eta = 1.0; \lambda = 0.0$, *Bottom right*: $\eta = 0.0; \lambda = 1.0$

- Purely stochastic runs ($\eta = \lambda = 0.0$) are restricted to relatively simple $C_{\mathbb{T}}$ and fail to reach the full $10,000$ generations. Without selection mechanisms to maintain effectiveness, the co-evolutionary dynamics are extremely volatile and collapse into an absorbing barrier relatively quickly. This condition represents a theoretical baseline for why sustainable co-evolutionary dynamics in our model require some selection for effectiveness.

- When $\mathbb{T}$ is purely deterministic ($\eta = 1.0$) and $\mathbb{S}$ is purely stochastic ($\lambda = 0.0$), we observe a long-tailed distribution of simulation runs: the majority collapse into an absorbing barrier, but a small number survive for significantly longer (with a single run reaching the end of the simulation). This demonstrates that maximally powerful selection pressures on technological systems can overcome the stochastic perturbations of randomly evolving search spaces. Strong selection pressure on technological systems allow the dynamics to effectively track and exploit randomly changing search spaces, leading to both high effectiveness and the potential for sustained growth in complexity (albeit noisy).
- Inverting the pressures so that $\eta = 0.0$ and $\lambda = 1.0$ does not lead to comparable dynamics. Instead, the long-term survivability is greatly diminished, with zero runs reaching the end of the simulation and only one run was capable of exceeding 1750 generations. This links up with the general observations in our paper about an asymmetry between η and λ.
- In a perfectly deterministic scenario ($\eta = \lambda = 1.0$), we see that a significant portion of runs very quickly converge to a stable state and essentially halt for the remainder of the simulation. This traps societies with technological systems of relatively low complexity ($\max_{\{C_{\mathbb{T}}\}} = 4$). As discussed in the main paper (for $\eta = \lambda = 0.99$), we can think of this as a type of emergent local optima where $\mathbb{T}$ and $\mathbb{S}$ rapidly co-evolve toward one another and lead to long-term stasis. However, unlike $\eta = \lambda = 0.99$, the absence of stochastic perturbations prevents escape from these equilibria.

In general, the comparison between $\eta = 1.0, \lambda = 0.0$ and $\eta = 0.0, \lambda = 1.0$ reveals a fundamental asymmetry in the model. Having strong selection pressures on technological systems ($\eta = 1.0$ and $\lambda = 0.0$) is more effective at sustaining long-term co-evolution than the inverse scenario ($\eta = 0.0$ and $\lambda = 1.0$). The extreme conditions demonstrate that neither pure randomness ($\eta = \lambda = 0.0$) nor perfect selection ($\eta = \lambda = 1.0$) alone can sustain open-ended evolution. Purely deterministic dynamics leads to stasis while purely stochastic dynamics results in collapse. The highly curtailed sustainability of purely stochastic runs highlights the critical role of resource constraints in shaping co-evolutionary trajectories. Without a sustained production of resources, societies rapidly hit absorbing barriers even though they are provided with a generous initial resource endowment ($R_{\text{endow}} = 100$). Finally, for the minority of runs that survive for extended periods (see $\eta = 1.0; \lambda = 0.0$), gains in complexity can enhance long-term sustainability when coupled with strong selection. Complex technological systems are more resilient to individual stochastic perturbations in search spaces because any single change represents a proportionally smaller disruption to overall effectiveness.

2.3 Different initial resource endowments (R_{endow})

In the main paper, we only considered resources endowments of $R_{\text{endow}} = 100$. However, we can consider a series of counterfactual scenarios with lower resource endowments, which provides us with information about the survivability of the same simulation runs under different initial endowments (for R code, see: https://github.com/j-winters/bitw0r1d/blob/main/supplementary/counterfactual.R). Below, we systematically vary initial endowments across five levels, $R_{\text{endow}} \in [1, 25, 50, 75, 100]$, and produce heatmaps for $C_{\mathbb{T}}$ at point of termination (see Figure 5) and mean number of generations reached (see Figure 6).

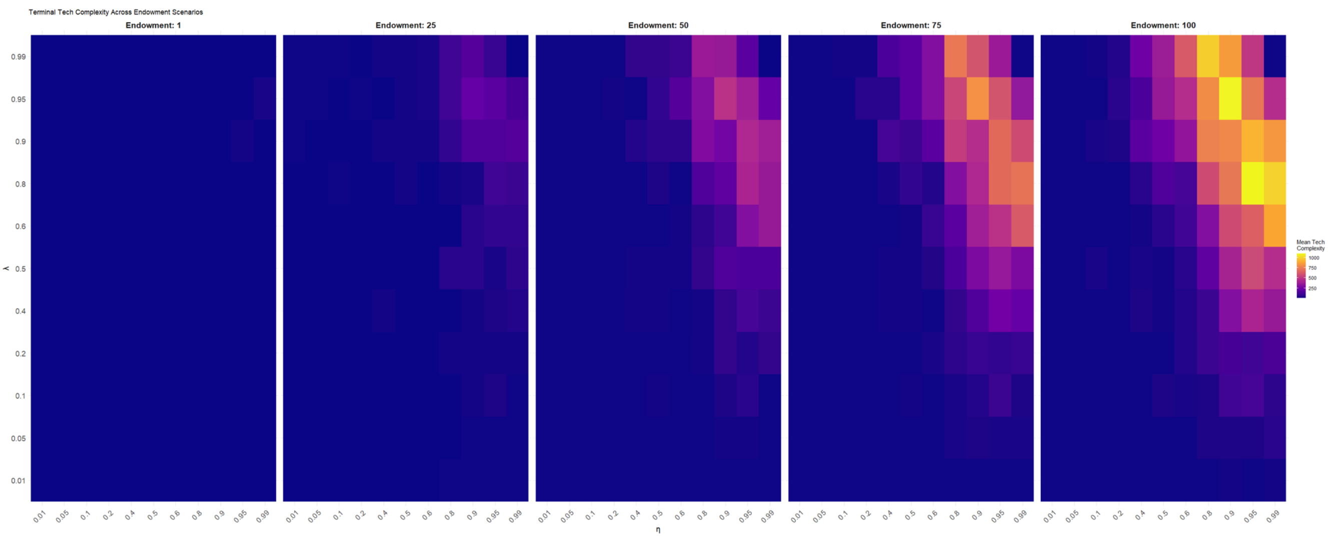

Figure 5: Heatmaps showing mean complexity of technological systems achieved at termination points of simulation runs across different initial resource endowments. Each panel represents the parameter space with η (selection pressure on technological systems) on the x-axis and λ (selection pressure on search spaces) on the y-axis. Color intensity indicates mean technological complexity, with darker blue representing lower complexity and brighter yellow representing higher complexity.

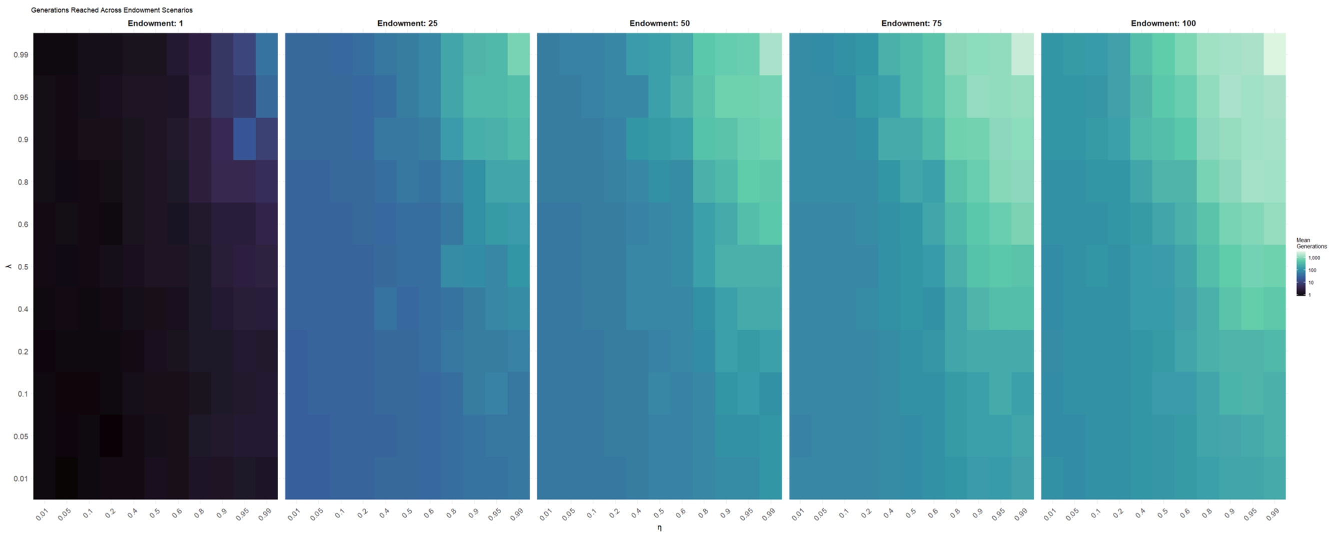

Figure 6: Heatmaps showing mean generations survived before termination across different initial resource endowments. Each panel represents the parameter space with η (selection pressure on technological systems) on the x-axis and λ (selection pressure on search spaces) on the y-axis. Color intensity indicates mean survival duration, with darker colors representing shorter survival and brighter colors representing longer survival.

Under severely constrained initial conditions ($R_{\text{endow}} = 1$), the co-evolutionary dynamics collapse across virtually the entire parameter space. Both technological complexity and survival times remain uniformly low regardless of η and λ. Only when selection pressures are extremely powerful, i.e., $\eta \to 1.0$ and $\lambda \to 1.0$, do we observe instances of sustainable co-evolutionary dynamics in the long-term. On average, increasing the initial endowment generally leads to higher levels of sustainability and greater levels of complexity.

Bibliography

Adami, C. (2002) "What is complexity?," *BioEssays*, 24(12), pp. 1085–1094.

Arthur, W.B. and Polak, W. (2006) "The evolution of technology within a simple computer model," *Complexity*, 11(5), pp. 23–31. Available at: https://doi.org/10.1002/cplx.20130.

Banzhaf, W. *et al.* (2016) "Defining and simulating open-ended novelty: requirements, guidelines, and challenges," *Theory in Biosciences*, 135(3), pp. 131–161. Available at: https://doi.org/10.1007/s12064-016-0229-7.

Buskell, A., Enquist, M. and Jansson, F. (2019) "A systems approach to cultural evolution," *Palgrave Communications*, 5(1).

Charbonneau, Strachan, J. and Winters, J. (2023) "The problem with solutions: A commentary of "Blind alleys and fruitful pathways in the comparative study of cultural cognition" by Andrew

Whiten," *Physics of Life Reviews*, 44, pp. 61–63. Available at: https://doi.org/10.1016/j.plrev.2022.11.012.

Coccia, M. and Watts, J. (2020) "A theory of the evolution of technology: Technological parasitism and the implications for innovation magement," *Journal of Engineering and Technology Management*, 55, p. 101552.

Corominas-Murtra, B., Seoane, L. and Solé, R. (2018) "Zipf's law, unbounded complexity and open-ended evolution," *arXiv:1612.01605 [physics, q-bio]* [Preprint]. Available at: http://arxiv.org/abs/1612.01605 (Accessed: January 18, 2022).

Enquist, M., Ghirlanda, S. and Eriksson, K. (2011) "Modelling the evolution and diversity of cumulative culture," *Philosophical Transactions of the Royal Society B: Biological Sciences*, 366(1563), pp. 412–423. Available at: https://doi.org/10.1098/rstb.2010.0132.

Kauffman, S. and Levin, S. (1987) "Towards a general theory of adaptive walks on rugged landscapes," *Journal of theoretical Biology*, 128(1), pp. 11–45.

Levenshtein, V.I. (1975) "On the minimal redundancy of binary error-correcting codes," *Information and Control*, 28(4), pp. 268–291.

Mesoudi, A. (2011) "Variable Cultural Acquisition Costs Constrain Cumulative Cultural Evolution," *PLoS ONE*. Edited by R. Bentley, 6(3), p. e18239. Available at: https://doi.org/10.1371/journal.pone.0018239.

Winters, J. (2019) "Escaping optimization traps: the role of cultural adaptation and cultural exaptation in facilitating open-ended cumulative dynamics," *Palgrave Communications*, 5(1), p. 149. Available at: https://doi.org/10.1057/s41599-019-0361-3.