

Derivatives Markets: Advanced Modeling and Strategies

Cheat sheet for MITx 15.435x Derivatives Markets: Advanced Modeling and Strategies.

Week 1: Forward Contracts

Forward contract basics

Forward Contract

- A **forward contract** is an agreement between two counterparties to trade a pre-specified amount of goods or securities at a pre-specified future date, T , for a pre-specified price, F_0 .
- The **Profit/Loss (P/L)** at the contract maturity T for each counterparty is:
 $P/L_{long} = N(S_T - F_0)$, $P/L_{short} = N(F_0 - S_T)$
- Price of a zero coupon bond with face value Z : $P = e^{-rT} Z$
 $f(0, T_1, T_2)$ denotes the **forward rate** between time T_1 and T_2 , as of time 0:
 $f(0, T_1, T_2) = \frac{T_2 r_{T_2} - T_1 r_{T_1}}{T_2 - T_1}$
- Long forward positions are equivalent to borrowing and going long in the underlying asset
- Forward short positions are equivalent to lending and going short the underlying

Pricing formulas

Pricing formulas

- An **arbitrage opportunity** is a trading strategy that either (1) Yields a positive profit today, and zero cash flows in the future; or (2) Costs nothing today and yields a positive profit in the future
- The Law of One Price:** Securities with identical payoffs must have the same price
- Stock** with known dividend D at time $t < T$: $F_0 = (P_{S,0} - De^{-rt})e^{rT}$
Stock with known dividend yield q : $F_0 = P_{S,0}e^{(r-q)T}$
- Bond** with coupon C at time $t < T$: $F_0 = (P_{B,0} - Ce^{-rt})e^{rT}$
- Currencies.** $r_{\$}$ ($r_{\text{€}}$) the USD (EUR) risk-free rate. S_t is the exchange rate (USD per EUR) at time t : $F_0 = S_0 e^{(r_{\$} - r_{\text{€}})T}$

Forward prices for commodities

- Forward price with lump-sum storage cost U : $F_{0,T} = (S_0 + PV(U))e^{rT}$
- Forward price with proportional storage cost u : $F_{0,T} = S_0 e^{(r+u)T}$
- Forward price with convenience yield y : $F_{0,T} = S_0 e^{(r-y)T}$
- Forward price with proportional storage cost and convenience yield:
 $F_{0,T} = S_0 e^{(r+u-y)T}$
- Contango** is a pattern of forward prices that increases with contract maturity
- Backwardation** is a pattern of forward prices over time that decreases with contract maturity

Key concepts for hedging and speculating

Valuing a forward contract over time

- Suppose that $K = F_0$ the original delivery price, initial value of contract $f_0 = 0$.
- Value of a **long** forward contract at time t : $f_{long,t,T} = (F_t - K)e^{-r(T-t)}$
- Value of a **short** forward contract at time t : $f_{short,t,T} = (K - F_t)e^{-r(T-t)}$
- Basis** is the difference between the spot and forward price of a security or commodity.

- Cross-hedging** involves using a contract type to hedge which differs from the security or commodity being hedged.
- The **hedge ratio** is the relative number of forward contracts to units of the asset being hedged that maximizes the effectiveness of the hedge:
 $N_S \mathbb{E}[dS] = N_F \mathbb{E}[dF]$ then: $\frac{N_S}{N_F} = \frac{\mathbb{E}[dF]}{\mathbb{E}[dS]}$. If long in spot then short in forwards, and vice versa.

Recommended Resources

- MITx 15.435x Derivatives Markets: Advanced Modeling and Strategies
Lecture Slides
- John Hull's, Options Futures and Other Derivatives, 10th edition
- Bruce Tuckman and Angel Serrat, Fixed Income Securities; Tools for Today's
Markets, 3rd Edition (BTAS)
- LaTeX File (github.com/j053g/cheatsheets/15.435x)

Last Updated October 7, 2021