# **Mathematical Methods for Quantitative Finance**

Cheat sheet for MITx 15.455x Mathematical Methods for Quantitative Finance.

## Week 5: Continuous-Time Finance (contd.)

# **Black-Scholes equation**

#### Summary of some key formulas

- Itô process: dX = adt + bdB
- Itô formula:

$$\begin{split} \mathrm{d}F &= \frac{\partial F}{\partial t} \mathrm{d}t + \frac{\partial^2 F}{\partial X^2} \mathrm{d}X + \frac{b^2}{2} \frac{\partial F}{\partial X} \mathrm{d}t \\ &= \left( \frac{\partial F}{\partial t} + a \frac{\partial F}{\partial X} + \frac{b^2}{2} \frac{\partial^2 F}{\partial X^2} \right) \mathrm{d}t + b \frac{\partial F}{\partial X} \mathrm{d}B \end{split}$$

- • Stock price: d $S = \mu S \mathrm{d}t + \sigma \mathrm{d}B \implies \mathrm{d}(\log S) = \left(\mu - \frac{\sigma^2}{2}\right) \mathrm{d}t + \sigma \mathrm{d}B$
- Black-Scholes:  $\Delta = \partial V/\partial S$  ,  $\mathrm{d}\pi = r\pi\mathrm{d}t$  ,

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

## **Recommended Resources**

- MITx 15.455x MITx 15.455x Mathematical Methods for Quantitative Finance [Lecture Slides]
  - (https://learning.edx.org/course/course-v1:MITx+15.455x+3T2020/home)
- Tsay, Analysis of Financial Time Series (3e), Wiley. (Tsay)
- Capinski and Zastawniak, Mathematics for Finance, Springer. (CZ)
- Olver, Introduction to Partial Differential Equations (2016), Springer. (Olver)
- Campbell, Lo, and MacKinlay, Econometrics of Financial Markets (1997), Princeton. (CLM)
- Lang, Introduction to Linear Algebra (2e), Springer (Lang)
- Axler, Linear Algebra Done Right (3e), Springer (Axler)
- LaTeX File (github.com/j053g/cheatsheets/15.455x)

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