Derivatives Markets: Advanced Modeling and Strategies

Cheat sheet for MITx 15.453x Derivatives Markets: Advanced Modeling and Strategies.

Week 1: Forward Contracts

Forward contract basics

Forward Contract

- A **forward contract** is an agreement between two counterparties to trade a pre-specified amount of goods or securities at a pre-specified future date, T, for a pre- specified price, F_0.
- The **Profit/Loss (P/L)** at the contract maturity T for each counterparty is: $P/L_{long} = N(S_T - F_0)$, $P/L_{short} = N(F_0 - S_T)$
- Price of a zero coupong bond with face value Z: $P = e^{-r_T T} Z$
  $f(0, T_1, T_2)$ denotes the **forward rate** between time T_1 and T_2, as of time 0 :
  $f(0, T_1, T_2) = \frac{T_2 r_{T_2} - T_1 r_{T_1}}{T_2 - T_1}$
- Long forward positions are equivalent to borrowing and going long in the underlying asset
- Forward short positions are equivalent to lending and going short the underlying

Pricing formulas

Pricing formulas

- An **arbitrage opportunity** is a trading strategy that either (1) Yields a positive profit today, and zero cash flows in the future; or (2) Costs nothing today and yields a positive profit in the future
- **The Law of One Price**: Securities with identical payoffs must have the same price
- **Stock** with known dividend D at time $t < T$: $F_0 = (P_{S,0} - De^{-rt})e^{rT}$
  Stock with known dividend yield q: $F_0 = P_{S,0}e^{(r-q)T}$
- **Bond** with coupon C at time $t < T$: $F_0 = (P_{B,0} - Ce^{-rt})e^{rT}$
- **Currencies**. $r_\$$ ($r_€$) the USD (EUR) risk-free rate. S_t is the exchange rate (USD per EUR) at time t: $F_0 = S_0 e^{(r_\$ - r_€)T}$

Forward prices for commodities

- Forward price with lump-sum storage cost U: $F_{0,T} = (S_0 + PV(U))e^{rT}$
- Forward price with proportional storage cost u: $F_{0,T} = S_0 e^{(r+u)T}$
- Forward price with convenience yield y: $F_{0,T} = S_0 e^{(r-y)T}$
- Forward price with proportional storage cost and convenience yield: $F_{0,T} = S_0 e^{(r+u-y)T}$
- **Contango** is a pattern of forward prices that increases with contract maturity
- **Backwardation** is a pattern of forward prices over time that decreases with contract maturity

Key concepts for hedging and speculating

Valuing a forward contract over time

- Suppose that $K = F_0$ the original delivery price, initial value of contract $f_0 = 0$.
- Value of a **long** forward contract at time t : $f_{long,t,T} = (F_t - K)e^{-r(T-t)}$
- Value of a **short** forward contract at time t : $f_{short,t,T} = (K - F_t)e^{-r(T-t)}$
- **Basis** is the difference between the spot and forward price of a security or commodity.
- **Cross-hedging** involves using a contract type to hedge which differs from the security or commodity being hedged.
- The **hedge ratio** is the relative number of forward contracts to units of the asset being hedged that maximizes the effectiveness of the hedge: $N_S \mathbb{E}[\mathrm{d}S] = N_F \mathbb{E}[\mathrm{d}F]$ then: $\frac{N_S}{N_F} = \frac{\mathbb{E}[\mathrm{d}F]}{\mathbb{E}[\mathrm{d}S]}$. If long in spot then short in forwards, and vice versa.

Recommended Resources

- MITx 15.453x Derivatives Markets: Advanced Modeling and Strategies Lecture Slides
- John Hull's, Options Futures and Other Derivatives, 10th edition
- Bruce Tuckman and Angel Serrat, Fixed Income Securities; Tools for Today's Markets, 3rd Edition (BTAS)
- LaTeX File (`github.com/j053g/cheatsheets/15.453x`)

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