

# COMBINATIONAL LOGIC DESIGN

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## 2.1 EXERCISES

Any problem noted with an asterisk (\*) represents an especially challenging problem.

### Section 2.2: Switches

- 2.1. A microprocessor in 1980 used about 10,000 transistors. How many of those microprocessors would fit in a modern chip having 3 billion transistors?  
 $3,000,000,000 / 10,000 = 300,000$  microprocessors
- 2.2 The first Pentium microprocessor had about 3 million transistors. How many of those microprocessors would fit in a modern chip having 3 billion transistors?  
 $3,000,000,000 / 3,000,000 = 1,000$  microprocessors
- 2.3 Describe the concept known as Moore's Law.  
Integrated circuit density doubles approximately every 18 months.
- 2.4 Assume for a particular year that a particular size chip using state-of-the-art technology can contain 1 billion transistors. Assuming Moore's Law holds, how many transistors will the same size chip be able to contain in ten years?  
Approximately 100 billion transistors ( $10 \text{ years} * 12 \text{ months/year} / 18 \text{ months/doubling} = 6.667$  doublings.  $1 \text{ billion} * 2^{6.667} = 101.617$  billion).
- 2.5 Assume a cell phone contains 50 million transistors. How big would such a cell phone be if the phone used vacuum tubes instead of transistors, assuming a vacuum tube has an volume of 1 cubic inch?  
 $50,000,000 \text{ transistors} * 1 \text{ in}^3/\text{transistor} = 50,000,000 \text{ in}^3$  (nearly 30,000 cubic feet - as large as a house)

- 2.6 A modern desktop processor may contain 1 billion transistors in a chip area of  $100 \text{ mm}^2$ . If Moore's Law continues to apply, what would be chip area for those 1 billion transistors after 9 years? What percentage is that area of the original area? Name a product into which the smaller chip might fit whereas the original chip would have been too big.

Doubling chip capacity every 18 months also suggests halving of size every 18 months of the same number of transistors. 9 years / 18 months is 108 months / 18 months = 6 halvings.  $100 \text{ mm}^2 * (1/2)^6 = 100 \text{ mm}^2 / 64 = 1.56 \text{ mm}^2$ .  $1.56 \text{ mm}^2 / 100 \text{ mm}^2 = 1.56\%$  of the original area. A product into which such a small chip might now fit is a hearing aid, for example.

### Section 2.3: The CMOS Transistor

- 2.7 Describe the behavior of the CMOS transistor circuit shown in Figure 2.77, clearly indicating when the transistor circuit conducts.

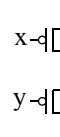


Figure 2.77

When  $x$  is a logical 0, the top transistor will conduct, otherwise the top transistor will not conduct. Likewise, when  $y$  is a logical 0, the bottom transistor will conduct and not conduct otherwise. Thus, the circuit conducts only when  $x$  is 0 and  $y$  is 0.

- 2.8 If we apply a voltage to the gate of a CMOS transistor, why doesn't the current flow to the transistor's source or drain?

An insulator exists between the gate and the source-drain channel, prohibiting current from flowing to the transistor's source or drain.

- 2.9 Why does applying a positive voltage to the gate of a CMOS transistor cause the transistor to conduct between source and drain?

The positive voltage at the gate attracts electrons into the channel between source and drain. Those electrons are enough to change the channel from non-conducting to conducting.

### Section 2.4: Boolean Logic Gates—Building Blocks for Digital Circuits

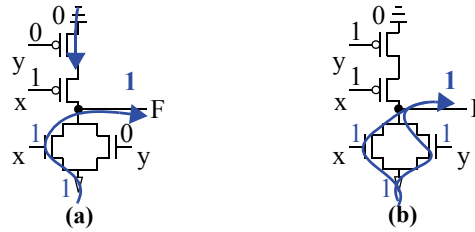
- 2.10 Which Boolean operation, AND, OR or NOT, is appropriate for each of the following:

- Detecting motion in any motion sensor surrounding a house (each motion sensor outputs 1 when motion is detected).
- Detecting that three buttons are being pressed simultaneously (each button outputs 1 when a button is being pressed).
- Detecting the absence of light from a light sensor (the light sensor outputs 1 when light is sensed).

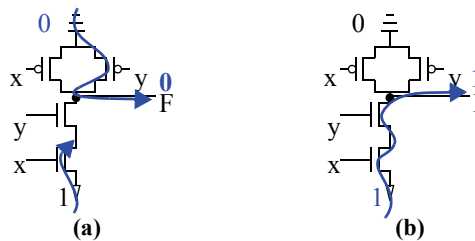
- OR
- AND
- NOT

- 2.11 Convert the following English problem statements to Boolean equations. Introduce Boolean variables as needed.
- A flood detector should turn on a pump if water is detected and the system is set to enabled
  - A house energy monitor should sound an alarm if it is night and light is detected inside a room but motion is not detected.
  - An irrigation system should open the sprinkler's water valve if the system is enabled and neither rain nor freezing temperatures are detected.
- a)  $\text{Pump} = \text{WaterDetected} \text{ AND } \text{SystemEnabled}$   
 b)  $\text{Alarm} = \text{Night} \text{ AND } \text{LightInsideDetected} \text{ AND NOT } \text{MotionDetected}$   
 c)  $\text{WaterValveOpen} = \text{SystemEnabled} \text{ AND NOT } (\text{RainDetected} \text{ OR } \text{FreezingTemperaturesDetected})$
- 2.12 Evaluate the Boolean equation  $F = (a \text{ AND } b) \text{ OR } c \text{ OR } d$  for the given values of variables a, b, c, and d:
- $a=1, b=1, c=1, d=0$
  - $a=0, b=1, c=1, d=0$
  - $a=1, b=1, c=0, d=0$
  - $a=1, b=0, c=1, d=1$
- a)  $F = (1 \text{ AND } 1) \text{ OR } 1 \text{ OR } 0 = 1 \text{ OR } 1 \text{ OR } 0 = 1$   
 b)  $F = (0 \text{ AND } 1) \text{ OR } 1 \text{ OR } 0 = 0 \text{ OR } 1 \text{ OR } 0 = 1$   
 c)  $F = (1 \text{ AND } 1) \text{ OR } 0 \text{ OR } 0 = 1 \text{ OR } 0 \text{ OR } 0 = 1$   
 d)  $F = (1 \text{ AND } 0) \text{ OR } 0 \text{ OR } 0 = 0 \text{ OR } 0 \text{ OR } 0 = 0$
- 2.13 Evaluate the Boolean equation  $F = a \text{ AND } (b \text{ OR } c) \text{ AND } d$  for the given values of variables a, b, c, and d:
- $a=1, b=1, c=0, d=1$
  - $a=0, b=0, c=0, d=1$
  - $a=1, b=0, c=0, d=0$
  - $a=1, b=0, c=1, d=1$
- a)  $F = 1 \text{ AND } (1 \text{ OR } 0) \text{ AND } 1 = 1 \text{ AND } 1 \text{ AND } 1 = 1$   
 b)  $F = 0 \text{ AND } (0 \text{ OR } 0) \text{ AND } 1 = 0 \text{ AND } 0 \text{ AND } 1 = 0$   
 c)  $F = 1 \text{ AND } (0 \text{ OR } 0) \text{ AND } 0 = 1 \text{ AND } 0 \text{ AND } 0 = 0$   
 d)  $F = 1 \text{ AND } (0 \text{ OR } 1) \text{ AND } 1 = 1 \text{ AND } 1 \text{ AND } 1 = 1$
- 2.14 Evaluate the Boolean equation  $F = a \text{ AND } (b \text{ OR } (c \text{ AND } d))$  for the given values of variables a, b, c, and d:
- $a=1, b=1, c=0, d=1$
  - $a=0, b=0, c=0, d=1$
  - $a=1, b=0, c=0, d=0$
  - $a=1, b=0, c=1, d=1$
- a)  $F = 1 \text{ AND } (1 \text{ OR } (0 \text{ AND } 1)) = 1 \text{ AND } (1 \text{ OR } 0) = 1 \text{ AND } 1 = 1$   
 b)  $F = 0 \text{ AND } (0 \text{ OR } (0 \text{ AND } 1)) = 0 \text{ AND } (0 \text{ OR } 0) = 0 \text{ AND } 0 = 0$   
 c)  $F = 1 \text{ AND } (0 \text{ OR } (0 \text{ AND } 0)) = 1 \text{ AND } (0 \text{ OR } 0) = 1 \text{ AND } 0 = 0$   
 d)  $F = 1 \text{ AND } (0 \text{ OR } (1 \text{ AND } 1)) = 1 \text{ AND } (0 \text{ OR } 1) = 1 \text{ AND } 1 = 1$

2.15 Show the conduction paths and output value of the OR gate transistor circuit in Figure 2.12 when: (a)  $x = 1$  and  $y = 0$ , (b)  $x = 1$  and  $y = 1$ .



2.16 Show the conduction paths and output value of the AND gate transistor circuit in Figure 2.14 when: (a)  $x = 1$  and  $y = 0$ , (b)  $x = 1$  and  $y = 1$ .

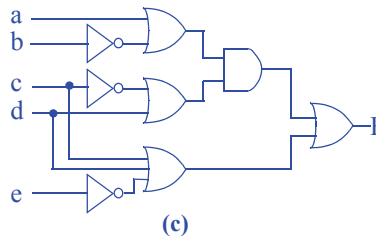
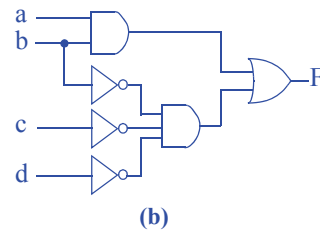
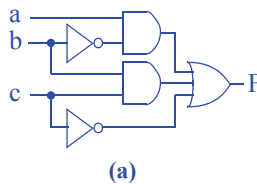


2.17 Convert each of the following equations directly to gate-level circuits:

a.  $F = ab' + bc + c'$

b.  $F = ab + b'c'd'$

c.  $F = ((a + b') * (c' + d)) + (c + d + e')$

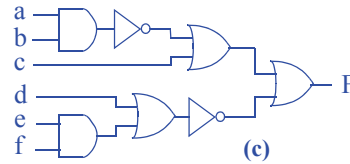
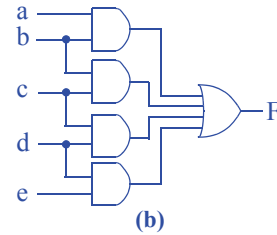
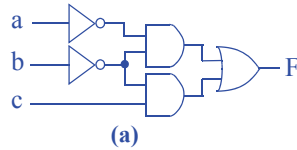


2.18 Convert each of the following equations directly to gate-level circuits:

a.  $F = a'b' + b'c$

$$b.F = ab + bc + cd + de$$

$$c.F = ((ab)' + (c)) + (d + ef)'$$

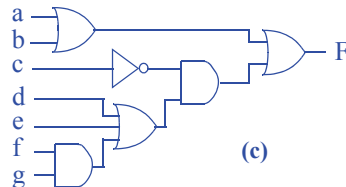
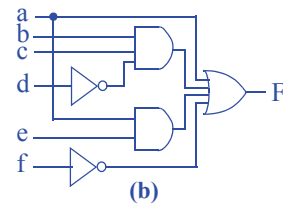
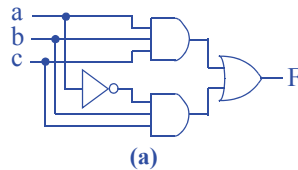


2.19 Convert each of the following equations directly to gate-level circuits:

$$a.F = abc + a'bc$$

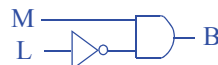
$$b.F = a + bcd' + ae + f'$$

$$c.F = (a + b) + (c' * (d + e + fg))$$

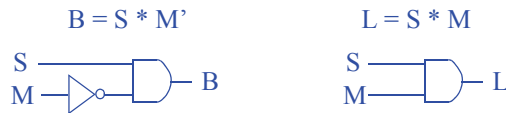


2.20 Design a system that sounds a buzzer inside a home whenever motion outside is detected at night. Assume a motion sensor has an output M that indicates whether motion is detected (M=1 means motion detected) and a light sensor with output L that indicates if light is detected (L=1 means light is detected). The buzzer inside the home has a single input B that when 1 sounds the buzzer. Capture the desired system behavior using an equation, and then convert the equation to a circuit using AND, OR, and NOT gates.

$$B = M * L'$$



- 2.21 A DJ (“disc jockey,” meaning someone who plays music at a party) would like a system to automatically control a strobe light and disco ball in a dance hall depending on whether music is playing and people are dancing. A sound sensor has output S that when 1 indicates that music is playing, and a motion sensor has output M that when 1 indicates that people are dancing. The strobe light has an input L that when 1 turns the light on, and the disco ball has an input B that when 1 turns the ball on. The DJ wants the disco ball to turn on only when music is playing and nobody is dancing, and wants the strobe light to turn on only when music is playing and people are dancing. Create equations describing the desired behavior for B and for L, and then convert each to a circuit using AND, OR, and NOT gates.



- 2.22 We want to concisely describe the following situation using a Boolean equation. We want to fire a football coach (by setting  $F=1$ ) if he is mean (represented by  $M=1$ ). If he is not mean, but has a losing season (represented by the Boolean variable  $L=1$ ), we want to fire him anyways. Write an equation that translates the situation directly to a Boolean equation for F, without any simplification.

$$F = M + (M' * L)$$

### Section 2.5: Boolean Algebra

- 2.23 For the function  $F = a + a'b + acd + c'$ :

- List all the variables.
- List all the literals.
- List all the product terms.

a) a, b, c, d

b) a, a', b, a, c, d, c'

c) a, a'b, acd, c'

- 2.24 For the function  $F = a'd' + a'c + b'cd' + cd$ :

- List all the variables.
- List all the literals.
- List all the product terms.

a) a, b, c, d

b) a', d', a', c, b', c, d', c, d

c) a'd', a'c, b'cd', cd

- 2.25 Let variables T represent being tall, H being heavy, and F being fast. Let's consider anyone who is not tall as short, not heavy as light, and not fast as slow. Write a Boolean equation to represent the following:

- You may ride a particular amusement park ride only if you are either tall and light, or short and heavy.

- b. You may NOT ride an amusement park ride if you are either tall and light, or short and heavy. Use algebra to simplify the equation to sum of products.
- c. You are eligible to play on a particular basketball team if you are tall and fast, or tall and slow. Simplify this equation.
- d. You are NOT eligible to play on a particular football team if you are short and slow, or if you are light. Simplify to sum of products form.
- e. You are eligible to play on both the basketball and football teams above, based on the above criteria. Hint: combine the two equations into one equation by ANDing them.
- a)  $\text{Ride} = TH' + T'H$   
 b)  $\text{Ride} = (TH' + T'H)' = (TH')'(T'H)' = (T' + H)(T + H') = T'H' + TH$   
 c)  $\text{Basketball} = TF + TF' = T(F + F') = T(1) = T$   
 d)  $\text{Football} = (T'F' + H')' = (T'F')'H = (T + F)H = TH + FH$   
 e)  $\text{BasketballAndFootball} = T(TH + FH) = TTH + TFH = TH + TFH = TH(1 + F) = TH$ . In other words, only people who are both tall and heavy can play on both teams.
- 2.26 Let variables S represent a package being small, H being heavy, and E being expensive. Let's consider a package that is not small as big, not heavy as light, and not expensive as inexpensive. Write a Boolean equation to represent the following:
- a. Your company specializes in delivering packages that are both small and inexpensive (a package must be small AND inexpensive for us to deliver it); you'll also deliver packages that are big but only if they are expensive.
- b. A particular truck can be loaded with packages only if the packages are small and light, small and heavy, or big and light. Simplify the equation.
- c. Your above-mentioned company buys the above-mentioned truck. Write an equation that describes the packages your company can deliver. Hint: Appropriately combine the equations from the above two parts.
- a)  $\text{Deliver} = SE' + S'E$   
 b)  $\text{Load} = SH' + SH + S'H' = SH' + SH + SH' + S'H' = S + H'$   
 c)  $\text{Packages} = \text{Deliver} * \text{Load} = (SE' + S'E)(S + H') = SSE' + SS'E + H'SE' + H'S'E = SE' + 0 + H'SE' + H'S'E = (1 + H')SE' + H'S'E = SE' + S'EH'$ . In other words, you can deliver small inexpensive packages, or large expensive light packages.
- 2.27 Use algebraic manipulation to convert the following equation to sum-of-products form:  $F = a(b + c)(d') + ac'(b + d)$
- $$F = (ab + ac)d' + ac'b + ac'd$$
- $$F = abd' + acd' + ac'b + ac'd$$
- 2.28 Use algebraic manipulation to convert the following equation to sum-of-products form:  $F = a'b(c + d') + a(b' + c) + a(b + d)c$
- $$F = a'bc + a'bd' + ab' + ac + (ab + ad)c$$
- $$F = a'bc + a'bd' + ab' + ac + abc + acd$$
- $$F = a'bc + a'bd' + ab' + ac$$
- 2.29 Use DeMorgan's Law to find the inverse of the following equation:  $F = abc + a'b$ . Reduce to sum-of-products form. Hint: Start with  $F' = (abc + a'b)'$ .
- $$F' = (abc + a'b)'$$

$$F' = (abc)'(a'b)'$$

$$F' = (a' + b' + c')(a'' + b')$$

$$F' = (a' + b' + c')(a + b')$$

$$F' = a(a' + b' + c') + b'(a' + b' + c')$$

$$F' = 0 + ab' + ac' + a'b' + b' + b'c'$$

$$F' = (a + a')b' + b' + ac' + \cancel{b'c'} \text{ (The } b' \text{ term makes all other terms with } b' \text{ redundant)}$$

$$F' = b' + ac'$$

2.30 Use DeMorgan's Law to find the inverse of the following equation:  $F = ac' + abd' + acd$ . Reduce to sum-of-products form.

$$F' = (ac' + abd' + acd)'$$

$$F' = (ac')'(abd')'(acd)'$$

$$F' = (a' + c'')(a' + b' + d'')(a' + c' + d')$$

$$F' = (a' + c)(a' + b' + d)(a' + c' + d')$$

$$F' = (a' + a'b' + a'd + a'c + b'c + cd)(a' + c' + d')$$

$$F' = a' + a'c' + a'd' + a'b' + a'b'c' + a'b'd' + a'd + \cancel{a'c'c'} + a'cd' + a'b'c + \cancel{b'c'c'} + b'cd' + a'cd + \cancel{c'cd} + \cancel{c'dd'} \text{ (The } a' \text{ term makes all other terms with } a' \text{ redundant)}$$

$$F' = a' + b'cd'$$

## Section 2.6: Representations of Boolean Functions

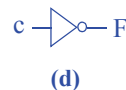
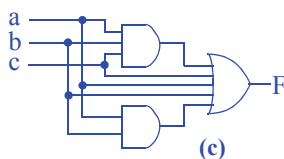
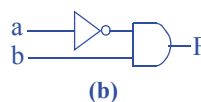
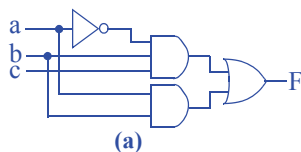
2.31 Convert the following Boolean equations to a digital circuit:

a.  $F(a, b, c) = a'bc + ab$

b.  $F(a, b, c) = a'b$

c.  $F(a, b, c) = abc + ab + a + b + c$

d.  $F(a, b, c) = c'$



2.32 Create a Boolean equation representation of the digital circuit in Figure 2.78.

$$F = (ab' + b)'$$

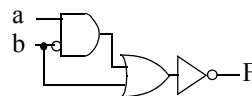


Figure 2.78



2.33 Create a Boolean equation representation for the digital circuit in Figure 2.79.

$F = (ab' + b) + a'c$

2.34 Convert each of the Boolean equations in Exercise 2.31 to a truth table.

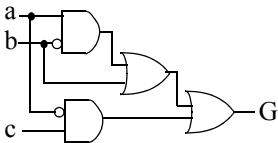


Figure 2.79

Inputs			Outputs
a	b	c	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

(a)

Inputs			Outputs
a	b	c	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

(b)

Inputs			Outputs
a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

(c)

Inputs			Outputs
a	b	c	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

(d)

2.35 Convert each of the following Boolean equations to a truth table:

- a.  $F(a, b, c) = a' + bc'$
- b.  $F(a, b, c) = (ab)' + ac' + bc$
- c.  $F(a, b, c) = ab + ac + ab'c' + c'$
- d.  $F(a, b, c, d) = a'bc + d'$

Inputs			Outputs
a	b	c	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(a)

Inputs			Outputs
a	b	c	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

(b)

Inputs			Outputs
a	b	c	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

(c)

Inputs				Outputs
a	b	c	d	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

(d)

2.36 Fill in Table 2.8's columns for the equation:  $F = ab + b'$

Table 2.8

Inputs					Output
a	b	ab	b'	ab+b'	F
0	0	0	1	1	1
0	1	0	0	0	0
1	0	0	1	1	1
1	1	1	0	1	1

- 2.37 Convert the function  $F$  shown in the truth table in Table 2.9 to an equation. Don't minimize the equation.

$$F = a'b'c + a'bc' + a'bc + ab'c + abc' + abc$$

- 2.38 Use algebraic manipulation to minimize the equation obtained in Exercise 2.37

$$F = a'b'c + a'bc' + a'bc + ab'c + abc' + abc$$

$$F = a'(b'c + bc' + bc) + a(b'c + bc' + bc)$$

$$F = a'(b'c + b(c' + c)) + a(b'c + b(c' + c))$$

$$F = a'(b'c + b) + a(b'c + b)$$

$$F = (a' + a)(b'c + b)$$

$$F = b'c + b$$

- 2.39 Convert the function  $F$  shown in the truth table in Table 2.10 to an equation. Don't minimize the equation.

$$F = a'b'c' + a'bc' + ab'c' + ab'c + abc'$$

- 2.40 Use algebraic manipulation to minimize the equation obtained in Exercise 2.39

$$F = a'b'c' + a'bc' + ab'c' + ab'c + abc'$$

$$F = a'(b'c' + bc') + a(b'c' + b'c + bc')$$

$$F = a'((b' + b)c') + a(b'c' + c) + bc'$$

$$F = a'c' + a(b' + bc')$$

- 2.41 Convert the function  $F$  shown in the truth table in Table 2.11 to an equation. Don't minimize the equation.

$$F = a'b'c + abc' + abc$$

- 2.42 Use algebraic manipulation to minimize the equation obtained in Exercise 2.41.

$$F = a'b'c + abc' + abc$$

$$F = a'b'c + ab(c' + c)$$

$$F = a'b'c + ab$$

- 2.43 Create a truth table for the circuit of Figure 2.78

Table 2.9

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Table 2.10

a	b	c	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Table 2.11

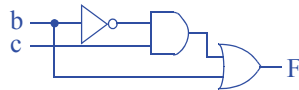
a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Inputs		Outputs
a	b	F
0	0	1
0	1	0
1	0	0
1	1	0

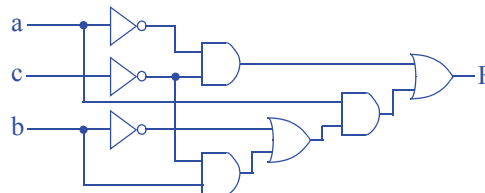
2.44 Create a truth table for the circuit of Figure 2.79.

Inputs					Outputs
a	b	c	$ab' + b$	$a'c$	F
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	0	1

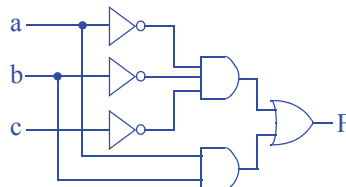
2.45 Convert the function F shown in the truth table in Table 2.9 to a digital circuit.



2.46 Convert the function F shown in the truth table in Table 2.10 to a digital circuit.



2.47 Convert the function F shown in the truth table in Table 2.11 to a digital circuit.



2.48 Convert the following Boolean equations to canonical sum-of-minterms form:

- a.  $F(a,b,c) = a'bc + ab$   
 b.  $F(a,b,c) = a'b$   
 c.  $F(a,b,c) = abc + ab + a + b + c$   
 d.  $F(a,b,c) = c'$   
 a)  $F(a,b,c) = a'bc + abc' + abc$   
 b)  $F(a,b,c) = a'bc' + a'bc$   
 c)  $F(a,b,c) = a'b'c + a'bc' + a'bc + ab'c' + ab'c + abc' + abc$   
 d)  $F(a,b,c) = a'b'c' + a'bc' + ab'c' + abc'$
- 2.49 Determine whether the Boolean functions  $F = (a + b)' * a$  and  $G = a + b'$  are equivalent, using: (a) algebraic manipulation, and (b) truth tables.
- a) Convert the two functions to canonical sum-of-minterms form:  
 $F = (a + b)' * a$   
 $F = a'b'a$   
 $F = 0$   
 $G = a + b'$   
 $G = ab' + ab + a'b'$   
 F and G are not equivalent.

(b)	Inputs		Outputs	Inputs		Outputs
	a	b	F	a	b	G
	0	0	0	0	0	1
	0	1	0	0	1	0
	1	0	0	1	0	1
	1	1	0	1	1	1

- 2.50 Determine whether the Boolean functions  $F = ab'$  and  $G = (a' + ab)'$  are equivalent, using: (a) algebraic manipulation, and (b) truth tables.
- a) Convert the two functions to canonical sum-of-minterms form:  
 $F = ab'$   
 $G = (a' + ab)'$   
 $G = (a)(ab)'$   
 $G = a(a' + b')$   
 $G = 0 + ab'$   
 $G = ab'$   
 F and G are equivalent.

(b)	Inputs		Outputs	Inputs		Outputs
	a	b	F	a	b	G
	0	0	0	0	0	0
	0	1	0	0	1	0
	1	0	1	1	0	1
	1	1	0	1	1	0

- 2.51 Determine whether the Boolean function  $G = a'b'c + ab'c + abc' + abc$  is equivalent to the function represented by the circuit in Figure 2.80.

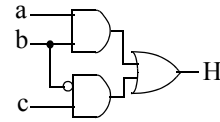


Figure 2.80

The circuit can be converted to the equation  $H = ab + b'c$ . That equation can be algebraically expanded to canonical sum-of-minterms form as  $H = ab(c'+c) + (a'+a)b'c = abc' + abc + a'b'c + ab'c$ , which is equivalent to  $G$ .

- 2.52 Determine whether the two circuits in Figure 2.81 are equivalent circuits using: (a) algebraic manipulation, and (b) truth tables.

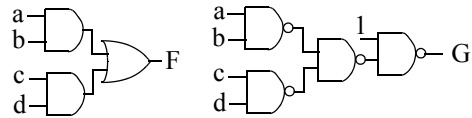


Figure 2.81

a)  $F = ab + cd$  and  $G = 1*((ab)' * (cd)')'$

In canonical sum-of-minterms

form,  $F = a'b'cd + a'bcd + ab'cd + abc'd' + abc'd + abcd' + abcd$  and  $G = a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'bc'd' + a'bc'd + ab'c'd' + ab'c'd + ab'cd'$ .  $F$  and  $G$  are not equivalent ( $F \neq G'$ )

b)

Inputs				Outputs
a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(a)

Inputs				Outputs
a	b	c	d	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

(b)

2.53 \*Figure 2.82 shows two circuits whose inputs are unlabeled.

- a. Determine whether the two circuits are equivalent. Hint: Try all possible labellings of the inputs for both circuits.

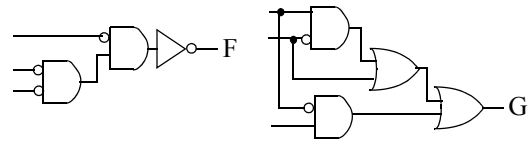


Figure 2.82

(No solution provided for challenge problem)

- b. How many circuit comparisons would need to be performed to determine if two circuits with 10 unlabeled inputs are equivalent?

(No solution provided for challenge problem)

### Section 2.7: Combinational Logic Design Process

2.54 A museum has three rooms, each with a motion sensor ( $m_0$ ,  $m_1$ , and  $m_2$ ) that outputs 1 when motion is detected. At night, the only person in the museum is one security guard who walks from room to room. Create a circuit that sounds an alarm (by setting an output A to 1) if motion is ever detected in more than one room at a time (i.e., in two or three rooms), meaning there must be one or more intruders in the museum. Start with a truth table.

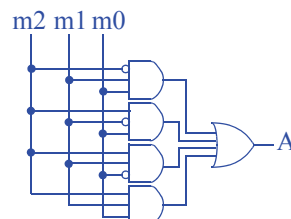
#### Step 1 - Capture the function

Inputs			Outputs
$m_2$	$m_1$	$m_0$	A
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

#### Step 2A - Create equations

$$A = m_2'm_1m_0 + m_2m_1'm_0 + m_2m_1m_0' + m_2m_1m_0$$

#### Step 2B- Implement as a gate-based circuit



- 2.55 Create a circuit for the museum of Exercise 2.54 that detects whether the guard is properly patrolling the museum, detected by *exactly* one motion sensor being 1. (If no motion sensor is 1, the guard may be sitting, sleeping, or absent).

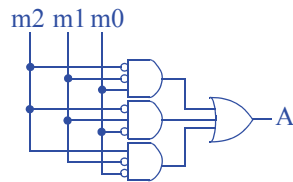
**Step 1 - Capture the function**

Inputs			Outputs
m2	m1	m0	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

**Step 2A - Create equations**

$$A = m2'm1'm0 + m2'm1m0' + m2m1'm0'$$

**Step 2B- Implement as a gate-based circuit**





- 2.56 Consider the museum security alarm function of Exercise 2.54, but for a museum with 10 rooms. A truth table is not a good starting point (too many rows), nor is an equation describing when the alarm should sound (too many terms). However, the inverse of the alarm function can be straightforwardly captured as an equation. Design the circuit for the 10 room security system, by designing the inverse of the function, and then just adding an inverter before the circuit's output.

### Step 1 - Capture the function

The inverse function detects that motion is detected by exactly one motion sensor, or no motion sensor detecting motion; all the other possibilities are for two or more sensors detecting motion. Thus, the inverse function can be written as:

$A' =$

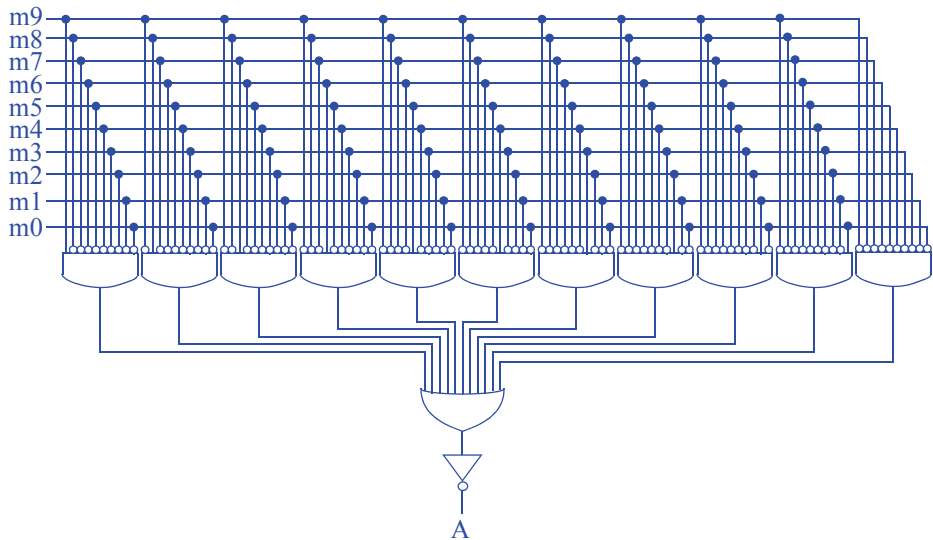
$$\begin{aligned} & m_9 m_8' m_7' m_6' m_5' m_4' m_3' m_2' m_1' m_0' + m_9' m_8 m_7' m_6' m_5' m_4' m_3' m_2' m_1' m_0' + \\ & m_9' m_8' m_7 m_6' m_5' m_4' m_3' m_2' m_1' m_0' + m_9' m_8' m_7' m_6 m_5' m_4' m_3' m_2' m_1' m_0' + \\ & m_9' m_8' m_7' m_6' m_5 m_4' m_3' m_2' m_1' m_0' + m_9' m_8' m_7' m_6' m_5' m_4 m_3' m_2' m_1' m_0' + \\ & m_9' m_8' m_7' m_6' m_5' m_4' m_3 m_2' m_1' m_0' + m_9' m_8' m_7' m_6' m_5' m_4' m_3' m_2 m_1' m_0' + \\ & m_9' m_8' m_7' m_6' m_5' m_4' m_3' m_2' m_1 m_0' + m_9' m_8' m_7' m_6' m_5' m_4' m_3' m_2' m_1' m_0' \end{aligned}$$

The first term is for motion sensor  $m_9$  detecting motion and all others detecting no motion, the second term is for  $m_8$ , and so on. That last term is for no sensor detecting motion.

### Step 2A - Create equations

Already done.

### Step 2B- Implement as a gate-based circuit



2.57 A network router connects multiple computers together and allows them to send messages to each other. If two or more computers send messages simultaneously, the messages “collide” and the messages must be resent. Using the combinational design process of Table 2.5, create a collision detection circuit for a router that connects 4 computers. The circuit has 4 inputs labeled M0 through M3 that are 1 when the corresponding computer is sending a message and 0 otherwise. The circuit has one output labeled C that is 1 when a collision is detected and 0 otherwise.

### Step 1 - Capture the function

A truth table is convenient for this problem.

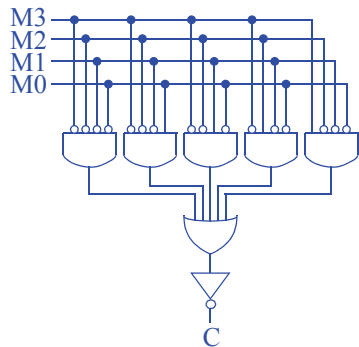
Inputs				Outputs
M3	M2	M1	M0	C
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

### Step 2A - Create equation

We note that there are more 1s in the output column than there are 0s. Thus, we choose to create an equation for the inverse of the function, and we'll then add an inverter at the output. The problem could also be solved by creating a (longer) equation for the function itself rather than the inverse.

$$C' = M3'M2'M1'M0' + M3'M2'M1'M0 + M3'M2'M1M0' + M3'M2M1'M0' + M3M2'M1'M0'$$

Step 2B- Implement as a gate-based circuit



2.58 Using the combinational design process of Table 2.5, create a 4-bit prime number detector. The circuit has four inputs, N3, N2, N1, and N0 that correspond to a 4-bit number (N3 is the most significant bit) and one output P that is 1 when the input is a prime number and that is 0 otherwise.

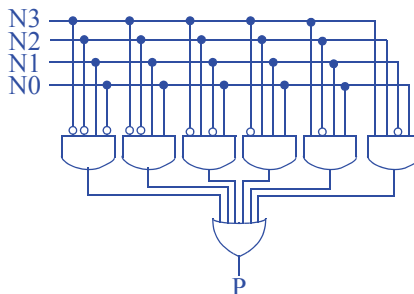
Step 1 - Capture the function

The prime numbers in the range 0-15 are 2, 3, 5, 7, 11, and 13. Rows whose input binary number correspond to those numbers have P set to a 1; the other rows get 0.

Inputs				Outputs
N3	N2	N1	N0	P
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Step 2A - Create equations

$$P = N3'N2'N1N0' + N3'N2'N1N0 + N3'N2N1'N0 + N3'N2N1N0 + N3N2'N1N0 + N3N2N1'N0$$

**Step 2B - Implement as a gate-based circuit**

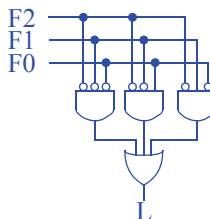
- 2.59 A car has a fuel-level detector that outputs the current fuel-level as a 3-bit binary number, with 000 meaning empty and 111 meaning full. Create a circuit that illuminates a “low fuel” indicator light (by setting an output  $L$  to 1) when the fuel level drops below level 3.

**Step 1 - Capture the function**

Inputs			Outputs
F2	F1	F0	L
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

**Step 2A -Create equations**

$$L = F2'F1'F0' + F2'F1'F0 + F2'F1F0'$$

**Step 2B- Implement as a gate-based circuit**

- 2.60 A car has a low-tire-pressure sensor that outputs the current tire pressure as a 5-bit binary number. Create a circuit that illuminates a “low tire pressure” indicator light (by setting an output  $T$  to 1) when the tire pressure drops below 16. Hint: you might find it easier to create a circuit that detects the inverse function. You can then just append an inverter to the output of that circuit.

**Step 1 - Capture the function**

The inverse function outputs 1 if the input is 16 or greater. For a 5-bit number, we know that any number 16 or greater has a 1 in the leftmost bit, which we'll name P4. Any number less than 16 will have a 0 in P4. Thus, an equation that detects 16 or greater is just:

$$T' = P4$$

### Step 2A - Create equations

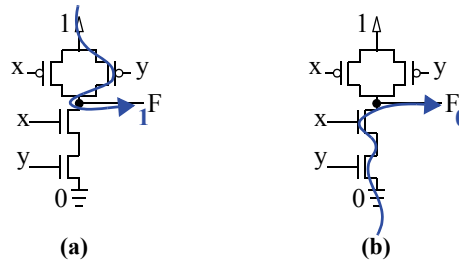
Already done

### 3 - Implement as a gate-based circuit

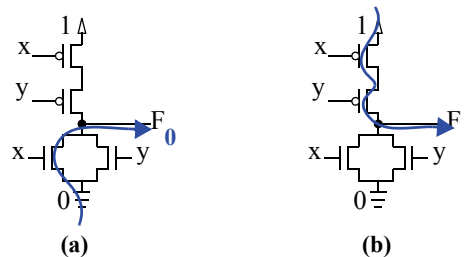


## Section 2.8: More Gates

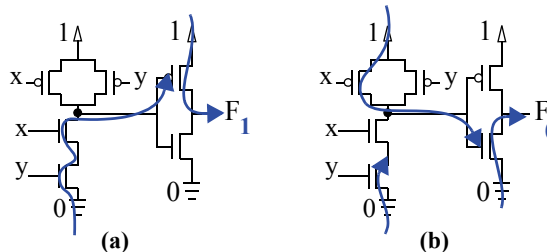
- 2.61 Show the conduction paths and output value of the NAND gate transistor circuit in Figure 2.54 when: (a)  $x = 1$  and  $y = 0$ , (b)  $x = 1$  and  $y = 1$ .



- 2.62 Show the conduction paths and output value of the NOR gate transistor circuit in Figure 2.54 when: (a)  $x = 1$  and  $y = 0$ , (b)  $x = 0$  and  $y = 0$ .



- 2.63 Show the conduction paths and output value of the AND gate transistor circuit in Figure 2.55 when: (a)  $x = 1$  and  $y = 1$ , (b)  $x = 0$  and  $y = 1$ .



- 2.64 Two people, denoted using variables A and B, want to ride with you on your motorcycle. Write a Boolean equation that indicates that exactly one of the two people can come (A=1 means A can come, A=0 means A can't come). Then use XOR to simplify your equation.

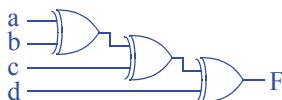
$$F = A'B + AB'$$

$$F = A \text{ XOR } B$$

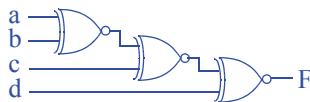
- 2.65 Simplify the following equation by using XOR wherever possible:  $F = a'b + ab' + cd' + c'd + ac$ .

$$F = (a \text{ XOR } b) + (c \text{ XOR } d) + ac$$

- 2.66 Use 2-input XOR gates to create a circuit that outputs a 1 when the number of 1s on inputs a, b, c, d is odd.

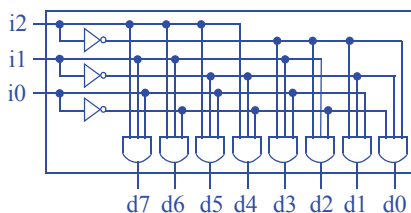


- 2.67 Use 2-input XOR or XNOR gates to create a circuit that detects if an even number of the inputs a, b, c, d are 1s.

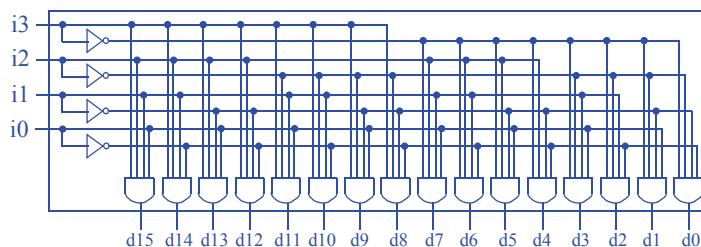


## Section 2.9: Decoders and Muxes

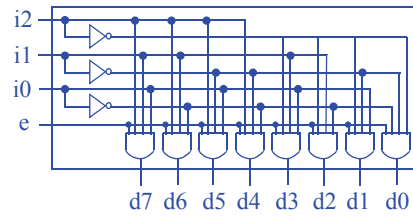
- 2.68 Design a 3x8 decoder using AND, OR and NOT gates.



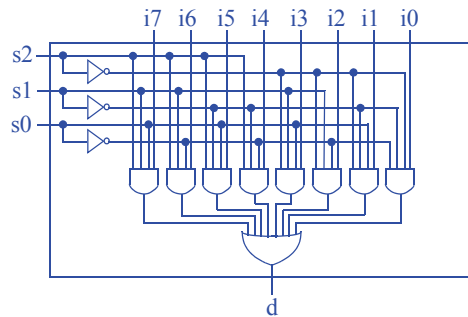
- 2.69 Design a 4x16 decoder using AND, OR and NOT gates.



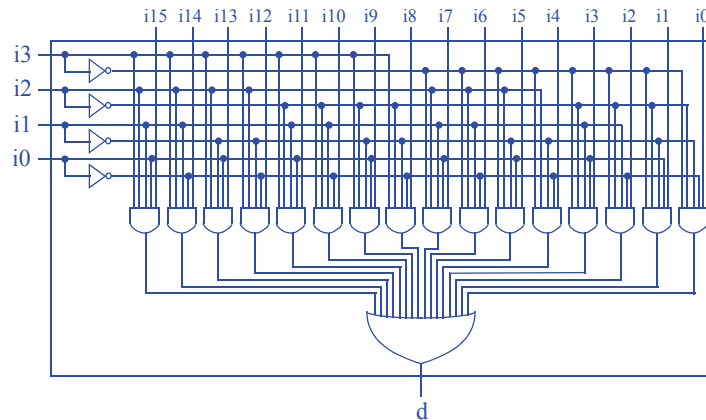
2.70 Design a 3x8 decoder with enable using AND, OR and NOT gates.



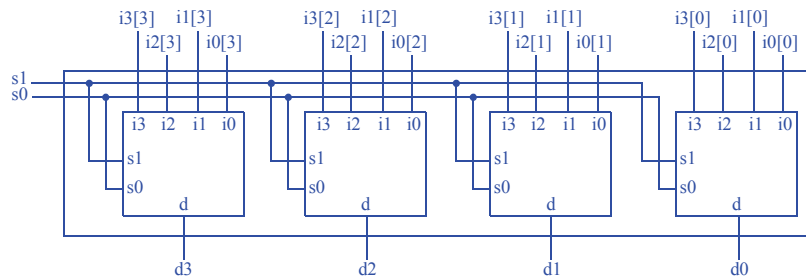
2.71 Design an 8x1 multiplexer using AND, OR and NOT gates.



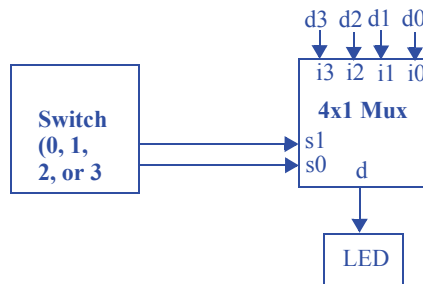
2.72 Design a 16x1 multiplexer using AND, OR and NOT gates.



2.73 Design a 4-bit 4x1 multiplexer using four 4x1 multiplexors.

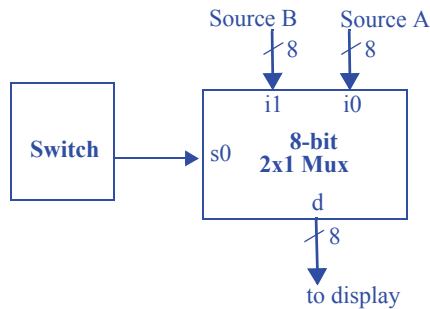


2.74 A house has four external doors each with a sensor that outputs 1 if its door is open. Inside the house is a single LED that a homeowner wishes to use to indicate whether a door is open or closed. Because the LED can only show the status of one sensor, the homeowner buys a switch that can be set to 0, 1, 2, or 3 and that has a 2-bit output representing the switch position in binary. Create a circuit to connect the four sensors, the switch, and the LED. Use at least one mux (a single mux or an N-bit mux) or decoder. Use block symbols with a clearly defined function, such as “2x1 mux,” “8-bit 2x1 mux,” or “3x8 decoder”; do not show the internal design of a mux or decoder..

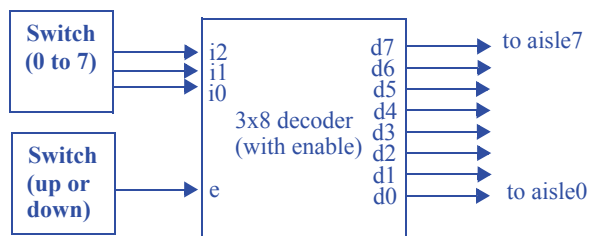




- 2.75 A video system can accept video from one of two video sources, but can only display one source at a given time. Each source outputs a stream of digitized video on its own 8-bit output. A switch with a single bit output chooses which of the two 8-bit streams will be passed on a display's single 8-bit input. Create a circuit to connect the two video sources, the switch, and the display. Use at least one mux (a single mux or an N-bit mux) or decoder. Use block symbols with a clearly defined function, such as "2x1 mux," "8-bit 2x1 mux," or "3x8 decoder"; do not show the internal design of a mux or decoder.



- 2.76 A store owner wishes to be able to indicate to customers that the items in one of the store's eight aisles are temporarily discounted ("on sale"). The store owner thus mounts a light above each aisle, and each light has a single bit input that turns on the light when 1. The store owner has a switch that can be set to 0, 1, 2, 3, 4, 5, 6, or 7, and that has a 3-bit output representing the switch position in binary. A second switch can be set up or down and has a single bit output that is 1 when the switch is up; the store owner can set this switch down if no aisles are currently discounted. Use at least one mux (a single mux or an N-bit mux) or decoder. Use block symbols each with a clearly defined function, such as "2x1 mux," "8-bit 2x1 mux," or "3x8 decoder"; do not show the internal design of a mux or decoder.



**Section 2.10: Additional Considerations**

2.77 Determine the critical path of the specified circuit. Assume that each AND and OR gate has a delay of 1 ns, each NOT gate has a delay of 0.75 ns, and each wire has a delay of 0.5 ns.

a. The circuit of Figure 2.37.

The path from input c to output F has a delay of  $0.5 + 0.75 + 0.5 + 1 + 0.5 = 3.25$  ns.

The path from input h to output F has a delay of  $0.5 + 1 + 0.5 + 1 + 0.5 = 3.5$  ns.

The path from input p to output F has a delay of  $0.5 + 1 + 0.5 + 1 + 0.5 = 3.5$  ns.

The longest path is 3.5 ns. Thus, the circuit's critical path is 3.5 ns.

b. The circuit of Figure 2.41.

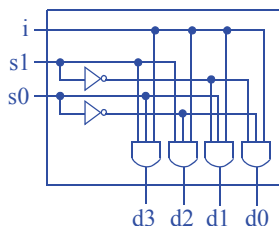
The path from input a to output F has a delay of  $0.5 + 1 + 0.5 + 0.75 + 0.5 + 1 + 0.5 = 4.75$  ns.

The path from input b to output F is identical to that from input a: 4.75 ns.

The path from input c to output F has a delay of  $0.5 + 0.75 + 0.5 + 1 + 0.5 = 3.25$  ns.

The longest path is 4.75 ns. Thus, the circuit's critical path is 4.75 ns.

2.78 Design a 1x4 demultiplexer using AND, OR and NOT gates.



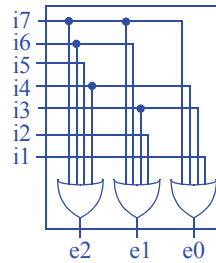
2.79 Design an 8x3 encoder using AND, OR and NOT gates. Assume that only one input will be asserted at any given time.

Inputs								Outputs		
i7	i6	i5	i4	i3	i2	i1	i0	e2	e1	e0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

$$e2 = I7 + I6 + I5 + I4$$

$$e1 = I7 + I6 + I3 + I2$$

$$e0 = I7 + I5 + I3 + I1$$



2.80 Design a 4x2 priority encoder using AND, OR and NOT gates. If every input is 0, the output should be “00”.

Inputs				Outputs	
i3	i2	i1	i0	e1	e0
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	1	0
0	1	1	1	1	0
1	0	0	0	1	1
1	0	0	1	1	1
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

$$e1 = i3 + i2$$

$$e0 = i3 + i2' i1$$

