

```

In[529]:= WD = DigitCount[#, 2, 1] &;
computeW[a_, outType_: 0] := Module[{ad, b, genAD, weights},
  ad = FromDigits[#, 2] & /@ a;
  b = Tuples[{0, 1}, a // Length];
  genAD = Fold[BitXor] /@ ((Times[ad, #] &) /@ b) // Union;
  weights = WD /@ genAD;
  If[outType == 0,
    weights // Tally,
    {weights, genAD}
  ]
]
weightsHistogram = BarChart[#[[;;, 2]], ChartLabels->#[[;;,1]]]&;

runOnA[A_] := Module[{w, dataStr, hist, max},
  w = computeW[A];
  max = Max[w[[;;, 1]]];
  hist = Range[0, max] // Tally;
  (hist[[#[[1]] + 1, 2]] = 0) & /@ hist;
  (hist[[#[[1]] + 1, 2]] = #[[2]]) & /@ w;
  hist
]

In[517]:= genMat[seq_: {3}, height_: 3, width_: 3] := Module[{l, h = height, m, e, i, j = 1},
  (* seq == {4,3,2,1} *)
  h = Max[seq~Join~{h}];
  l = Total@seq;
  l = Max[l, width];
  m = Table[0, {ii, 1, h}, {jj, 1, l}];

  For[i = 1, i <= Length@seq, i++,
    e = seq[[i]];
    If[e == 0, Continue[]];
    m[[1 ;; e, j ;; j + e - 1]] = DiagonalMatrix[Array[1 &, e]];
    j += e;
  ];
  m
]

In[520]:= genMat[{4, 3, 3, 0, 0}] // MatrixForm
t = Table[{i, j, k}, {i, 4, 1, -1}, {j, i, 0, -1}, {k, j, 0, -1}];

Out[520]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

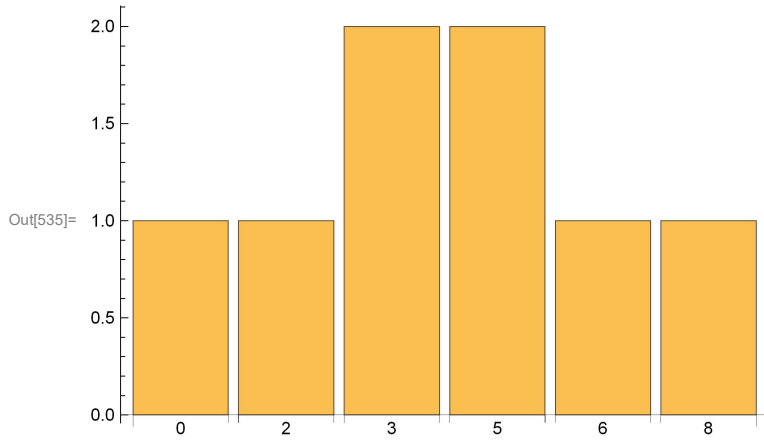

In[524]:= ms = genMat[#] & /@ Flatten[t, 2];

```

```
In[534]:= m = ms[[17]]; m // MatrixForm
computeW[ms[[17]]] // weightsHistogram
```

Out[534]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



```
In[536]:= (*genMat[{4,3,3,0,0}]]//MatrixForm;*)
```

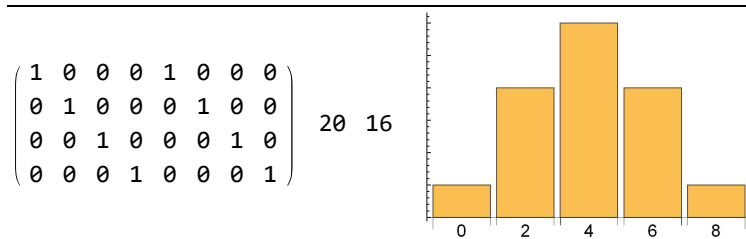
```
Clear[dispMatrixes];
dispMatrixes[ms_] := Module[{ws, ws1, ws2, mfms, pics},
  ws = computeW /@ ms;
  ws1 = Total[#[[;;, 1]]] & /@ ws;
  ws2 = Total[#[[;;, 2]]] & /@ ws;
  mfms = MatrixForm /@ ms;
  pics = weightsHistogram /@ ws;
  GridBox[MapThread[Join, {List /@ mfms, List /@ ws1, List /@ ws2, List /@ pics}],
    GridBoxDividers → {"Rows" → {{True}}}] // DisplayForm
  (*{ws}*)
]

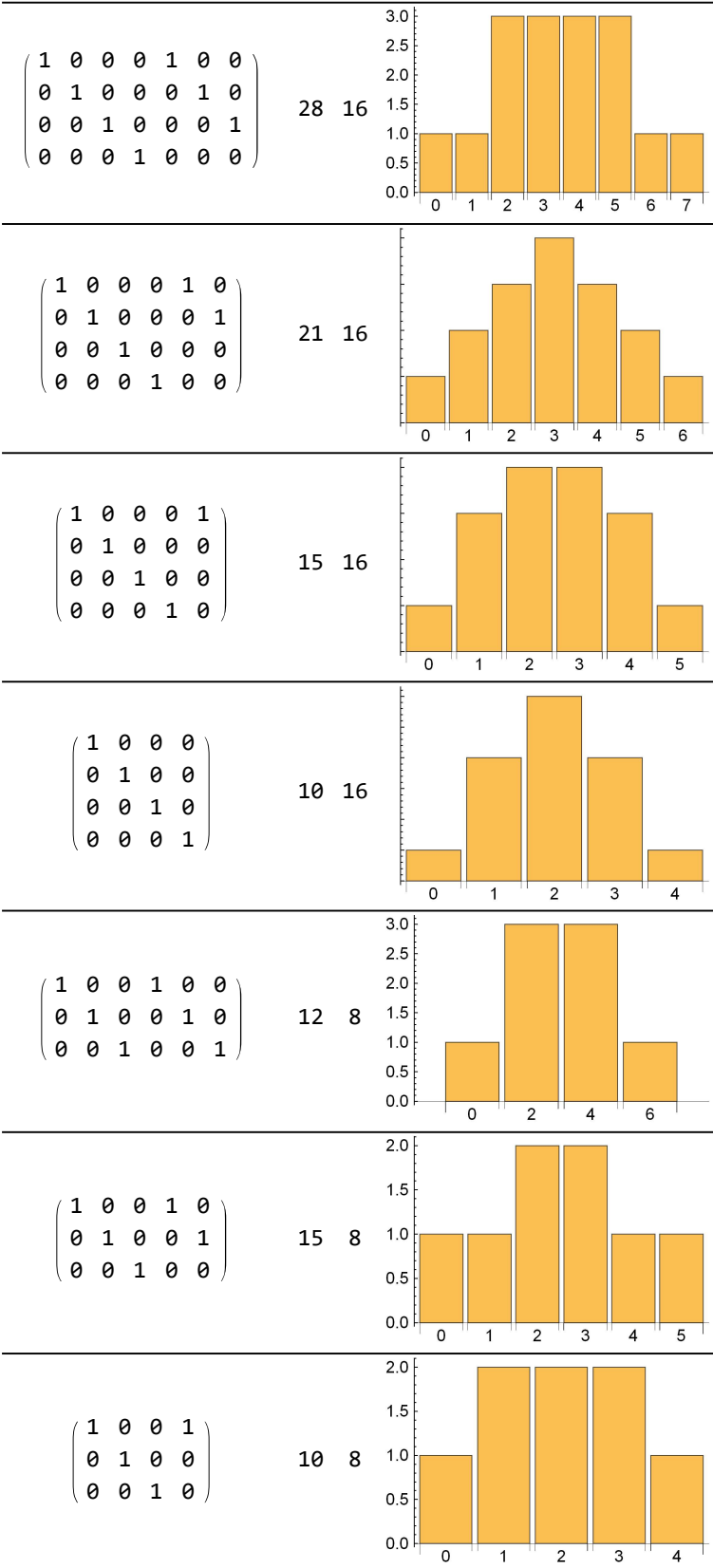
t = Table[{i, j}, {i, 4, 1, -1}, {j, i, 0, -1}];
ms = genMat[#] & /@ Flatten[t, 1];
```

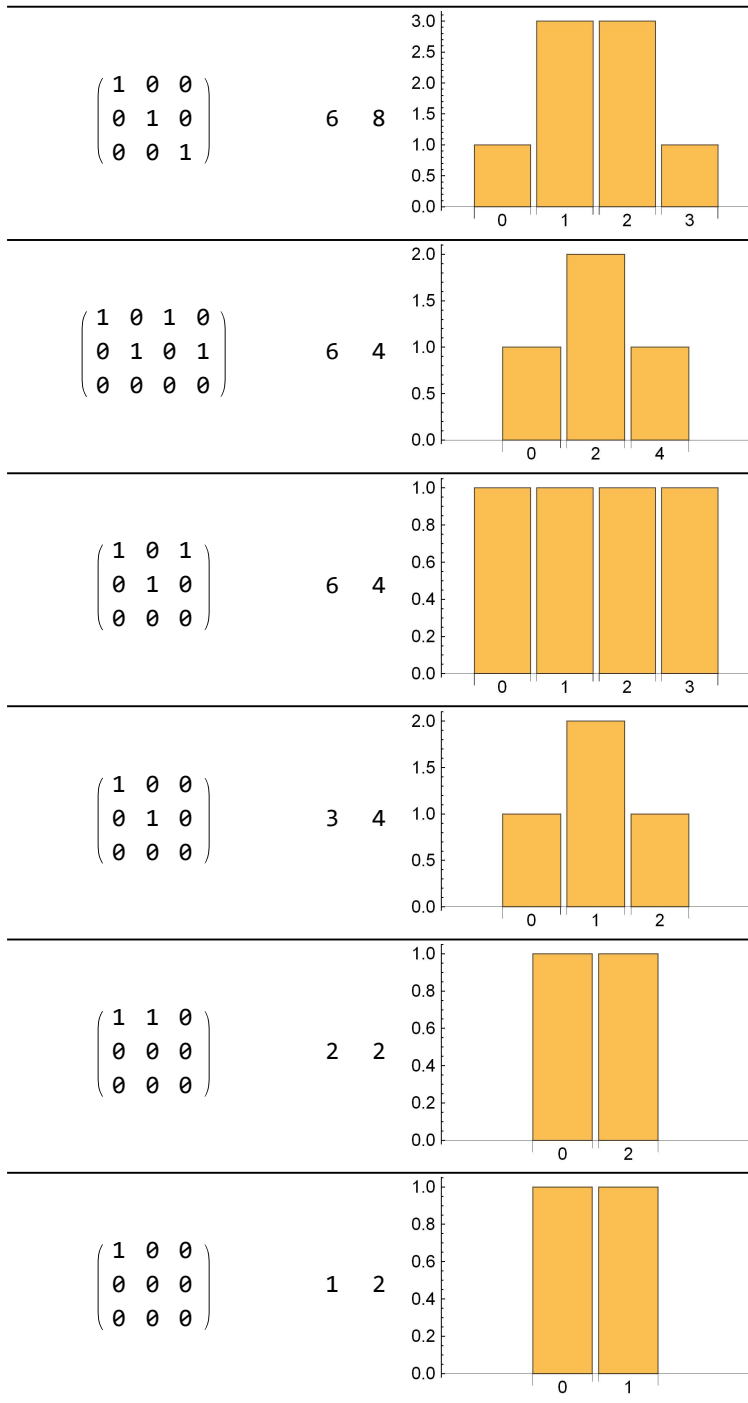
In[542]=

```
dispMatrixes[ms]
```

Out[542]//DisplayForm=







```
In[544]:= m = ms[[1]]; m // MatrixForm
```

```
Out[544]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

In[545]:= tt = Total /@ m
Out[545]= {2, 2, 2, 2}

In[546]:= Binomial[4, #] & /@ Range[0, 4]
Out[546]= {1, 4, 6, 4, 1}

In[547]:= (* binomials precalculation *)
For[i = 0, i ≤ 1000, i++,
  For[j = 0, j ≤ i, j++,
    BINOM[i, j] = Binomial[i, j]; (*&/@Range[0, n] *)
  ]
]

In[548]:= binomials[n_] := Binomial[n, #] & /@ Range[0, n]
binomials[n_] := BINOM[n, #] & /@ Range[0, n]


In[550]:= binomials[3]
Out[550]= {1, 3, 3, 1}

(* calc *)
BxN[seq_, n_] := Map[ (PadRight[{#}, n, 0] &), seq] // Flatten // #[[ ;; -n]] &

In[552]:= BxN[{1, 3, 1}, 1]
BxN[{1, 3, 1}, 3]
Out[552]= {1, 3, 1}
Out[553]= {1, 0, 0, 3, 0, 0, 1}

In[554]:= totalSeq[sequences_] := Module[{l = Max[Length /@ sequences], res},
  res = Array[0 &, l];
  (res += PadRight[#, l, 0]) & /@ sequences;
  res
]

In[558]:= totalSeq[{{1, 0, 0, 99}, {2, 100}, {1}}]
Out[558]= {4, 100, 0, 99}

In[564]:= {BxN[binomials[30], 2]} // ArrayPlot
Out[564]= 

```

```

In[566]:= (* sum histograms seq and seqx1, *)
totalSeqX[seq_, seqx1_] := Module[{pos},
  (*seqx1=Sign/@seqx;*)
  pos = Position[seqx1, 1] // Flatten;
  seqx1[[pos]];
  PadLeft[seq, Length@seq + # - 1, 0] & /@ pos // totalSeq
]

```

```
In[568]:= totalSeqX[binomials[10], binomials[1]]
          binomials[11]
```

```
Out[568]= {1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1}
```

```
Out[569]= {1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1}
```

```
In[571]:= m = genMat[{4, 4}]; m // MatrixForm
          tt = Total /@ m
```

```
Out[571]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Out[572]= {2, 2, 2, 2}
```

```
In[573]:= BxN[binomials[1], 2]
```

```
Out[573]= {1, 0, 1}
```

```
In[574]:= histSumDiagonal[seq_] := Module[{sequences},
          sequences = BxN[binomials[1], #] & /@ seq;
          Fold[totalSeqX, sequences[[1]], sequences[[2 ;;]]]
        ]
```

```
In[575]:= histSumDiagonal[tt]
```

```
Out[575]= {1, 0, 4, 0, 6, 0, 4, 0, 1}
```

```
In[576]:= histSumDiagonalOPT1[seq_] := Module[{tally, elem, k, seq1, seq2, sequences, fold},
          tally = SortBy[seq // Tally, Last];
          {elem, k} = (Last@tally);

          seq1 = BxN[binomials[k], elem];
          seq2 = Select[seq, # != elem &];
          sequences = {seq1} ~Join~ (BxN[binomials[1], #] & /@ seq2);
          fold = Fold[totalSeqX, sequences[[1]], sequences[[2 ;;]]]
          (*{seq1, seq2, sequences, fold}*)
        ]
```

```
In[577]:= tt2 = {2, 2, 2, 2}
          histSumDiagonalOPT1[tt2]
          histSumDiagonal[tt2]
```

```
Out[577]= {2, 2, 2, 2}
```

```
Out[578]= {1, 0, 4, 0, 6, 0, 4, 0, 1}
```

```
Out[579]= {1, 0, 4, 0, 6, 0, 4, 0, 1}
```

```

(* test the speed for basis of 999 vectors *)
r1 = AbsoluteTiming[Total /@ genMat[{999, 99}] // histSumDiagonal];
r2 = AbsoluteTiming[Total /@ genMat[{999, 99}] // histSumDiagonalOPT1];

In[582]:= r1[[1]]
          r2[[1]]
Out[582]= 0.731216

Out[583]= 0.244575

In[619]:= dispMatrixes2[ms_] := Module[{ws, wsbin, mfms, pics},
  ws = computeW /@ ms;
  wsbin = (histSumDiagonalOPT1[Total /@ #]) & /@ ws;

  mfms = MatrixForm /@ ws;
  pics = weightsHistogram /@ ws;
  GridBox[MapThread[Join, {List /@ mfms, List /@ wsbin, List /@ pics}],
    GridBoxDividers → {"Rows" → {{True}}}] // DisplayForm
  (*{ws}*)
]

In[601]:= r1 = AbsoluteTiming[Total /@ genMat[{1}, 1, 1] // histSumDiagonal]
Out[601]= {0.000293034, {1, 1}}

In[596]:= genMat[{1}] // Total
Out[596]= {1, 0, 0}

In[599]:= histSumDiagonalOPT1[{2}]
Out[599]= {1, 0, 1}

In[604]:= ms[[-1]]
Out[604]= {{1, 0, 0}, {0, 0, 0}, {0, 0, 0}}

In[605]:= t = Table[{i, j, k}, {i, 4, 1, -1}, {j, i, 0, -1}, {k, j, 0, -1}];
ms = genMat[#, 1, 1] & /@ Flatten[t, 2];

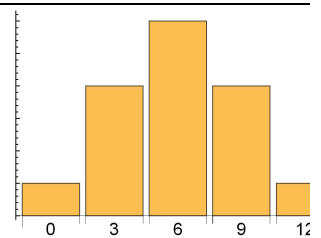
(* Test bruteforce histogram and binomial-way histogram. They are the same. *)
dispMatrixes2[ms]

```

Out[620]//DisplayForm=

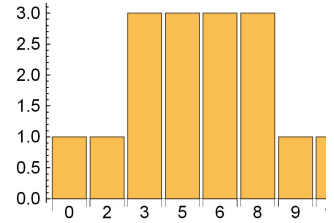
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

{1, 0, 0, 4, 0, 0, 6, 0, 0, 4, 0, 0, 1}



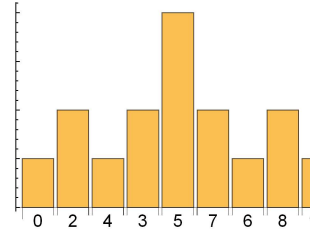
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$\{1, 0, 1, 3, 0, 3, 3, 0, 3, 1, 0, 1\}$



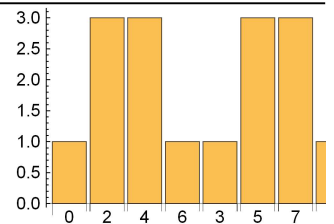
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$\{1, 0, 2, 2, 1, 4, 1, 2, 2, 0, 1\}$



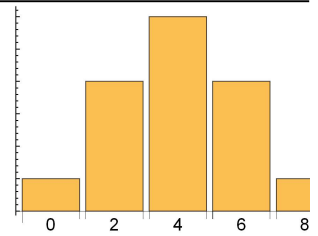
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$\{1, 0, 3, 1, 3, 3, 1, 3, 0, 1\}$



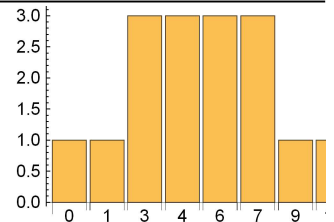
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\{1, 0, 4, 0, 6, 0, 4, 0, 1\}$



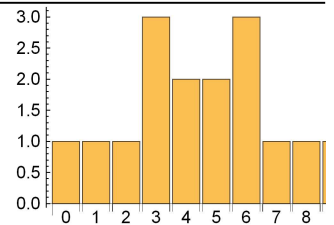
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\{1, 1, 0, 3, 3, 0, 3, 3, 0, 1, 1\}$



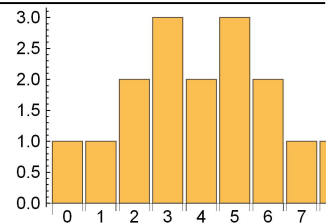
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\{1, 1, 1, 3, 2, 2, 3, 1, 1, 1\}$

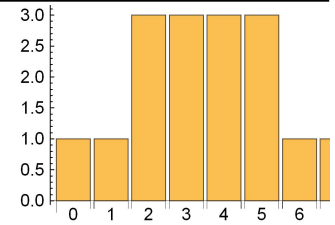


$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

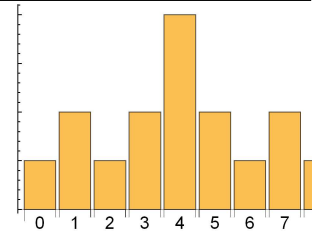
$\{1, 1, 2, 3, 2, 3, 2, 1, 1\}$



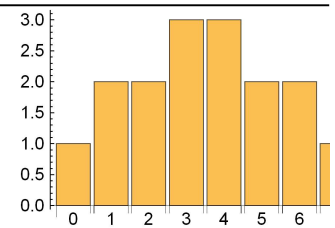
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

 $\{1, 1, 3, 3, 3, 3, 1, 1\}$


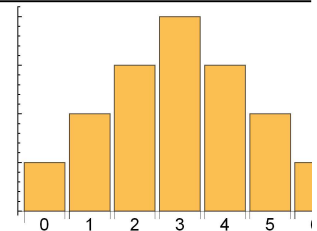
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

 $\{1, 2, 1, 2, 4, 2, 1, 2, 1\}$


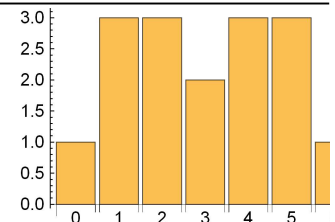
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

 $\{1, 2, 2, 3, 3, 2, 2, 1\}$


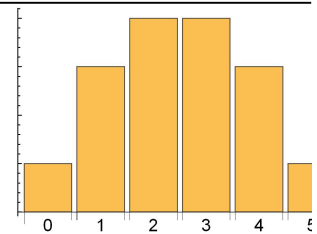
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

 $\{1, 2, 3, 4, 3, 2, 1\}$


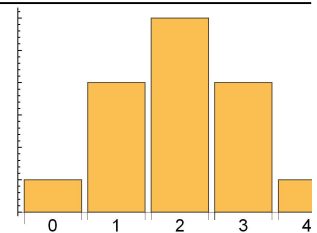
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

 $\{1, 3, 3, 2, 3, 3, 1\}$


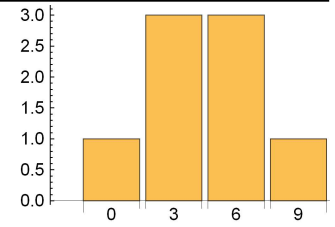
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

 $\{1, 3, 4, 4, 3, 1\}$


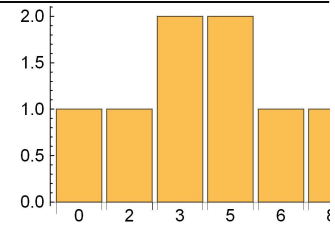
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

 $\{1, 4, 6, 4, 1\}$


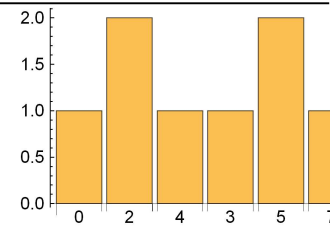
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

 $\{1, 0, 0, 3, 0, 0, 3, 0, 0, 1\}$


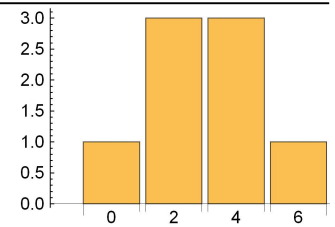
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

 $\{1, 0, 1, 2, 0, 2, 1, 0, 1\}$


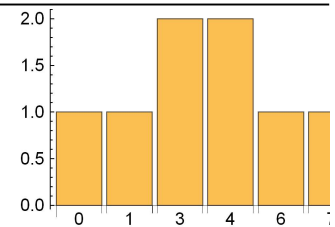
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

 $\{1, 0, 2, 1, 1, 2, 0, 1\}$


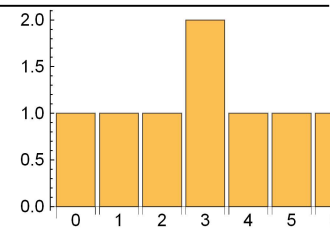
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

 $\{1, 0, 3, 0, 3, 0, 1\}$


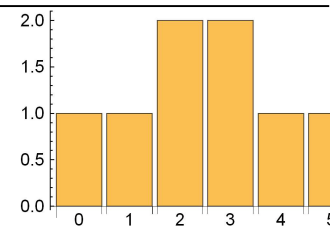
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

 $\{1, 1, 0, 2, 2, 0, 1, 1\}$


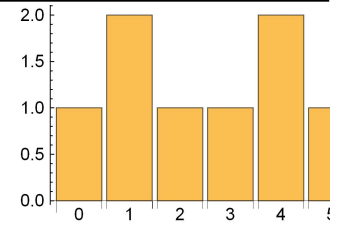
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

 $\{1, 1, 1, 2, 1, 1, 1\}$


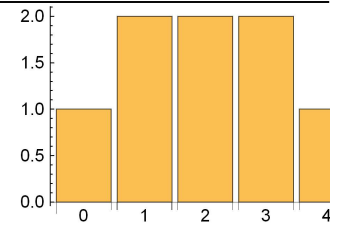
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

 $\{1, 1, 2, 2, 1, 1\}$


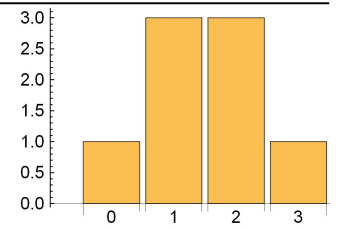
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

 $\{1, 2, 1, 1, 2, 1\}$


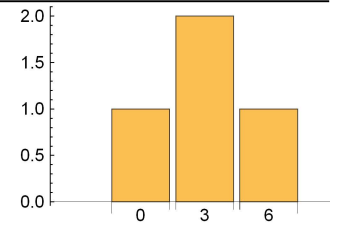
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

 $\{1, 2, 2, 2, 1\}$


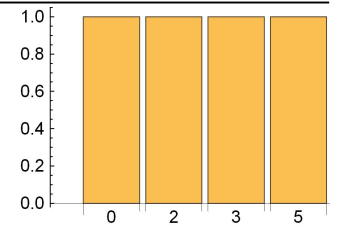
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

 $\{1, 3, 3, 1\}$


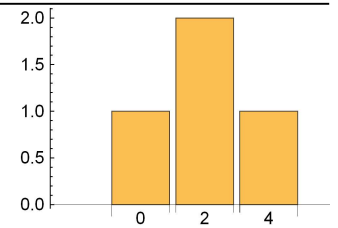
$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

 $\{1, 0, 0, 2, 0, 0, 1\}$


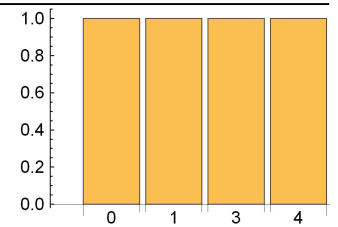
$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

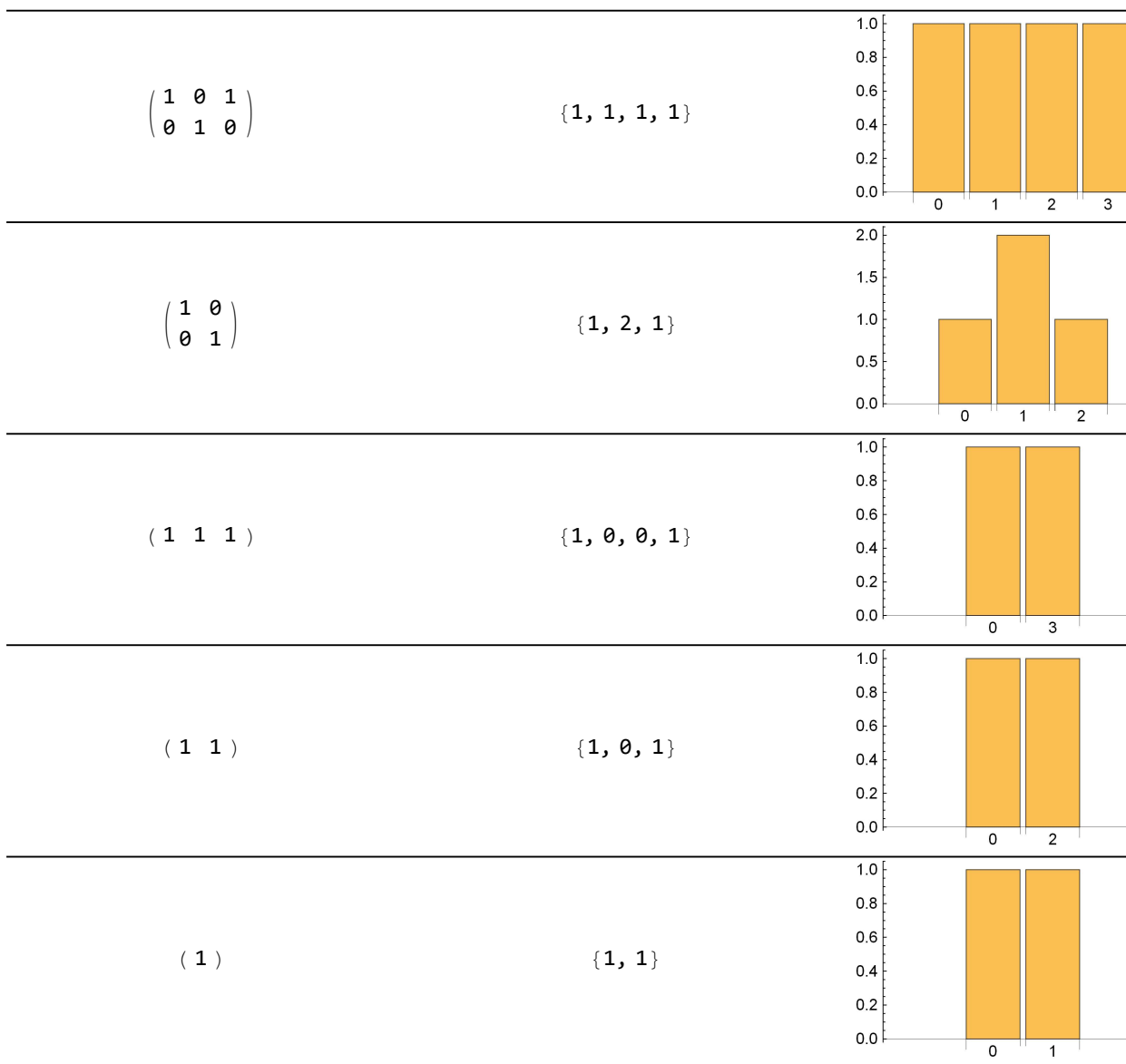
 $\{1, 0, 1, 1, 0, 1\}$


$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

 $\{1, 0, 2, 0, 1\}$


$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

 $\{1, 1, 0, 1, 1\}$




```
In[642]:= m = genMat[{16, 10, 4}]; m // MatrixForm
```

[illegible]

```
t1 = AbsoluteTiming [computeW[m] [[ ;; , 2]] ]
t2 = AbsoluteTiming [histSumDiagonal[Total /@m]]
t3 = AbsoluteTiming [histSumDiagonalOPT1[Total /@m]]
t1[[1]] / t2[[1]]
t1[[1]] / t3[[1]]
```

$$\text{Out[655]} = \{1.95995, \{1, 6, 21, 60, 144, 300, 566, 972, 1536, 2264, 3114, 4020, 4897, 5622, 6105, 6280, 6105, 5622, 4897, 4020, 3114, 2264, 1536, 972, 566, 300, 144, 60, 21, 6, 1\}\}$$

Out[656]= {0.0014266, {1, 6, 21, 60, 144, 300, 566, 972, 1536, 2264, 3114, 4020, 4897, 5622, 6105, 6280, 6105, 5622, 4897, 4020, 3114, 2264, 1536, 972, 566, 300, 144, 60, 21, 6, 1}}

$$\text{Out}[657] = \{0.00139191, \{1, 6, 21, 60, 144, 300, 566, 972, 1536, 2264, 3114, 4020, 4897, 5622, 6105, 6280, 6105, 5622, 4897, 4020, 3114, 2264, 1536, 972, 566, 300, 144, 60, 21, 6, 1\}\}$$

Out[658]= 1373.87

Out[659]= 1408.1