SPACE MISSION DESIGN AND OPERATIONS

EE-585 - SPRING 2024

# Mini-Project: A trip to Mars and back

*From:* ***Joshua Cohen-Dumani***

*To:* Claude Nicollier

May 13, 2024

# Numerical Data

| Name | Abbreviation | Value | Unit |
|---|---|---|---|
| Dry Mass B | $m_B$ | 20 x $10^3$ | kg |
| Dry Mass C | $m_C$ | 10x $10^3$ | kg |
| Dry Mass D | $m_A$ | 5 x $10^3$ | kg |
| Propellant | $I_{sp}$ | 380 | s |
| Gravitational acceleration on Earth's surface | $g_0$ | 9.81 | $m\dot{s}^{-2}$ |
| Earth's orbit | $E_O$ | 1.496 x $10^{11}$ | m (1 AU) |
| Earth's radius | $E_R$ | 1.496 x $10^{11}$ | m |
| Mars' orbit | $M_O$ | 1.524 | AU |
| Mars' radius | $M_R$ | 3.3895 x $10^6$ | m |
| Earth's gravitational parameter | $\mu_e$ | 3.986 x $10^{14}$ | $m^3\dot{s}^{-2}$ |
| Sun's gravitational parameter | $\mu_m$ | 1.327 x $10^{20}$ | $m^3\dot{s}^{-2}$ |
| Mars' gravitational parameter | $\mu_s$ | 4.283 x $10^{13}$ | $m^3\dot{s}^{-2}$ |
| LEO altitude | $h_{LEO}$ | 500 x $10^3$ | m |
| LMO altitude | $h_{LMO}$ | 300 x $10^3$ | m |

# Contents

# 1 Context

In this assignment, we are tasked with calculating the propellant needed in order to complete a trip from Earth LEO to Mars and back. We will be doing so using several maneuvers, which are summarised qualitatively in figure 1.

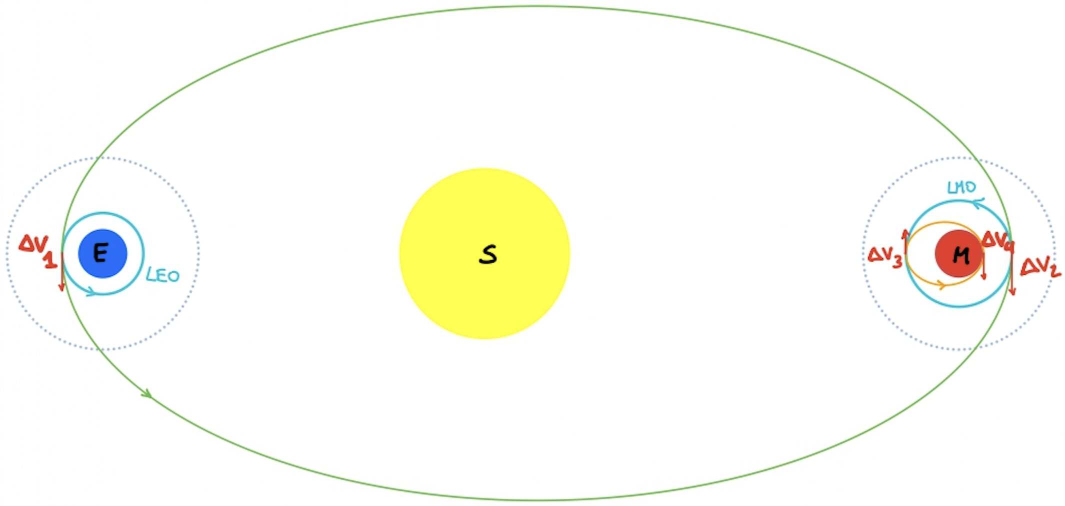


Figure 1: Summary of the $\Delta v$ burns

# 2 One-way journey to Mars

Our first task is to determine the total propellant needed to fuel a one-way trip from Earth LEO to Mars' surface, where our refueling station is set.

For this, we will use several formula in a somwewhat repetitive manner, they are defined like this:

Velocity on a circular orbit:

$$V_c = \sqrt{\frac{\mu}{r}} \tag{1}$$

Velocity at apogee ($V_a$)on an elliptical orbit:

$$V_a = \sqrt{\frac{2\mu r_a}{rp(ra + rp)}} \tag{2}$$

and at perigee ($V_p$):

$$V_p = \sqrt{\frac{2\mu r_p}{rp(ra + rp)}} \tag{3}$$

## 2.1 From LEO to MTO

Our first maneuver is to go from Low Earth Orbit (LEO) to Mars Transfer Orbit (MTO) using a purely heliocentric Hohmann transfer.

First, we need to find $v_d = \sqrt{(v_d^{\infty})^2 + (V_{Erd})^2}$, with $v_d^{\infty} = \Delta v_{tran}$ the velocity needed to reach the elliptical path to Mars, and $V_{Erd}$ the escape velocity.

From $v_d$ (which we defined above using the conservation of mechanical energy, given during week 6), we can get the first term of our $\Delta v_1$. The second component, $v_{LEO}$, is obtained with:

$$v_{LEO} = \frac{\mu_E}{E_R + h_{LEO}} \tag{4}$$

Thus, we can calculate $\Delta v_{tran}$:

$$\Delta v_{tran} = \sqrt{\frac{2\mu_s M_O}{E_O(E_O + M_O}} - \sqrt{\frac{\mu_s}{E_O}} = 2945[\frac{m}{s}] \tag{5}$$

From which, $\Delta V_1$:

$$\Delta V_1 = \sqrt{(v_{tran})^2} + 2\frac{\mu_E}{E_R + h_{LEO}} = 3549[\frac{m}{s}] \tag{6}$$

# 3 From MTO to LMO

Now that we have successfully exited Earth's Sphere of Influence, we now need to reach Low Mars Orbit (LMO).

Let's first start by calculating the entry speed, given by the difference of our speed at the aphelion minus the speed of Mars:

$$v_a^\infty = \sqrt{\frac{2\mu_s E_O}{M_O(E_O + M_O}} - \sqrt{\frac{\mu_s}{M_O}} = -2649.84[m/s] \tag{7}$$

Next, we also know that the insertion velocity is given by:

$$|\Delta v_{p,\vec{insert}}| = \sqrt{\frac{2\mu}{r_p} + (v_a^\infty)^2} - \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a_i}} \tag{8}$$

With p for periapsis (perigee), and $a_i$ the semi-major axis of the elliptical orbit at insertion. In our case we will be entering a roughly circular orbit, and thus $a = M_R + h_{LMO}$ (the radius of our orbit). And thus we can calculate $\Delta V_2$:

$$\Delta V_2 = \sqrt{\frac{2\mu_M}{M_R + h_{LMO}} + (v_a^\infty)^2} - \sqrt{\frac{\mu_M}{M_R + h_{LMO}}} = 2092[m/s] \tag{9}$$

# 4 From LMO to Mars

Now that we are in Mars's SOI, we now need to do a third maneuver in order to deorbit and reach Mars' surface. This calculation is quite straightforward and can be thought of as a Hohmann transfer. Thus being the velocity at apogee (of the new, smaller ellipse) minus the circular, initial velocity:

$$\Delta V_3 = \sqrt{\frac{2\mu_M M_R}{(M_R + h_{LMO})(2M_R + h_{LMO})}} - \sqrt{\frac{\mu_M}{M_R + h_{LMO}}} = -73[m/s] \tag{10}$$

## 5 Landing maneuver

We are now very close to landing on the surface. We just need to do one last maneuver, in order to slow down enough to land successfully (ie with final velocity of 0). We can for this use the equation 3:

$$\Delta V_4 = -\sqrt{\frac{2\mu_M(M_R + h_{LMO})}{M_R(2M_R + h_{LMO}}} = -3629[m/s] \tag{11}$$

## 6 Calculating the required propellant

We can use the Tsiolkovsky equation to calculate the required propellant now that we have a good overview of the $\Delta V$ required at each step.

$$\Delta V = g \cdot I_{sp} \cdot ln(\frac{m_i}{m_f}) \tag{12}$$

From the given assumptions in the project prompt, we know that the initial mass $m_i$ is simply given by the sum of the final mass $m_f$ and the propellant mass $m_p$. Thus we have (using 12):

$$m_p = m_i - m_f = m_f \cdot (e^{\frac{\Delta V}{g_0 \cdot I_{sp}}} - 1) \tag{13}$$

Let's summarize our findings so far, in order to have a clear view:

| | $\Delta V$ [m/s] | Vehicle Composition |
|---|---|---|
| $\Delta V_1$ | 3549 | BCD |
| $\Delta V_2$ | 2092 | BCD |
| $\Delta V_3$ | 73 | CD |
| $\Delta V_4$ | 3629 | CD |

Since the masses can be calculated recursively, we will start with the propellant needed for $\Delta V_4$, then moving to $\Delta V_3$, etc.

$m_{p,4}$, with $m_{f,4} = m_C + M_D$ is then:

$$m_{p,4} = m_{f,4} \cdot (e^{\frac{\Delta V_4}{g_0 \cdot I_{sp}}} - 1) = 24711[kg] \tag{14}$$

$m_{p,3}$, with $m_{f,3} = m_C + M_D + m_{p,4}$ is:

$$m_{p,3} = m_{f,3} \cdot (e^{\frac{\Delta V_3}{g_0 \cdot I_{sp}}} - 1) = 785[kg] \tag{15}$$

$m_{p,2}$, with $m_{f,2} = m_C + M_D + m_{p,3} + m_{p,4}$ is:

$$m_{p,2} = m_{f,2} \cdot (e^{\frac{\Delta V_2}{g_0 \cdot I_{sp}}} - 1) = 45534[kg] \tag{16}$$

and finally, $m_{p,1}$, with $m_{f,1} = m_C + M_D + m_{p,2} + m_{p,3} + m_{p,4}$ is:

$$m_{p,1} = m_{f,1} \cdot (e^{\frac{\Delta V_1}{g_0 \cdot I_{sp}}} - 1) = 168694[kg] \tag{17}$$

# 7 Return trip

We now need to calculate the propellant needed for the return trip. Fortunately, the $\Delta V$ values are the same, as our trip is symmetrical. However, the values are not quite the same because we are leaving some modules behind.

Let's again start from the end, so with only our Earth-Mars Transfer Vehicle, and $\Delta V_1$ and $\Delta V_2$ burns. For $\Delta V_3$ and $\Delta V_4$, we will only be using our ascent vehicle.

For $m_{p,1}$, we have $m_{f,1} = m_B$, and thus:

$$m_{p,1} = m_{f,1} \cdot (e^{\frac{\Delta V_1}{g_0 \cdot I_{sp}}} - 1) = 31820[kg] \tag{18}$$

For $m_{p,2}$, we have $m_{f,2} = m_B + m_{p,1}$, and thus:

$$m_{p,2} = m_{f,2} \cdot (e^{\frac{\Delta V_2}{g_0 \cdot I_{sp}}} - 1) = 39004[kg] \tag{19}$$

For $m_{p,3}$, we have $m_{f,3} = m_D + m_{p,1} + m_{p,2}$, and thus:

$$m_{p,3} = m_{f,3} \cdot (e^{\frac{\Delta V_3}{g_0 \cdot I_{sp}}} - 1) = 1499[kg] \tag{20}$$

Finally, for $m_{p,3}$, we have $m_{f,3} = m_D + m_{p,1} + m_{p,2}$, and:

$$m_{p,4} = m_{f,4} \cdot (e^{\frac{\Delta V_4}{g_0 \cdot I_{sp}}} - 1) = 127381[kg] \tag{21}$$

# 8 Results and Discussion

## 8.1 Result table

We can summarize our results in the following table:

| | $\Delta V$ [m/s] | Vehicle Composition | $m_p$ [kg] | $m_f$ [kg] | $m_i$ [kg] |
|---|---|---|---|---|---|
| Earth to Mars | | | | | |
| $\Delta V_1$ | 3549 | BCD | 168 694 | 106 030 | 274 724 |
| $\Delta V_2$ | 2092 | BCD | 45 534 | 60 496 | 106 030 |
| $\Delta V_3$ | 73 | CD | 785 | 40 496 | 39 711 |
| $\Delta V_4$ | 3629 | CD | 24 711 | 39 711 | 15 000 |
| Return trip | | | | | |
| $\Delta V_1$ | 3549 | B | 31 820 | 20 000 | 51 820 |
| $\Delta V_2$ | 2092 | B | 39 004 | 51820 | 90 824 |
| $\Delta V_3$ | 73 | D | 1499 | 75 824 | 77 323 |
| $\Delta V_4$ | 3629 | D | 127 381 | 77 323 | 204 704 |
| | | Total | Earth to Mars | 239 724 | |
| | | Total | Return Trip | 199 704 | |

We will need 239 724 kg of propellant for the one-way trip from Earth to Mars, which is 87.26% of the $m_i$ when leaving Earth. For the return trip, we will need 199 704 kg, which is 97.6% of our take-off mass from Mars (with the caveat that we are here considering the weight of only the ascent vehicle, but there is also the Transfer Vehicle which has stayed in orbit).

## 8.2 Discussion

When compared with a mission plan where the fuel is loaded onboard at the beginning of the mission, there are several benefits and drawbacks to consider.

Let's start with the benefits of using this mission plan with only enough fuel to go one way, and refueling on Mars. The main advantage is that you are taking off with less fuel, thus less weight, which allows you to save a lot of fuel. It is thus cheaper and more efficient during Earth take-off to only take the minimum amount of fuel possible (our calculations don't even take into account the take-off from Earth to LEO, where significant resources are also expended).

However, there are also a few drawbacks. First of all, if you had enough fuel for the return trip in the Transfer Vehicle, you would not need to take off a second time from Mars using the ascent vehicle (we would have a much lighter ascent vehicle, and thus a much cheaper second take-off). Secondly, there is also the fact that in order to set up this refueling station, several unmanned (and perhaps more...) previous missions has to be executed, which should be at least partly taken in consideration.

Thus, while instinctively it makes more sense to go with our proposed mission plan, where we oly bring enough for one way and refuel on Mars, this is not so obvious when considering the total cost of the operation (including previous preparation work), and a more in-depth analysis is warranted. It is however probable that taking off with 200 extra tonnes (almost doubling the total initial weight) is suboptimal and that the cost of setting up a refueling station is worth it (especially in the long term, where it can be reusable many times).