Math 123 Homework 1
(Tuesday, May 8th)
Solutions

2 a) $1+2+3+\cdots+250 = \dfrac{250\cdot 251}{2} = \boxed{31{,}375}$

b.) $200+201+202+\cdots+500$

$$\left.\begin{aligned}&= 1+2+3+\cdots+199+200+201+\cdots+500\\ &\quad -(1+2+3+\cdots+199)\end{aligned}\right\}\text{Dave's Party Principl}$$

$$= \frac{500\cdot 501}{2} - \frac{199\cdot 200}{2}$$

$$= 125\,250 - 19900 = \boxed{105{,}350}$$

Here is a second solution...

$$200+201+202+\cdots+500$$

$$= 200+(200+1)+(200+2)+\cdots+(200+300)$$

$$\begin{aligned}= 200 &+ 200 + 200 + \cdots + 200\\ &+ 1 + 2 + \cdots + 300\end{aligned}$$

$$= 200\cdot\underset{\substack{\uparrow\\ \text{Not } 300}}{301} + \frac{300\cdot 301}{2} = \boxed{105{,}350}$$

c) $5+10+15+\cdots+3335 = 5\cdot 1+5\cdot 2+5\cdot 3+\cdots+5\cdot 66$

$$= 5\cdot(1+2+3+\cdots+667) = 5\cdot\frac{667\cdot 668}{2}$$

$$= \boxed{1{,}113{,}890}$$

d) Using the formula

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

we get

$$1^2+2^2+3^2+\cdots+571^2 = \frac{571\cdot 572\cdot 1143}{6}$$

$$\boxed{= 62{,}219{,}586}$$

e.) $50^2+51^2+52^2+\cdots+100^2$

$$\left.\begin{aligned}&= 1^2+2^2+\cdots+49^2+50^2+51^2+\cdots+100^2\\ &\quad -(1^2+2^2+\cdots+49^2)\end{aligned}\right\}\text{Dave's party principle}$$

$$= \frac{100\cdot(100+1)(2\cdot 100+1)}{6} - \frac{49\cdot(49+1)(2\cdot 49+1)}{6}$$

$$= 338350 - 40425$$

$$= \boxed{297{,}925}$$

f) $1^2+3^2+5^2+\cdots+959^2$

$$\left.\begin{aligned}&= 1^2+2^2+3^2+4^2+5^2+\cdots+959^2\\ &\quad -(2^2+4^2+6^2+\cdots+958^2)\end{aligned}\right\}\text{Dave's party principle}$$

$$= \frac{959\cdot 960\cdot 1919}{6} - (2^2\cdot 1^2+2^2\cdot 2^2+2^2\cdot 3^2+\cdots+2^2\cdot 479^2)$$

common factor

$$= 294{,}451{,}360 - 2^2\cdot(1^2+2^2+3^2+\cdots+479^2)$$

$$= 294{,}451{,}360 - 4\cdot\frac{479\cdot 480\cdot 959}{6}$$

$= 294{,}451{,}360 - 146{,}995{,}520$

$$\boxed{= 147{,}455{,}840}$$

3. a) $1 + 5 + 9 + \cdots + (4n+1)$

$= 1 + (4\cdot 1 + 1) + (4\cdot 2 + 1) + \cdots + (4\cdot n + 1)$

$= 1 + 1 + 1 + \cdots + 1$ ← $n+1$ 1's

$+ 4\cdot 1 + 4\cdot 2 + \cdots + 4\cdot n$

$= (n+1) + 4\cdot(1 + 2 + \cdots + n)$

$= (n+1) + \cancel{4}^{2} \cdot \dfrac{n(n+1)}{\cancel{2}}$

$= n + 1 + 2n^2 + 2n$

$$\boxed{= 2n^2 + 3n + 1 = (2n+1)(n+1)}$$

Either answer is fine.

b) $2 + 5 + 8 + \cdots + (3n+2)$

$= 2 + (3\cdot 1 + 2) + (3\cdot 2 + 2) + \cdots + (3\cdot n + 2)$

$= 2 + 2 + 2 + \cdots + 2$ ← $n+1$ 2's

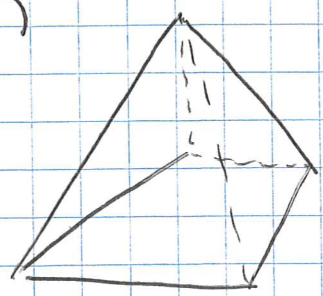
$+ 3\cdot 1 + 3\cdot 2 + \cdots + 3\cdot n$

$= 2\cdot(n+1) + 3\cdot(1 + 2 + \cdots + n)$

$= 2\cdot(n+1) + 3\cdot\dfrac{n(n+1)}{2} = \dfrac{4(n+1)}{2} + \dfrac{3n(n+1)}{2}$

$= \dfrac{4n + 4 + 3n^2 + 3n}{2} = \boxed{\dfrac{3n^2 + 7n + 4}{2}}$

4. a)



The pyramid is made from one square and 4 triangles so

$F = 5$.

$$E = \frac{\text{total \# of edges on all faces}}{2} = \frac{3\cdot 4 + 4}{2} = 8$$

(or just count them)

$V = 5$ (just count them).

b) When we truncate the top vertex becomes a square, the triangles become hexagons, the four base vertices become triangles and the square base becomes an octagon.

So $F = 1 + 4 + 4 + 1 = 10$.

$$E = \frac{\text{total \# of edges on all faces}}{2}$$

$$= \frac{1\cdot 4 + 4\cdot 6 + 4\cdot 3 + 8\cdot 1}{2}$$

$$= 24$$

$$V = \frac{\text{total \# of vertices on all faces}}{3}$$

← once we have truncated we always divide by 3 here

$$= \frac{1\cdot 4 + 4\cdot 6 + 4\cdot 3 + 8\cdot 1}{3}$$

$$= 16$$

c) The square becomes an octagon
The 4 hexagons become 12-gons
The 4 triangles become hexagons
The octagon becomes a 16-gon
The 16 vertices become triangles

So $F = 1 + 4 + 4 + 1 + 16 = 26$

$$E = \frac{1 \cdot 8 + 4 \cdot 12 + 4 \cdot 6 + 1 \cdot 16 + 16 \cdot 3}{2}$$

$$= \frac{144}{2} = 72$$

$$V = \frac{1 \cdot 8 + 4 \cdot 12 + 4 \cdot 6 + 1 \cdot 16 + 16 \cdot 3}{3}$$

$\leftarrow$ once we have truncated this will be 3

$$= \frac{144}{3} = 48$$