MATH 123 MIDTERM # 2

- This exam has 10 questions.
- There are 52 marks available.
- Show your work. Marks are awarded for completeness, clarity and correctness.
- You may have one hour and fifty minutes to complete the exam.

NAME:____

1. a) Use the rules of inference and the laws of logic to show that the following argument is valid:

$$\begin{array}{l} p \to \neg q \\ q \vee r \\ \underline{\neg r \qquad} \\ \therefore \neg(p \vee r) \end{array}$$

b) Prove that the following argument is invalid by finding truth values for p, q, r, s, and t such that all of the premises are true but the conclusion is false.

$$\begin{array}{l} \neg q \wedge s \\ p \to s \\ t \to r \\ \underline{\neg p \vee \neg t} \\ \therefore s \to (r \wedge \neg q) \end{array}$$

(6 marks)

2. Let $U = \{c, o, w, b, r, a, i, n\}$, $A = \{b, a, r, n\}$, $B = \{c, a, r, b\}$, and $C = \{r, a, w\}$. Determine each of the following:

a) $(A - C) \Delta \overline{B}$

b) $\overline{A \cap \overline{C} \cup (B - A)}$

(4 marks)

3. Use an <u>elementwise proof</u> to prove that $\overline{A} \cap B = \overline{A \cup \overline{B}}$.

(4 marks)

4. Determine the number of 5 element subsets of $\{a,b,c,d,e,f,g,h\}$ that

a) contain a and b.

b) contain a but not b.

c) contain a or b.

(6 marks)

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5. Let $A=\{2,4,6,8....\}$ and $B=\{-3,-5,-7,-9,...\}$. Find an explicit function (i.e. a formula), $f:A\to B$ such that f is both 1-1 and onto (You do not need to prove your answer is correct).

(3 marks)

6. Let $A=\{2,4,6,8,\ldots\}$ and let $B=\{\ldots,-5,-3,-1,1,3,5,\ldots\}$ and define $f:A\to B$ by

$$f(n)=\begin{cases} -\dfrac{n}{2} & \text{if } \quad 4 \nmid n \\ \dfrac{n}{2}-1 & \text{if } \quad 4 \mid n \end{cases}$$

a) Prove that f is 1-1.

b) Find a value of n so that $f(n)=135$.

c) Find a value of n so that $f(n)=-211$.

(8 marks)

7. Use the principle of mathematical induction to prove the following:

$$3+7+11+\cdots+(4n-1)=2n^2+n \text{ for all } n\geq 1.$$

(6 marks)

8. A sequence is defined as follows:

$$a_0 = 1,$$
$$a_1 = 3,$$
$$a_n = 2a_{n-1} + 3a_{n-2} \text{ for } n \geq 2.$$

a) Compute a_2 and a_3.

b) Use the strong form of mathematical induction to prove that $a_n = 3^n$ for all $n \geq 1$.

(6 marks)

9. Use the definition of divides and a direct proof to prove the following statement:

If $a|b$ and $a|c$ then $a^2 | b^2 + 3bc + 2c^3$.

(5 marks)

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10. Use a proof by contradiction to prove the following statement:

Let n, a, b and c be positive integers. If $n = abc$ then at least one of a, b or c is less than or equal to $\sqrt[3]{n}$.

(4 marks)