CSCI-B490: Quantum Programming
Homework 6
Due: Thur, Feb 27

1 Instructions

The questions marked as "challenge research questions" are optional and can be turned in any time before dead week for extra credit.

2 Exercises on discrete Fourier transforms

Let $\mathbf{f} \in \mathbb{C}^N$ be a vector of N complex numbers $(f_0, f_1, \cdots, f_{N-1})$. Let $\omega = e^{2\pi i/N}$ be the principal Nth root of unity (technically ω_N but we omit the subscript when it's clear from context). The Discrete Fourier Transform (DFT) of $\mathbf{f}$ is a vector $\hat{\mathbf{f}}$ whose kth element is defined by:

$$\hat{f}_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \omega^{kj} f_j$$

The action of DFT on the vector $\mathbf{f}$ can be described by a matrix.

- **Question I.** Write the DFT matrix for $N = 2, 3, 4$, and 5.
- **Question II.** A *unitary* matrix is defined to be a complex-valued square matrix whose inverse is equal to its conjugate transpose. Verify that all four matrices above are unitary.
- **Question III.** The straightforward implementation of DFT in a classical programming language will have complexity $O(N^2)$. Implement it in Python.
- **Challenge Research Question I.** Can you implement a reversible version of the above algorithm in Qiskit?
- **Challenge Research Question II** The best known classical implementation of the DFT, known as the *Fast Fourier Transform* (FFT) has complexity $O(N \log N)$. Can you implement it? **Hint.** The simplest approach is to consider instances when N is a power of 2 and use the divide-and-conquer algorithm of Cooley-Tukey.
- **Challenge Research Question III** Can you implement a reversible version of the FFT in Qiskit?
- **Question IV.** Describe a quantum circuit to implement DFT following the presentation in the 2/20 lecture. Argue that the implementation has complexity $O(\log^2 N)$. **Hint.** Assuming each gate takes one time unit, this essentially is asking you to count the number of gates.

3 Exercises on superpositions

Do exercises 2.2, 2.3 and 2.4 from Rieffel and Polak. You will need the following definitions:

$$|\mathbf{i}\rangle := \frac{1}{\sqrt{2}}\big(\,|0\rangle + i\,|1\rangle\,\big)$$
$$|-\mathbf{i}\rangle := \frac{1}{\sqrt{2}}\big(\,|0\rangle - i\,|1\rangle\,\big)$$

Definition. *Let H be a two-dimensional complex Hilbert space. Let h_1, h_2 be unit vectors in H. h_1 and h_2 are* equivalent as quantum states *if there exists a modulus one complex number c such that $h_1 = ch_2$.*