M482 Homework 2

Joshua Larkin

January 2019

1 Practice with definitions.

(G_1) [image here]
Formally, G_1 is the pairing of the set of states $S = \{a, b, c, d\}$ with the relation $R = \{(a, a), (b, b), (c, c), (d, d), (b, a), (b, c), (c, d)\}$.

reflexive True, all nodes a, b, c, d bear R to themselves.

symmetric False, counter-example: bRc but cRb does not exist.

anti-symmetric Yes, since there is no symmetry, the only time aRb and bRa holds is when $a = b$. We know the graph is reflexive, so $a = b$ holds for all nodes.

transitive False, counter-example: bRc and cRd but bRd does not exist.

euclidean False, in the case of b when bRa and bRb, aRb does not exist.

equivalence relation False, only reflexive, i.e. not symmetric and not transitive.

partial function False, counter-example: bRa and bRc but $a \neq c$.

(G_2) [image here]
Formally, G_2 is the pairing of the set of states $S = \{a, b, c\}$ with the relation $R = \{(a, b), (a, c), (b, c)\}$.

reflexive False, there is no node $s \in S$ such that sRs.

symmetric False, counter-example: aRb but bRa does not exist.

anti-symmetric True, vacuously because $xRy \wedge yRx$ is false since no such nodes x, y exist.

transitive True, aRb and bRc and aRc, and $bRc \wedge cRb$ is false so vacuously true, and similar case for c.

euclidean $\forall a, b, c.\ aRb \wedge aRc \Rightarrow bRc$.

x	y	z	xRy	xRz	$xRy \wedge xRz : (P)$	$yRz : (Q)$	$P \Rightarrow Q$
a	b	c	T	T	T	T	T
a	c	b	T	T	T	T	T
b	a	c	F	T	F	T	T
b	c	a	T	F	F	T	T
c	a	b	F	F	F	T	T
c	b	a	F	F	F	F	T

equivalence relation

partial function

(G_3) [image here]
Formally, G_3 is the pairing of the set of states $S = \{a, b\}$ with the relation $R = \{(a, a), (a, b), (b, a)\}$.

reflexive

symmetric

anti-symmetric

transitive

euclidean

euclidean $\forall a, b, c.\ aRb \wedge aRc \Rightarrow bRc$.

x	y	z	xRy	xRz	$xRy \wedge xRz : (P)$	$yRz : (Q)$	$P \Rightarrow Q$
a	b	c	T	T	T	T	T
a	c	b	T	T	T	T	T
b	a	c	F	T	F	T	T
b	c	a	T	F	F	T	T
c	a	b	F	F	F	T	T
c	b	a	F	F	F	F	T

equivalence relation

partial function

(G_4) [image here]
Formally, G_4 is the pairing of the set of states $S = \{a\}$ with the relation $R = \{(a, a)\}$.

reflexive

symmetric

anti-symmetric

transitive

euclidean

euclidean $\forall a, b, c.\ aRb \wedge aRc \Rightarrow bRc$.

x	y	z	xRy	xRz	$xRy \wedge xRz : (P)$	$yRz : (Q)$	$P \Rightarrow Q$
a	b	c	T	T	T	T	T
a	c	b	T	T	T	T	T
b	a	c	F	T	F	T	T
b	c	a	T	F	F	T	T
c	a	b	F	F	F	T	T
c	b	a	F	F	F	F	T

equivalence relation

partial function

(G_5) [image here]
Formally, G_5 is the pairing of the set of states $S = \{a, b, c\}$ with the relation $R = \{(a, b), (b, a), (b, c), (c, b)\}$.

reflexive

symmetric

anti-symmetric

transitive

euclidean

euclidean $\forall a, b, c.\ aRb \wedge aRc \Rightarrow bRc$.

x	y	z	xRy	xRz	$xRy \wedge xRz : (P)$	$yRz : (Q)$	$P \Rightarrow Q$
a	b	c	T	T	T	T	T
a	c	b	T	T	T	T	T
b	a	c	F	T	F	T	T
b	c	a	T	F	F	T	T
c	a	b	F	F	F	T	T
c	b	a	F	F	F	F	T

equivalence relation

partial function

(G_6) [image here]
Formally, G_6 is the pairing of the set of states $S = \{a, b, c\}$ with the relation $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c, b)\}$.

reflexive

symmetric

anti-symmetric

transitive

euclidean $\forall a, b, c.\ aRb \wedge aRc \Rightarrow bRc.$

x	y	z	xRy	xRz	$xRy \wedge xRz : (P)$	$yRz : (Q)$	$P \Rightarrow Q$
a	b	c	T	T	T	T	T
a	c	b	T	T	T	T	T
b	a	c	F	T	F	T	T
b	c	a	T	F	F	T	T
c	a	b	F	F	F	T	T
c	b	a	F	F	F	F	T

equivalence relation

partial function

(G_7) [image here]
Formally, G_7 is the pairing of the set of states $S = \{a, b, c\}$ with the relation $R = \{(a, a), (b, b), (c, c), (b, a), (b, c)\}$.

reflexive

symmetric

anti-symmetric

transitive

euclidean $\forall a, b, c.\ aRb \wedge aRc \Rightarrow bRc.$

x	y	z	xRy	xRz	$xRy \wedge xRz : (P)$	$yRz : (Q)$	$P \Rightarrow Q$
a	b	c	T	T	T	T	T
a	c	b	T	T	T	T	T
b	a	c	F	T	F	T	T
b	c	a	T	F	F	T	T
c	a	b	F	F	F	T	T
c	b	a	F	F	F	F	T

equivalence relation

partial function

(G_8) [image here]
Formally, G_8 is the pairing of the set of states $S = \{a, b, c\}$ with the relation $R = \{(a, b), (a, c), (b, b), (b, c), (c, c), (c, b)\}$.

reflexive

symmetric

anti-symmetric

transitive

euclidean $\forall a, b, c.\ aRb \land aRc \Rightarrow bRc.$

x	y	z	xRy	xRz	$xRy \land xRz : (P)$	$yRz : (Q)$	$P \Rightarrow Q$
a	b	c	T	T	T	T	T
a	c	b	T	T	T	T	T
b	a	c	F	T	F	T	T
b	c	a	T	F	F	T	T
c	a	b	F	F	F	T	T
c	b	a	F	F	F	F	T

equivalence relation

partial function

2 True or false, and why?

The statements below are about relations on a fixed set X. For each statement below, say whether it is true or false. If it is true, give a short proof. If it is false, give a counter-example.

1. Every reflexive relation on X is serial.
2. Every reflexive relation on X which is reflexive and transitive is also symmetric.
3. Every reflexive relation on X which is Euclidean, symmetric and transitive is reflexive.
4. Every reflexive relation on X which is Euclidean and reflexive is symmetric.

3 Multiply two graphs

4 True or false, and why?

For each of the following assertions, tell whether it is true or false. (As always in a problem like this, "true" means *true for all graphs.* So "false" means *false for some graphs.*) Give a short proof of the true ones, and a counter-example for the false ones.

1. The product of two reflexive graphs is reflexive.
2. The product of two euclidean graphs is reflexive.
3. If the product of two graphs is reflexive, then both of the given graphs are reflexive.

4. If the product of two graphs is transitive, then both of the given graphs are transitive.

5. Let G be a one-point graph with no arrows. Then for all $H, G \times H$ looks like H.

5 Modal semantics with boxes and diamonds rather than K-operators

Attached.