502 Mini-Project #2 : Flow due to Oscillating Plate

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December 14, 2016

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1 Overview

The problem of an oscillating plate bounding a fluid has lots of applications in a broad range of engineering and medical fields. The are several examples of problems inclduing oscillating plates which will be discussed in this report.

2 Flow Due to an oscillating plate

2.1 Problem Overview

First lets consider an infinite flat plate containing an infinite dept of fluid relative to the affect range. The plate executes sinusoidal oscillations parallel to the itself as shown in the diagram **??**.

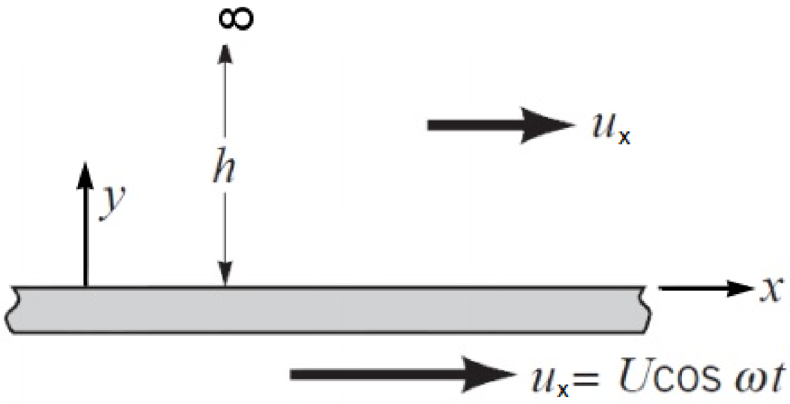

Figure 2.1: Calculated Order of Accuracy

The problem is to find a solution to the flow after all transients have died out. This simplifies the problem as there are no initial conditions to satisfy. This problem is often referred to as stokes second problem. We begin solving by finding the simplified governing equations from the generalized 2D differential navier stokes equations 2.2.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g_x + \eta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2.2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g_y + \eta\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{2.3}$$

This can written as a 1D problem as we are only concerned with the velocity profile u_x in the y direction, ie u_z and u_y are considered to be small and equal to zero. We don't expect a pressure gradient in the x direction because of the infinite dimension of the plate. A pressure gradient in the y direction will be due only to gravity which we are ignoring for this problem. These assumptions lead to many cancellations and result in the governing equations 2.4 used to solve the problem

$$\frac{\partial u}{\partial t} = \eta \frac{\partial^2 u}{\partial y^2} \tag{2.4}$$

The boundary conditions exist at the plate and far away from the plate. Near the plate we require a no slip condition, therefore the velocity of the flow at the plate is equal to its sinusoidal motion. The motion is expect to decay as one moves away from the plate were the fluid motion is zero. The boundary conditions are shown below.

$$u(0,t) = U\cos\omega t \tag{2.5}$$

$$u(0,\infty) = 0 \tag{2.6}$$

2.2 Separation

Based on the governing equations we expect the solution to take on the form of equation **??** which suggests this can be solved using separation of variables.

$$u = g(t)f(y) \tag{2.7}$$

Since the period of the oscillations of the plate introduces a time scale, no similarity solution exists to this problem. By considering the equations governing the fluid it may be expected that u_x will also oscillate with the same frequency relative to the movement of the plate, with however a phase shift due to the shear slip between fluid layers. In the steady state therefore the flow variables must have a periodicity equivalent to the periodicity of the motion of the plate. Therefore we will consider a separable solution of the form shown in equation 2.8.

$$u = e^{i\omega t}f(y) \tag{2.8}$$

The solution will be considered the real part of the right-hand side. Because $f(y)$ is complex the velocity of the fluid $u(y,t)$ is able to have a phase difference relative to the wall velocity $U\cos\omega t$. To find the solution equation 2.8 is substituted into the governing equation 2.4 which gives

$$i\omega f(y) = \eta \frac{d^2 f(y)}{dy^2} \tag{2.9}$$

This is an equation with constant coefficients and must have exponential solutions. Substitution of a solution of the form $f = exp(ky)$ gives $k = \sqrt{i\omega/\eta} \pm (i+1)\sqrt{\omega/2\eta}$ where the two square roots of i have been used. As a result the solution of equation 2.10 is

$$f(y) = Ae^{-(1+i)y\sqrt{\omega/2\eta}} + Be^{(1+i)y\sqrt{\omega/2\eta}} \tag{2.10}$$

The condition 2.6 requires that the solution for velocity must be zero at $y = \infty$, therefore we find B = 0 in order for this boundary to be satisfied. The solution 2.8 then becomes

$$u(y,t) = Ae^{i\omega t}e^{-(1+i)y\sqrt{\omega/2\eta}} \tag{2.11}$$

After applying the surface boundary condition 2.8 due to the oscillating plate we find A to be equal to U. After considering only the real part of Eq.**??**, we finally obtain the velocity distribution over the oscillating plate:

$$u(y,t) = Ue^{-\omega t}\cos\left(\omega t - y\sqrt{\omega/2\eta}\right) \tag{2.12}$$

2.3 SOLUTION

The cosine term in Eq. 2.12 ris a representation of a wave signal propagating in the direction of y , while the exponential term represents the amplitude decay in that propagating wave. The flow therefore resembles a damped wave as displayed in figure 2.2.

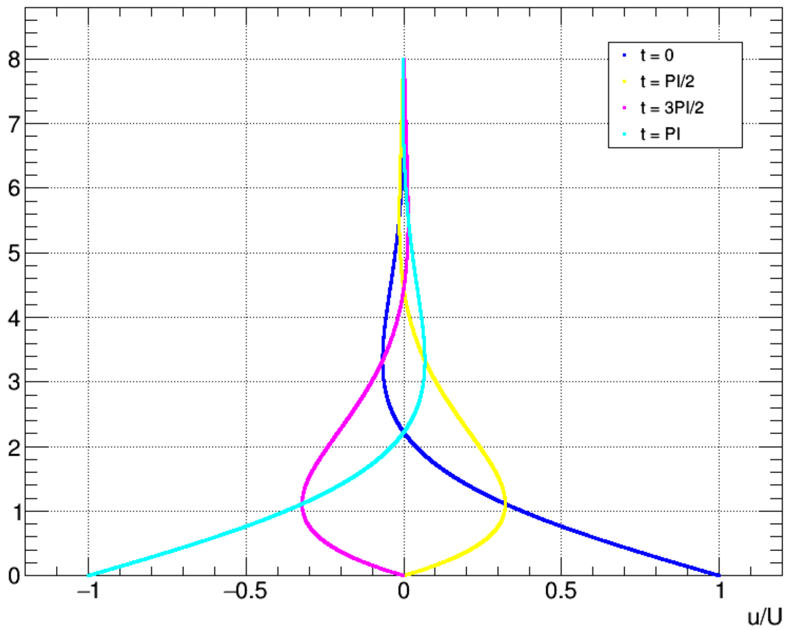

Figure 2.2: Calculated Order of Accuracy

On an interesting note the problem to be clear is purely a diffusion problem and not wave-propagation problem. This is because there are no restoring forces involved here. The apparent propagation is merely a result of the oscillating boundary condition and the shear coupling between fluid particles. The distance at which the wave penetrates into the fluid domain is known as the penetration dept. When we select a value for y far from the wall say, $y = 4\sqrt{\eta/\omega}$, the amplitude of u is $Uexp(-4\sqrt{2}) = O.O6U$ which is very small. Therefor we can say the influence of the wall is confined within a distance of approximately:

$$\delta \approx 4\sqrt{\eta/\omega} \tag{2.13}$$

This parameter is known as the depth of penetration dept [?]. This relations suggests that the distance over which the fluid feels the motion of the plate gets smaller as the frequency of the oscillations increases.

As discussed earlier we noted that the solution 2.12 cannot be represented by a single curve in terms of the non-dimensional variables. This is expected because the frequency of the boundary motion introduces a natural time scale l/o into the problem, thereby violating the requirements of self-similarity. There are two parameters in the governing set 2.4, namely, U and w. The parameter U can be eliminated by regarding u/U as the dependent variable.

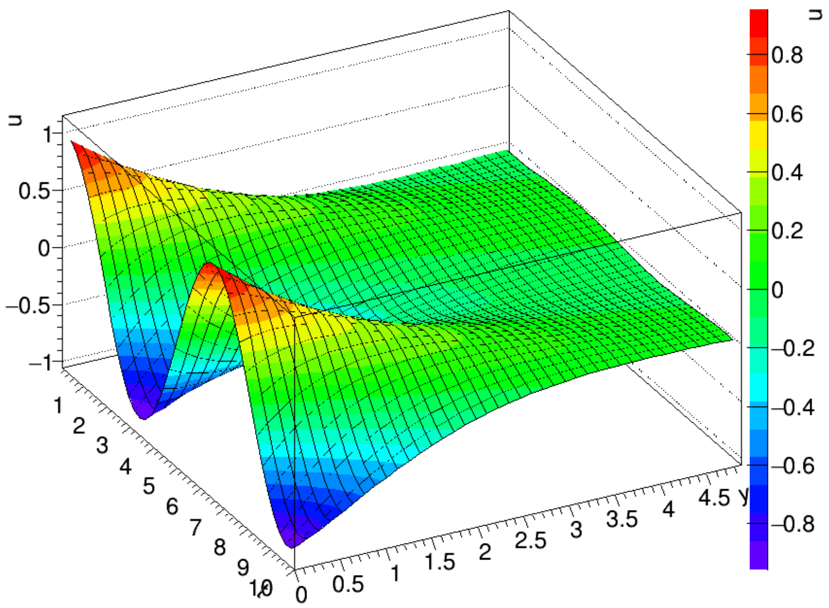

Figure 2.3: Calculated Order of Accuracy

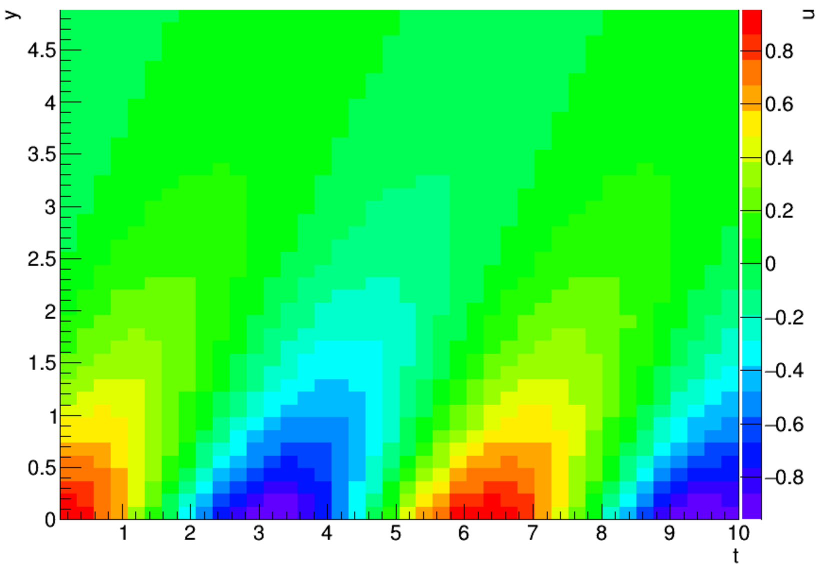

Figure 2.4: Calculated Order of Accuracy

An interesting point is that the oscillating plate has a constant diffusion distance $\delta = 4\sqrt{\eta/\omega}$ that is in contrast to the case of the impulsively started plate in which the diffusion distance increases with time. This can be understood from the governing equation 2.4. In the problem of sudden acceleration of a plate, $\frac{\partial^2 u_x}{\partial y^2}$ is positive for all y (see Figure 9.10), which results in a positive au/at everywhere. The monotonic acceleration signifies that momentum is constantly diffused outward, which results in an ever-increasing width of flow. In contrast, in thc casc of an oscillating plate, a2u/i3y2 (and therefore a u / a r) constantly changes sign in y and t . Therefore, momentum cannot diffuse outward monotonically, which results in a constant width of flow. The analogous problem in heat conduction is that of a semi-infinite solid, the surhce of which is subjected to a periodic fluctuation of temperature. The resulting solution, analogous to Eq. (9.59, has been used to estimate the effective "eddy" diffusiviry in thc upper layer of the ocean from measurements of the phase difference (that is, h e time lag between maxima) between the temperature fluctuations at two depths, generated by the diurnal cyclc of solar heating

2.4 Effect of Parameters

2.5 Applications

References

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