Homework 1

ECE 45 (Franceschetti)

Jiahong Long [redacted]

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1 Homework 1

1. Find the magnitude and phase of the following complex number

$$\frac{(1-j)(2e^{-j\pi/2})(\sin(1))(j^2)}{(2+2j)(-j\cos(1))(e^{j\pi})}$$

Let z denote the complex number. Then:

Magnitude. The magnitude of a quotient is the quotient of the magnitudes. Then,

$$\begin{aligned}
|z| &= \left|\frac{(1-j)(2e^{-j\pi/2})(\sin(1))(j^2)}{(2+2j)(-j\cos(1))(e^{j\pi})}\right| \\
&= \frac{\left|(1-j)(2e^{-j\pi/2})(\sin(1))(j^2)\right|}{\left|(2+2j)(-j\cos(1))(e^{j\pi})\right|} \\
&= \frac{\left|(\sqrt{2}e^{-j\pi/4})(2e^{-j\pi/2})(\sin(1)e^{j0})(-1e^{j0})\right|}{\left|(\sqrt{8}e^{j\pi/4})(\cos(1)e^{-j\pi/2})(1e^{j\pi})\right|} && \text{in polar form} \\
&= \frac{(\sqrt{2})(2)\sin(1)(-1)}{(2\sqrt{2})\cos(1)(1)} \\
&= -\tan(1).
\end{aligned}$$

Phase. The phase of a quotient is the difference of the phases. Then,

$$\begin{aligned}
\angle z &= \angle\frac{(1-j)(2e^{-j\pi/2})(\sin(1))(j^2)}{(2+2j)(-j\cos(1))(e^{j\pi})} \\
&= \angle\Big[(1-j)(2e^{-j\pi/2})(\sin(1))(j^2)\Big] - \angle\big[(2+2j)(-j\cos(1))(e^{j\pi})\big] \\
&= \angle\Big[(\sqrt{2}e^{-j\pi/4})(2e^{-j\pi/2})(\sin(1)e^{j0})(1e^{j\pi})\Big] - \angle\Big[(\sqrt{8}e^{j\pi/4})(\cos(1)e^{-j\pi/2})(1e^{j\pi})\Big] && \text{in polar form} \\
&= \left(-\frac{\pi}{4} - \frac{\pi}{2} + \pi\right) - \left(\frac{\pi}{4} - \frac{\pi}{2} + \pi\right) \\
&= \frac{\pi}{4} - \frac{3\pi}{4} \\
&= -\frac{\pi}{2}.
\end{aligned}$$

2. Let

$$X = 2 + j + 2e^{-j2\pi/3} - e^{j\pi/2}$$

We can simplify X:

$$\begin{aligned} X &= (2+j) + (2e^{-j2\pi/3}) - (e^{j\pi/2}) \\ &= (2+j) + 2(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) - (j) && \text{rectangular form} \\ &= 2 + j - 1 - j\sqrt{3} - j \\ &= 1 - j\sqrt{3} \end{aligned}$$

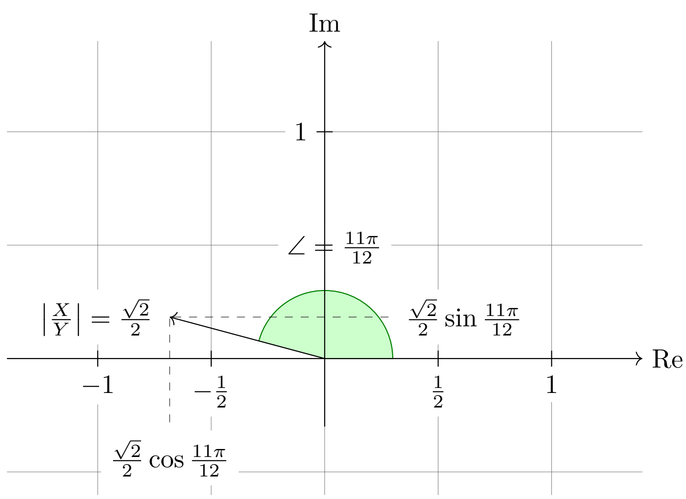
(a) Find the real portion of X.
From the above, $\Re[X] = 1$.

(b) Find the phase of the complex conjugate of X.
From the above, $X^* = 1 + j\sqrt{3}$. In polar form, we have $X = 2e^{j\pi/3}$. Then the phase is $\frac{\pi}{3}$.

(c) Let $Y = -2 + 2j$. Plot X/Y on the complex plane.
From the above, X in polar form must be $2e^{-j\pi/3}$. Similarly, Y in polar form is $|Y|e^{j\angle Y} = \sqrt{8}e^{j3\pi/4}$. It follows that

$$\frac{X}{Y} = \frac{2}{\sqrt{8}}e^{j(-\pi/3-3\pi/4)} = \frac{\sqrt{2}}{2}e^{j11\pi/12} =$$

Plotted:



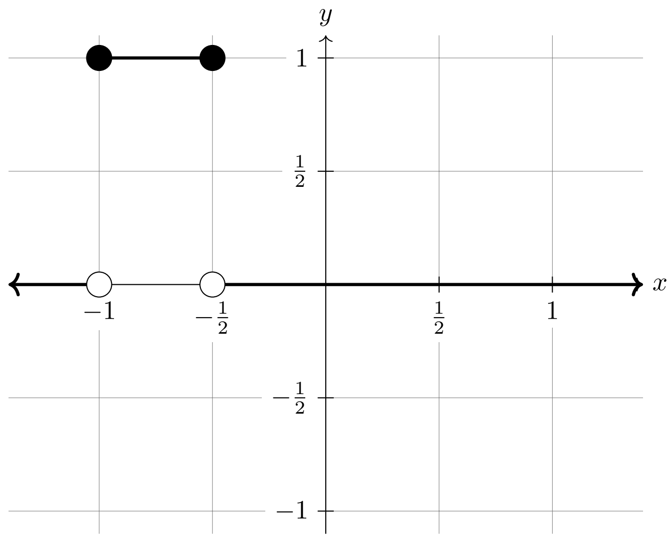
3. Let

$$f(x) = \begin{cases} 1 & \text{if } -1 \le x \le 0 \\ -1 & \text{if } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Plot $f(2x+1)$.
This composition is equivalent to a linear transformation of f:

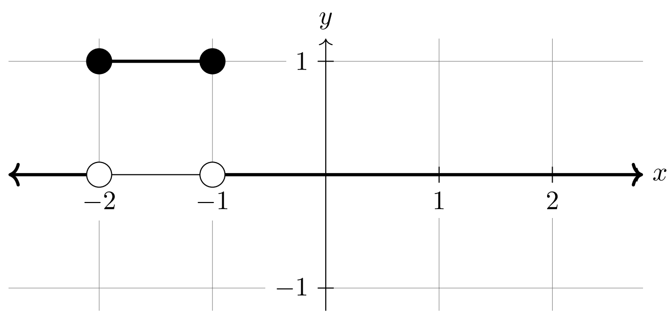
$$f \circ (2x+1) = \begin{cases} 1 & \text{if } -1 \le x \le \frac{-1}{2} \\ -1 & \text{if } \frac{-1}{2} < x < \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Hence:



(b) Plot the magnitude of $e^{jx}f(x)$.

The magnitude of a $f(x)e^{jx}$ is simply $f(x)$. Thus:



(c) If the energy of a signal $g(x)$ is defined to be

$$E\left[g(x)\right] = \int_{-\infty}^{\infty} |g(x)|^2 \; dx$$

how does the energy of $g(x)$ compare to the energy of $e^{jx}g(x)$?

Since the energy of a signal is defined in terms of the absolute value of the signal, there will be no difference between the energy of $g(x)$ and $g(x)e^{jx}$, since the e^{jx} term merely adds splits the *total* signal into real and an imaginary components, both of which are reconciled back to the original signal by taking their magnitude. Mathematically speaking, we see that

$$\begin{aligned}
E\left[g(x)e^{jx}\right] &= \int_{-\infty}^{\infty} \left|g(x)e^{jx}\right|^2 \; dx \\
&= \int_{-\infty}^{\infty} |g(x)|^2 \left|(\cos x + j\sin x)\right|^2 \; dx \\
&= \int_{-\infty}^{\infty} |g(x)|^2 \sqrt{\cos^2 x + \sin^2 x}^2 \; dx \\
E\left[g(x)\right] &= \int_{-\infty}^{\infty} |g(x)|^2 \; dx
\end{aligned}$$

4. Represent the following sinusoidal functions as phasors.

(a) $f_1(t) = 3\cos(4t) - 4\sin(4t)$

$$\begin{aligned} f_1(t) &= 3\cos(4t) - 4\sin(4t) \\ &= 3\cos(4t) - 4\cos(4t - \frac{\pi}{2}) \end{aligned}$$

By linearity of the function, $f_1(t)$ can be represented as the sum of two phasors (since both have $\omega = 4$):

$$\begin{aligned} \mathbf{F_1} &= 3e^{j0} - 4e^{-j\pi/2} \\ &= 3 - 4j. \end{aligned}$$

(b) $f_2(t) = 2\cos(\omega t) + \cos(\omega t + \pi/4)$
By linearity, we can use two phasors:

$$\begin{aligned} \mathbf{F_2} &= 2e^{j0} + e^{j\pi/4} \\ &= 2 + \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} \\ &= \frac{4+\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}. \end{aligned}$$

(c) $f_3(t) = \cos^2(t) - \sin^2(t)$
Using the half-angle formula, we shed the exponents:

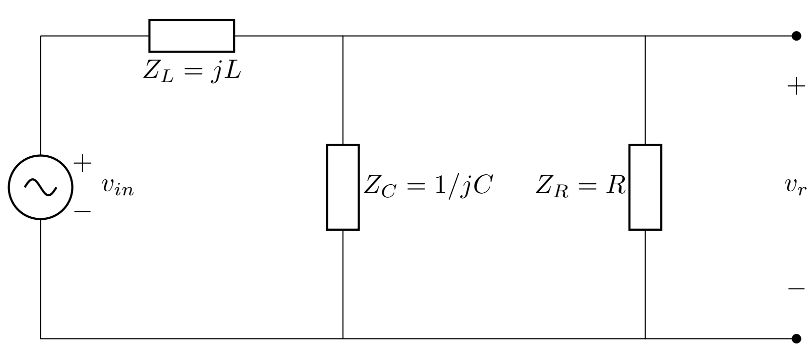
$$\begin{aligned} f_3(t) &= \cos^2(t) - \sin^2(t) \\ &= \frac{1+\cos(2t)}{2} - \frac{1-\cos(2t)}{2} \\ &= \frac{2\cos(2t)}{2} \\ &= \cos(2t) \end{aligned}$$

In this form, we arrive at $\mathbf{F_3} = 1e^{j0}$.

5. Find the voltage $v_r(t)$ in the circuit below, when

(a) $v_{in}(t) = 1/3$
Assume DC steady state. Since there no change in the current, voltage drop due to the inductor is zero. Since we are at steady state, the capacitor is full and becomes an open circuit. It follows that the only relevant circuit element at DC steady state is the resistor. Trivially, $v_r = v_{in} = \frac{1}{3}$.

(b) $v_{in}(t) = \sin(t)$
Assume sinusoidal steady state. We can rewrite $v_{in} = \cos(t - \frac{\pi}{2})$, where $\omega = 1$. Working in complex impedances, the circuit therefore becomes



In the complex frequency domain, the voltage source becomes

$$\mathbf{V}_{in} = 1e^{-j\pi/2}$$

The circuit is a current divider. Then, the voltage v_r across the resistor is

$$\begin{aligned}
\mathbf{V}_r = \mathbf{I}_r R &= R\left(\frac{1/jC}{R+1/jC}\right)\mathbf{I}_L \\
&= R\left(\frac{1}{RjC+1}\right)\mathbf{I}_L
\end{aligned}$$

To find $\mathbf{I}_L$, we take the equivalent impedance and solve using Ohm's law:

$$\begin{aligned}
\mathbf{V}_{in} &= \mathbf{I}_L\mathbf{Z}_t \\
\mathbf{I}_L &= \frac{\mathbf{V}_{in}}{\mathbf{Z}_t} \\
&= \frac{\mathbf{V}_{in}}{\frac{R}{1+jCR}+jL}
\end{aligned}$$

Substituting:

$$\begin{aligned}
\mathbf{V}_r &= R\left(\frac{1}{RjC+1}\right)\frac{\mathbf{V}_{in}}{\frac{R}{1+jCR}+jL} \\
&= R\left(\frac{1}{\cancel{RjC+1}}\right)\frac{\mathbf{V}_{in}\cancel{(RjC+1)}}{R+jL-RLC} \\
&= \frac{R\mathbf{V}_{in}}{(R-RLC)+jL} && \text{multiply by conjugate} \\
&= \frac{R\mathbf{V}_{in}}{(R-RLC)+jL}\cdot\frac{(R-RLC)-jL}{(R-RLC)-jL} \\
&= \frac{\mathbf{V}_{in}}{R^2(1-LC)^2+L^2}\cdot\left(R^2(1-LC)^2-jRL\right) && \mathbf{V}_{in} = e^{-j\pi/2} = -j \\
&= \frac{-j}{R^2(1-LC)^2+L^2}\cdot\left(R^2(1-LC)^2-jRL\right) \\
&= \frac{1}{R^2(1-LC)^2+L^2}\cdot\left(RL-j\left(R(1-LC)\right)^2\right)
\end{aligned}$$

Using the given values for R, L, and C yields

$$\begin{aligned}
\mathbf{V}_r &= \frac{1}{(1-1)^2+2^2}\cdot\left(2-j\left((1-1)\right)^2\right) \\
&= \frac{1}{4}\cdot(2) \\
&= \frac{1}{2} \\
&= \frac{1}{2}e^{j0}
\end{aligned}$$

and thus

$$v_r = \frac{1}{2}\cos(t)$$

(c) $v_{in}(t) = 1 + 2\sin(t-\pi)$

By linearity, superposition applies. Then the cumulative response of the system is equal to the sum of the responses to $v_{in} = 1$ and $v_{in} = 2\sin(t-\pi)$. The response of the system to $v_{in} = 1$ is a DC steady state response, so the capacitor is fully charged and is an open circuit, and there is no current change over the inductor and hence the back-EMF is zero. Then $v_{r1} = v_{in1} = 1$. Rewriting the second input as a phasor yields

$$\mathbf{V}_{in2} = 2e^{j(-\pi-\pi/2)} = 2e^{-3j\pi/2}$$

Using the equation for $\mathbf{V}_r\left(R, L, C, \mathbf{V_{in}}\right)$ from part (b), we have

$$\mathbf{V}_r = \frac{\mathbf{V}_{in}}{R^2(1-LC)^2 + L^2} \cdot \left(R^2(1-LC)^2 - jRL\right)$$

Using the given values for $\mathbf{V}_{in}$, R, L, and C yields

$$\begin{aligned}\mathbf{V}_r &= \frac{2e^{-3j\pi/2}}{(1-1)^2 + 2^2} \cdot \left((1-1)^2 - 2j\right) \\ &= \frac{2e^{-3j\pi/2}}{4} \cdot (-2j) \\ &= \frac{-4je^{-3j\pi/2}}{4} \\ &= -je^{-3j\pi/2} \\ &= -j(j) \\ &= 1\end{aligned}$$

Thus $v_{r2} = \cos(t)$. It follows that the cumulative response is

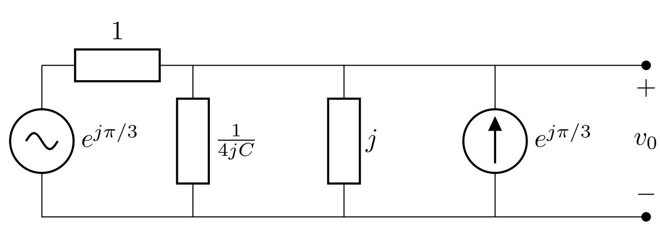
$$v_r = 1 + \cos(t)$$

6. Find the value of C for which the Norton equivalent i a curent source in parallel with only a resistor (ie. the Thevenin impedance is purely real).

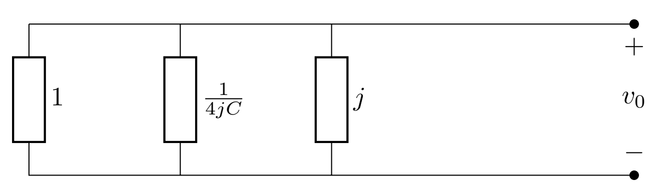
   In this circuit, we observe that $\omega = 4$. Then, $v(t)$ can be modelled with the phasor $e^{j\pi/3}$ and $i(t)$ can be modelled with the phasor $e^{j(5\pi/6-\pi/2)} = e^{j\pi/3}$. The passive components can then be modelled in complex impedances as

$$\begin{aligned}Z_R &= R = 1 \\ Z_C &= \frac{1}{4jC} \\ Z_L &= j\omega L = j\end{aligned}$$

   We can redraw the circuit as



   To find the Thevénin impedance, we replace the voltage source with a short and open-circuit the current source:



The equivalent impedance is calculated as

$$
\begin{aligned}
\frac{1}{Z_{eq}} &= 1 + 4jC + \frac{1}{j} \\
Z_{eq} &= \frac{1}{1 + 4jC + \frac{1}{j}} \\
&= \frac{j}{(1-4C)+j} && \text{multiply by conjugate} \\
&= \frac{j}{(1-4C)+j} \cdot \frac{(1-4C)-j}{(1-4C)-j} \\
&= \frac{1 + j(1-4C)}{(1-4C)^2+1} \\
&= \frac{1}{(1-4C)^2+1} + j\frac{1-4C}{(1-4C)^2+1} +
\end{aligned}
$$

For the Thevénin impedance to be purely real, we must set $C = \frac{1}{4}$, so the imaginary part of the impedance goes to zero.

7. Suppose $H(\omega)$ is the transfer function of a linear system and $H(\omega) = 1$ when $|\omega| < \pi$ and $H(\omega) = 0$ otherwise. If the input to the system is

$$
x(t) = \sum_{k=1}^{\infty} \frac{\cos\left(\frac{3\pi}{4}kt + \frac{\pi}{k}\right)}{k}
$$

then what is the output $y(t)$?

Our input x has angular frequency $\omega = \frac{3\pi}{4}$, where $|\omega| < \pi$. It follows, then that $H(\omega) = 1$. By the definition of the transfer function, we have

$$
H(\omega) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = 1
$$

which implies that the output phasor is identical to the input phasor, ie. the transfer function passes (and does not attenuate) the input where $\omega = \frac{3\pi}{4}$. Therefore, the output is directly equal to $x(t)$ and

$$
y(t) = x(t)
$$

8. Are the following steady-state input-output pairs consistent with the properties of RLC circuits (or more generally, LTI sytems)?

If a given LTI system has a transfer function $H(\omega)$ and an input $A\cos(\omega t + \Phi)$, then the output *necessarily* is of the form

$$
A\,|H(\omega)|\cos\left(\omega t + \Phi + \angle H(\omega)\right)
$$

With this in mind:

(a) $\cos(2t) \to H(\omega) \to 99\sin(2t-3)$

Yes. A sinusoidal input was mapped to a sinusoidal output, which is consistent with an LTI system.

(b) $\cos(4t) \to H(\omega) \to 1 + 4\cos(4t)$

No. A purely sinusoidal input with no intercept was mapped to a sinusoidal output superimposed with an intercept, which is inconsistent.

(c) $4 \to H(\omega) \to -8$

Yes. A constant input was mapped to a constant output.

(d) $4 \to H(\omega) \to 8j$

No. This transformation was non-linear, since the input was in the real domain, and the output in the imaginary domain. Additionally, the measured response of an RLC system is taken as the real part of a complex response, so this situation is impossible.

(e) $4 \to H(\omega) \to \cos(3t)$

No. A constant input was mapped to a sinusoidal output, which is a nonlinear transformation, and hence not possible in an LTI system.

(f) $\sin(\pi t) \to H(\omega) \to \cos(\pi t) + \sin(\pi t)$

Yes. The output can be rewritten using trigonometric identities as

$$\cos(\pi t) + \sin(\pi t) = \frac{1}{2}\sin(2\pi t)$$

which means a sinusoidal input was mapped to a sinusoidal output, consistent with an LTI system.

(g) $\sin(\pi t) \to H(\omega) \to \sin^2(\pi t)$

Yes. The output can be rewritten using trigonometric identities as

$$\sin^2(\pi t) = \frac{1 + \cos(2\pi t)}{2}$$

which means a sinusoidal input was mapped to a sinusoidal output, consistent with a linear time-invariant system.

(h) $0 \to H(\omega) \to 5$

No. A zero input was mapped to a constant output, which is inconsistent with an LTI system.

9. For each $k = 1, 2$ suppose $y_k(t)$ is the output when $x_k(t)$ is the input to an LTI system.

$$y_1(t) = \begin{cases} 1 & \text{if } 0 \le t < 2 \\ 0 & \text{otherwise} \end{cases} \qquad y_2(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ -1 & \text{if } 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) $z_1(t) = \begin{cases} -1 & \text{if } 2 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$

To achieve the boxcar effect of $z_1 = -1$ where $t \in [2, 4)$, we can use $-y_1$ shifted to the right two units. Hence a possible input is $-x_1(t-2)$.

(b) $z_2(t) = \begin{cases} 1 & \text{if } 1 \le t < 5 \\ 0 & \text{otherwise} \end{cases}$

To achieve this boxcar effect, we can use y_1 and shift t to the left by one, then stretch by a factor of two. Hence a possible input is $x_1\left(\frac{1}{2}(t-1)\right)$.

(c) $z_3(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$

To achieve this effect, we can shrink y_1 by a factor of 2. Hence a possible input is $x_1(2t)$.

(d) $z_4(t) = \begin{cases} 1 & \text{if } 3 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$

To achieve this effect, we can shift y_1 to the right by 3, then shrink it by a factor of 2. Hence a possible input is $x_1\left(2(t-3)\right)$.

(e) $z_5(t) = \begin{cases} 1 & \text{if } t \ge 0 \\ 0 & \text{otherwise} \end{cases}$

To achieve this effect, we can use an infinite series of inputs based on x_1, shifted by 2 each time. Then a possible input is

$$\sum_{i=0}^{\infty} x_1(t - 2i)$$

(f) $z_6(t) = 0$

To achieve this effect, we can superimpose x_1 and its negation. Then a possible input is $x_1(t) - x_1(t)$.

10. An LTI system with input $x(t)$ and output $y(t)$ is given by the differential equation

$$3\frac{d^4y(t)}{dt^4} - 2\frac{d^3x(t)}{dt^3} + y(t) = 2x(t) + \frac{d^2y(t)}{dt^2}$$

Find the steady state output when $x(t) = 1 + \cos(t) + \cos(2t)$.

In terms of $x(t)$, the system output is given by

$$3\frac{d^4y(t)}{dt^4} - \frac{d^2y(t)}{dt^2} + y(t) = 2x(t) + 2\frac{d^3x(t)}{dt^3}$$

Substituting $x(t)$, we get

$$3\frac{d^4y(t)}{dt^4} - \frac{d^2y(t)}{dt^2} + y(t) = 1 + \cos(t) + \sin(t) + \cos(2t) + 8\sin(2t)$$

We define $\mathcal{L}\{f(t)\} = F(s)$, and let the right-hand side be denoted by $g(t)$. Taking the Laplace transform of both sides yields

$$\begin{aligned} Y(s)\left(3s^4 - s^2 + 1\right) + y(0)\left(s - s^3\right) + y'(0)\left(1 - s^2\right) - sy'(0) - y'''(0) \\ = G(s) \end{aligned}$$

Solving for $Y(s)$, and assuming that the initial system state is zero

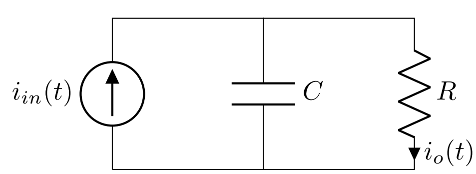
$$\begin{aligned} Y(s)\left(3s^4 - s^2 + 1\right) &= G(s) \\ Y(s) &= \frac{1}{(3s^4 - s^2 + 1)}G(s) \end{aligned}$$

By the Convolution theorem, we get that $y(t) = \mathcal{L}^{-1}\left(\frac{1}{3s^4-s^2+1}\right) * g(t) = \int_0^t \mathcal{L}^{-1}\left(\frac{1}{3s^4-s^2+1}\right)(t-v)g(v)\,dv$. Then, by properties of the inverse Laplace transform, we get that

$$y(t) = \frac{\sqrt{2}}{2}\sin\left(\frac{t\sqrt{2}}{2}\right) * (1 + \cos(t) + \sin(t) + \cos(2t) + 8\sin(2t))$$

11. What type of filter are the following circuits? Justify your answer by finding the magnitude of the transfer function of each circuit.

(a) i_{in} vs i_{out}:

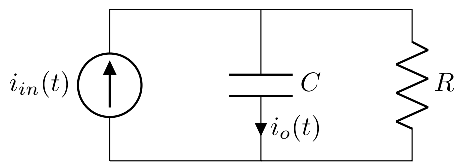


We can calculate the current over R using the current divider formula and subsequently find the transfer function:

$$\begin{aligned}
\mathbf{I}_o &= \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\mathbf{I}_{in} \\
&= \frac{1}{j\omega RC + 1}\mathbf{I}_{in} \\
\frac{\mathbf{I}_o}{\mathbf{I}_{in}} &= \frac{1}{j\omega RC + 1} = H(\omega) \\
|H(\omega)| &= \frac{1}{|j\omega RC + 1|} \\
&= \frac{1}{\sqrt{(\omega RC)^2 + 1}}
\end{aligned}$$

Clearly this is a *low pass filter*, since as $\omega \to \infty$, the denominator of the transfer function grows increasingly large, and the higher-frequency signals are attenuated.

(b) i_{in} vs i_{out}:



We can calculate the current over C using the current divider formula and subsequently find the transfer function:

$$\begin{aligned}
\mathbf{I}_o &= \frac{R}{R + \frac{1}{j\omega C}}\mathbf{I}_{in} \\
&= \frac{j\omega RC}{j\omega RC + 1}\mathbf{I}_{in} \\
\frac{\mathbf{I}_o}{\mathbf{I}_{in}} &= \frac{j\omega RC}{j\omega RC + 1}\mathbf{I}_{in} = H(\omega) \\
|H(\omega)| &= \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}
\end{aligned}$$

This is a *high-pass filter*, since as $\omega \to 0$ the effect of the 1 in the denominator becomes more pronounced. Inversely, as $\omega \to \infty$, the effect of the 1 becomes less and less relevant, and hence only lower frequencies are attenuated.

12. For each of the circuits in the previous problem, let $RC = 100$. If the circuit is a low-pass filter, find the frequency ω_c such that $|H(\omega)| < 0.01$ for all $\omega > \omega_c$. If the circuit is a high-pass filter, find the frequency ω_c such that $|H(\omega)| < 0.01$ for all $\omega < \omega_c$. If $RC = \beta$ for some $\beta > 0$, what is ω_c in terms of β?

(a) Low-pass filter. From above, substituting $RC = \beta$ we have

$$\begin{aligned}
|H(\omega)| &= \frac{1}{\sqrt{(\beta\omega)^2 + 1}} \\
0.01 &= \frac{1}{\sqrt{(\beta\omega_c)^2 + 1}} \\
\sqrt{(\beta\omega_c)^2 + 1} &= 100 \\
(\beta\omega_c)^2 + 1 &= 10000 \\
\beta\omega_c &= \sqrt{9999} \\
\omega_c &= \frac{\sqrt{9999}}{\beta}
\end{aligned}$$

For $\beta = 100$, we have $\omega_c = \frac{\sqrt{9999}}{100} \approx 0.99995$.

(b) High-pass filter. From above, substituting $RC = \beta$ we have

$$\begin{aligned}
|H(\omega)| &= \frac{\omega\beta}{\sqrt{(\omega\beta)^2 + 1}} \\
0.01 &= \frac{\omega_c\beta}{\sqrt{(\omega_c\beta)^2 + 1}} \\
\sqrt{(\omega_c\beta)^2 + 1} &= 100\omega_c\beta \\
(\omega_c\beta)^2 + 1 &= 10000(\omega_c\beta)^2 \\
(\omega_c\beta)^2\,(-9999) &= -1 \\
\omega_c\beta &= \sqrt{\frac{1}{9999}} \\
\omega_c &= \frac{1}{\beta}\sqrt{\frac{1}{9999}}
\end{aligned}$$

For $\beta = 100$, we have $\omega_c = \frac{1}{100}\sqrt{\frac{1}{9999}} \approx 1.00005 \cdot 10^4$.

2 MATLAB Homework

1. Decoding

(a) The deciphered message is "If you only knew the power of the dark side". Deciphering the message was done with the code

```
N = length(sum)
Fs = 11025
z = double.empty()
z(1) = sum(1)
for n = 2:N
    z(n) = sum(n) - sum(n-1)
end
sound(z, Fs)
```

(b) The plot generated; by MATLAB is

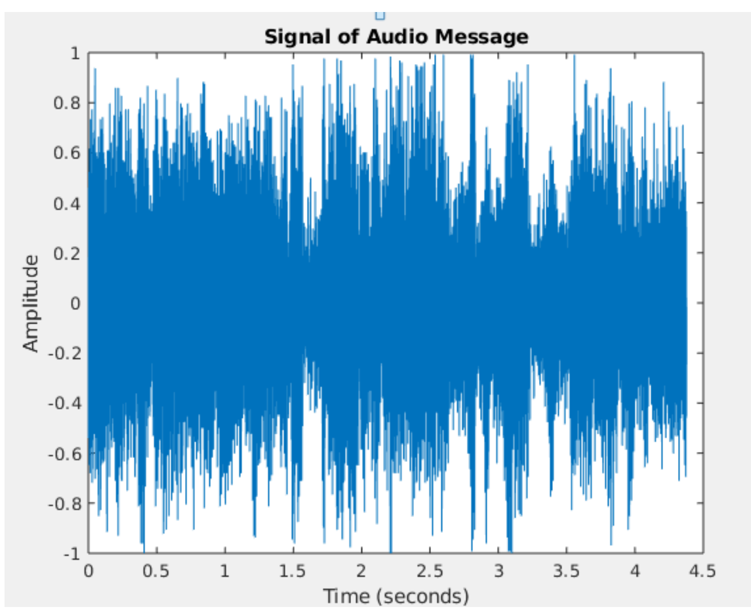


Figure 1: Time series plot of `sum` generated in MATLAB

(c) Setting F_s lower than 11025 slows the playback of the audio as well as pitch-shifts it down. Setting F_s greater than 11025 increases the playback speed, and pitch-shifts it up.

2. Series

(a) For each value of N in $\{1, 5, 10, 100\}$, each array was calculated as follows:

x_1:

```
x1 = zeros(1, 4000);
k = linspace(1, 4000, 4000);
for n = (1:N)
    l = pi * n * k / 500
    x1 = x1 + sin(l) / n
end
x1 = x1 * -1 / pi + 1 / 2
```

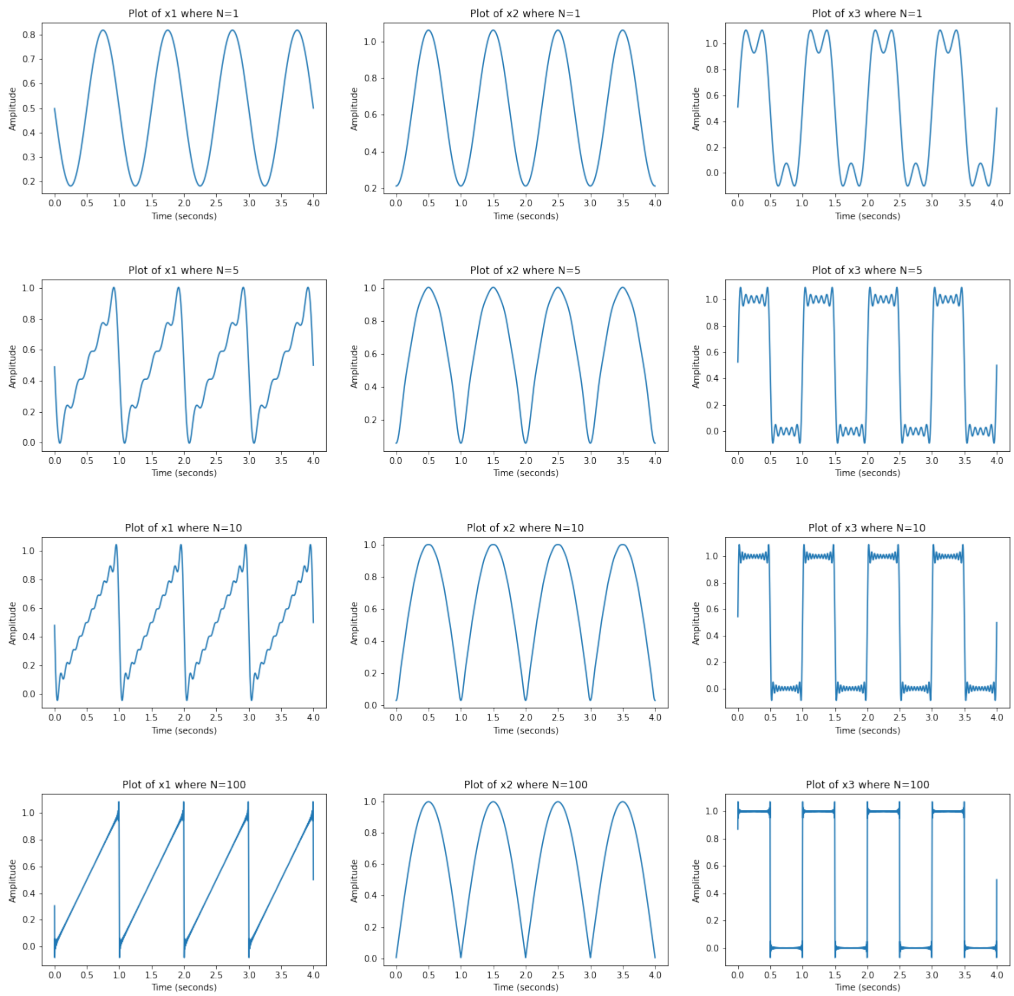
x_2:

```
x2 = zeros(1, 4000);
k = linspace(1, 4000, 4000);
for n = (1:N)
    l = pi * n * k / 500
    x2 = x2 + cos(l) / (4 * n^2 - 1)
end
x2 = x2 * -4 / pi + 2 / pi
```

x_3:

```
x3 = zeros(1, 4000);
k = linspace(1, 4000, 4000);
for n = linspace(1, 7999, 3999)
    l = pi * n * k / 500
    x3 = x3 + sin(l) / n
end
x3 = x3 * 2 / pi + 1 / 2
```

To generate each individual plot, the function `plot(t, xn)` was called for $n = 1, 5, 10, 100$.



(b) It appears that x_1 approximates the sawtooth wave function, x_2 approximates a form of periodic parabolic function, and x_3 approximates the square wave.