ECE 45 (FA20, Franceschetti)

Homework 3: The Fourier Transform

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1. Calculate the Fourier transform of the function

$$\Delta(t) = \begin{cases} 1 - 2|t| & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta(t)$ is the unit triangle function.

Observe that the Fourier tranform can be calculated with a split integral:

$$\begin{aligned} \hat{\Delta}(s) &= \int_{-\infty}^{\infty} \Delta(t) e^{-j\omega t} dt \\ &= \int_{-\frac{1}{2}}^{0} (1 + 2t)\, e^{-j\omega t} dt + \int_{0}^{\frac{1}{2}} (1 - 2t)\, e^{-j\omega t} dt \\ &= \underbrace{\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega t} dt}_{a} + \underbrace{\int_{-\frac{1}{2}}^{0} 2te^{-j\omega t} dt}_{b} - \underbrace{\int_{0}^{\frac{1}{2}} 2te^{-j\omega t} dt}_{b} \end{aligned}$$

The individual parts are calculated:

a :

$$\begin{aligned} \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega t} dt &= \left[\frac{-e^{-j\omega t}}{j\omega} \right]_{-\frac{1}{2}}^{\frac{1}{2}} \\ &= \frac{1}{j\omega} \left(e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}} \right) \\ &= \frac{1}{j\omega} 2j \sin \frac{\omega}{2} \\ &= \frac{2}{\omega} \sin \frac{\omega}{2} \end{aligned}$$

b : Begin with the general indefinite integral.

$$\begin{aligned} \int 2te^{-j\omega t} dt &= \frac{-2e^{-j\omega t}}{j\omega} + \frac{2}{j\omega} \int e^{-j\omega t} dt \\ &= \frac{-2te^{-j\omega t}}{j\omega} + \frac{2e^{-j\omega t}}{\omega^2} \\ &= e^{-j\omega t} \left(\frac{2}{\omega^2} - \frac{2t}{j\omega} \right) \end{aligned}$$

which implies that

$$\int_{-\frac{1}{2}}^{0} 2te^{-j\omega t}dt = e^{-j\omega t}\left(\frac{2}{\omega^2} - \frac{2t}{j\omega}\right)\Bigg|_{-\frac{1}{2}}^{0}$$
$$= \frac{2}{\omega^2} - e^{\frac{j\omega}{2}}\left(\frac{2}{\omega^2} + \frac{1}{j\omega}\right)$$

$c:$ With the indefinite integral from b, we have

$$-\int_{0}^{\frac{1}{2}} 2te^{-j\omega t}dt = -e^{-j\omega t}\left(\frac{2}{\omega^2} - \frac{2t}{j\omega}\right)\Bigg|_{-\frac{1}{2}}^{0}$$
$$= \frac{2}{\omega^2} - e^{-\frac{j\omega}{2}}\left(\frac{2}{\omega^2} - \frac{1}{j\omega}\right)$$

Summing all three, we have

$$\begin{aligned}
\hat{\Delta}(s) &= \frac{2}{\omega}\sin\frac{\omega}{2} + \frac{4}{\omega^2} - \left(\frac{2}{\omega^2}\cos\frac{\omega}{2} + \frac{1}{j\omega}\cos\frac{\omega}{2} + \frac{2}{\omega^2}j\sin\frac{\omega}{2} + \frac{1}{j\omega}j\sin\frac{\omega}{2}\right) \\
&\quad - \left(\frac{2}{\omega^2}\cos\frac{\omega}{2} - \frac{1}{j\omega}\cos\frac{\omega}{2} - \frac{2}{\omega^2}j\sin\frac{\omega}{2} + \frac{1}{j\omega}j\sin\frac{\omega}{2}\right) \\
&= \frac{4}{\omega^2} - \frac{4}{\omega^2}\cos\frac{\omega}{2} \\
&= \frac{4}{\omega^2}\left(1 - \cos\frac{\omega}{2}\right) \\
&= \frac{8}{\omega^2}\sin^2\frac{\omega}{4} \\
&= \frac{8\sin^2\frac{\omega}{4}}{2\cdot\frac{\omega^2}{16}} \\
\hat{\Delta}(s) &= \frac{1}{2}\operatorname{sinc}^2\frac{\omega}{4}
\end{aligned}$$

Thus we have found the Fourier transform of $\Delta(t)$.

2. Let A, W, and t_0 be real numbers such that $A, W > 0$, and suppose that $g(t)$ is given by the diagram. Show the Fourier transform of $g(t)$ is equal to

$$\frac{AW}{2}\operatorname{sinc}^2\frac{W\omega}{4}e^{-j\omega t_0}$$

without integrating.

Vacuously, the diagram indicates that $g(t)$ uses $\Delta(t)$ as a basis function where

- the function is shifted right by t_0,
- the function is stretched vertically to a height of A,
- and the function is stretched horizontally to a width of W.

It immediately follows that $g(t)$ can be written in terms of $\Delta(t)$ as follows:

$$g(t) = A\Delta(\frac{1}{W}t - t_0)$$

Applying the tranformations to the unit triangle in order of precedence, we can find $\hat{g}(s)$ using properties of the Fourier transform:

$$\begin{aligned}
\Delta(t) &\overset{\text{FT}}{\rightarrow} \hat{\Delta}(s) = \frac{1}{2}\operatorname{sinc}^2\frac{\omega}{4} \\
A\Delta(t) &\overset{\text{FT}}{\rightarrow} A\hat{\Delta}(s) = \frac{A}{2}\operatorname{sinc}^2\frac{\omega}{4} \\
A\Delta(\frac{1}{W}t) &\overset{\text{FT}}{\rightarrow} AW\hat{\Delta}(Ws) = \frac{AW}{2}\operatorname{sinc}^2\frac{W\omega}{4} && \text{for } W \geq 0 \\
A\Delta(\frac{1}{W}t - t_0) &\overset{\text{FT}}{\rightarrow} AW\hat{\Delta}(Ws)e^{-j\omega t_0} = \frac{AW}{2}\operatorname{sinc}^2\frac{W\omega}{4}e^{-j\omega t_0} && = \hat{g}(s)
\end{aligned}$$

QED.

3. Find the Fourier transform of the function

$$x(t) = \begin{cases} 1 & 1 \leq |t| \leq 3 \\ -1 & |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

using rectangle functions.

Define the Box function as follows:

$$\operatorname{Box}(T, t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

It follows that we can write $x(t)$ as a superposition of boxes:

$$x(t) = \operatorname{Box}(1, t+2) - \operatorname{Box}(1, t) + \operatorname{Box}(1, t-2)$$

Recall that the Fourier transform of a box is given by

$$\hat{\operatorname{Box}}(s) = \int_{-T}^{T} e^{-j\omega t}dt = 2T\operatorname{sinc}(\omega T)$$

It follows immediately from superposition arguments that

$$\begin{aligned}
\hat{x}(s) &= 2\operatorname{sinc}\omega \cdot e^{j\omega 2} - 2\operatorname{sinc}\omega + 2\operatorname{sinc}\omega \cdot e^{-j\omega 2} \\
\hat{x}(s) &= 2\operatorname{sinc}\omega\,(2\cos 2\omega - 1)
\end{aligned}$$

Hence we have found the Fourier transform of $x(t)$.

4. Find the inverse Fourier transform of the function

$$F(\omega) = \frac{12 + 7j\omega - \omega^2}{(\omega^2 - 2j\omega - 1)(-\omega^2 + j\omega - 6)}$$

using partial fractions.

By the partial fraction theorem, we can rewrite

$$\begin{aligned}
F(\omega) &= \frac{(4 + j\omega)(3 + j\omega)}{(\omega - j)^2(j\omega - 2)(j\omega + 3)} \\
&= \frac{4 + j\omega}{(\omega - j)^2(j\omega - 2)} \\
&= \frac{A}{\omega - j} + \frac{B}{(\omega - j)^2} + \frac{C}{j\omega - 2}
\end{aligned}$$

for complex numbers A, B, and C. Under a common denominator, we can write the equality:

$$12 + 7j\omega - \omega^2 = A(j\omega + 2j - \omega) + B(j\omega - 2) + C(\omega^2 - 2j\omega - 1)$$

Regrouping into like terms yields

$$12 + 7j\omega - \omega^2 = \omega^2 \left(Aj + c\right) + \omega \left(-A + Bj - 2jC\right) + \left(2jA - 2B - C\right)$$

which can be equated with the left-hand side per-term to yield the augmented coefficient matrix

$$\begin{bmatrix} j & 0 & 1 & 0 \\ -1 & j & -2j & j \\ 2j & -2 & -1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -\frac{2}{3}j \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{2}{3} \end{bmatrix}$$

and therefore

$$\begin{aligned} F(\omega) &= \frac{-\frac{2}{3}j}{\omega - j} - \frac{1}{(\omega - j)^2} - \frac{\frac{2}{3}}{j\omega - 2} \\ &= \frac{\frac{2}{3}}{j\omega + 1} + \frac{1}{(j\omega + 1)^2} - \frac{\frac{2}{3}}{j\omega - 2} \end{aligned}$$

and it follows immediately that

$$f(t) = \frac{2}{3}u(t)e^{-t} + tu(t)e^{-t} - \frac{2}{3}u(t)e^{2t}$$

from known Fourier transforms.

5. Suppose a function $f(t)$ has Fourier transform

$$F(\omega) = 2\pi j\omega e^{-|\omega|}$$

Is $f(t)$ purely real or imaginary? Is $f(t)$ odd or even? What is $f(0)$? Calculate $f(t)$ and verify these properties.

Suppose $f(t)$ is purely real. Then, $f^*(t) = f(t)$, and by the conjugation property of the Fourier transform, $F(\omega) = F^*(-\omega)$. Similarly, if $f(t)$ is purely imaginary, $f^*(t) = -f(t)$ and $F^*(\omega) = -F*(-\omega)$. We verify:

$$F^*(-\omega) = -2\pi j(-1)\omega e^{-|\omega|} = F(\omega)$$

which is vacuously equal to $F(\omega)$. Then $f(t)$ is not purely imaginary because $f(t)$ **is in fact purely real.**

Then, suppose $f(t)$ is odd. It follows that $f(-t) = -f(t)$ and by properties of the Fourier transform, $F(-\omega) = -F(\omega)$. Similarly, suppose $f(t)$ is even; then $f(-t) = f(t)$ and $F(-\omega) = F(\omega)$. We verify:

$$F(-\omega) = -2\pi j\omega e^{-|\omega|} = -F(\omega)$$

It immediately follows that $f(t)$ is not even because $f(t)$ **is odd.**

We conjecture that $f(0) = 0$, since $F(0) = 0$ and by the definition of the Fourier transform, $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$, where $\omega = 0$ the Fourier transform attains the value $f(0)$.

To find the inverse Fourier transform, note that $F(\omega) = (j\omega)\, 2\pi e^{-|\omega|}$. Suppose $G(\omega) = 2\pi e^{-|\omega|}$. Then $f(t) = \frac{dg}{dt}(t)$. We can solve for the inverse transform of $G(\omega)$:

$$\begin{aligned}
g(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi e^{-|\omega|} e^{j\omega t} d\omega \\
&= \int_{0}^{\infty} e^{-\omega} e^{j\omega t} d\omega + \int_{-\infty}^{0} e^{\omega} e^{j\omega t} d\omega \\
&= \left.\frac{-e^{-\omega(1-jt)}}{1-jt}\right|_{0}^{\infty} + \left.\frac{e^{\omega(1+jt)}}{1+jt}\right|_{-\infty}^{0} \\
&= \frac{2}{t^2+1}
\end{aligned}$$

Since $\frac{dg}{dt} = f(t)$, we have

$$\frac{dg}{dt} = \frac{d}{dt}\left[\frac{2}{t^2+1}\right] = f(t) = \frac{-4t}{(t^2+1)^2}$$

which is clearly an odd function (numerator contains an odd power of t), vanishes at $t = 0$, and is purely real.

6. Suppose $x(t)$ is the input to an LTI system with transfer function $H(\omega)$ and $y(t)$ is the output of the system, where

$$x(t) = e^{-|t|}\cos At \qquad \text{and} \qquad H(\omega) = 1 + e^{-j\omega} + e^{-3j\omega}$$

Find a real number $A > 0$ such that $y(0) = 1$. Is your answer unique?

It immediately follows from the definition of the transfer function that

$$\begin{aligned}
\hat{y}(s) &= \hat{x}(s)H(\omega) \\
&= \hat{x}(s) + \hat{x}(s)e^{-j\omega} + \hat{x}(s)e^{-j\omega 3}
\end{aligned}$$

Applying the time-shift property of the Fourier transform implies that $y(t)$ is given by

$$\begin{aligned}
y(t) &= x(t) + x(t-1) + x(t-3) \\
&= e^{-|t|}\cos(At) + e^{-|t-1|}\cos(At - A) + e^{-|t-3|}\cos(At - 3A)
\end{aligned}$$

Setting $y(0) = 1$, we solve:

$$\begin{aligned}
1 &= \cancelto{1}{e^{-|0|}\cos(0)} + e^{-|0-1|}\cos(-A) + e^{-|0-3|}\cos(-3A) \\
0 &= e^{-1}\cos(-A) + e^{-3}\cos(-3A) \\
&= e^{-1}\cos(A) + e^{-3}\cos(3A) \\
&= e^{2}\cos(A) + \cos(3A) \\
&= e^{2}\cos(A) + 4\cos^3(A) - 3\cos(A) \\
&= \cos(A)\left((e^2 - 3) + 4\cos^2(A)\right)
\end{aligned}$$

As the product vanishes, we have that either

$$\cos(A) = 0 \quad \text{or} \quad \cos^2(A) = -\frac{e^2-3}{4}$$

It follows that one such value of A is $A = \frac{\pi}{2}$; in fact, A can take on any value $A = n\pi + \frac{\pi}{2}$ for integer n. Hence this answer is not unique.

7. Suppose $g(t)$ is the input to an LTI system with transfer function $H(\omega)$, and $G(\omega)$ is the Fourier transform of $g(t)$. Find the output of the system $y(t)$.

Graphically, our transfer function $H(\omega)$ appears to attenuate all frequencies which are outside of the closed interval $[-5,5]$; while a gain of 0db and a phase shift of -1ω radians is applied to frequencies in the interval. It follows that the Fourier transform of the output $y(t)$ may be written as

$$\hat{y}(s) = \Delta(\frac{1}{2}\omega)e^{-j\omega^3}$$

where the unit triangle function is defined as $\Delta(t) = \begin{cases} 1+2t & -\frac{1}{2} \leq t \leq 0 \\ 1-2t & 0 \leq t \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$. Then, the inverse transform is given by

$$\begin{aligned} y(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{y}(s)e^{j\omega t}d\omega \\ &= \frac{1}{2\pi}\int_{-1}^{0}(1+t)e^{-j\omega}e^{j\omega t}d\omega + \frac{1}{2\pi}\int_{0}^{1}(1-t)e^{-j\omega}e^{j\omega t}d\omega \\ &= \frac{1+t}{2\pi}\int_{-1}^{0} e^{-j\omega}e^{j\omega t}d\omega + \frac{1-t}{2\pi}\int_{0}^{1} e^{-j\omega}e^{j\omega t}d\omega \\ &= \frac{1}{\pi}\int_{-1}^{1} e^{\omega(-j+jt)}d\omega \\ &= \frac{1}{\pi}\left[\frac{e^{\omega(-j+jt)}}{-j+jt}\right]_{-1}^{1} \\ &= \frac{1}{\pi}\left(\frac{e^{j(-1+t)}}{-j+jt} - \frac{e^{-j(-1+t)}}{-j+jt}\right) \\ y(t) &= \frac{2}{\pi(t-1)}\sin(t-1) \end{aligned}$$

and we have found $y(t)$.

8. Suppose we have a system for which the output $y(t)$ is

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

where the input is $x(t)$. Find $y(t)$ and its Fourier transform $Y(\omega)$ when the input is

$$x(t) = u(t+1) - 2u(t-1) + u(t-3)$$

where $u(t)$ is the unit step function.

We begin by deriving $\mathcal{F}\{u(t)\}$. Note that the Heaviside step function can be written, in the limit, as

$$u(t) = \lim_{T\to\infty} e^{-t/T}u(t)$$

which implies that the Fourier transform can be calculated in the limit as:

$$\mathcal{F}\{u(t)\} = \lim_{T\to\infty} \mathcal{F}\{u(t)e^{-t/T}\}$$

This Fourier transform is given by

$$\begin{aligned}
\mathcal{F}\{u(t)e^{-t/T}\} &= \int_{-\infty}^{\infty} u(t)e^{-t\left(\frac{1}{T}+j\omega\right)}dt \\
&= \int_{0}^{\infty} e^{-t\left(\frac{1}{T}+j\omega\right)}dt \\
&= \left[\frac{-e^{-t\left(\frac{1}{T}+j\omega\right)}}{\frac{1}{T}+j\omega}\right]_0^{\infty} \\
&= \frac{1}{\frac{1}{T}+j\omega}
\end{aligned}$$

Calculating the limit, we have

$$\begin{aligned}
\lim_{T\to\infty} \frac{1}{\frac{1}{T}+j\omega} &= \lim_{T\to\infty} \frac{\frac{1}{T}-j\omega}{\frac{1}{T^2}+\omega^2} \\
&= \lim_{T\to\infty} \left(\frac{T}{\frac{1}{T^2}+\omega^2} - \frac{j\omega T^2}{\omega^2 T^2}\right) \\
&= \pi\delta(\omega) + \frac{1}{j\omega}
\end{aligned}$$

It follows that by superposition arguments, the Fourier transform of $x(t)$ is given by

$$\hat{x}(s) = e^{j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right) - 2e^{-j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right) + e^{-3j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)$$

Since $x(t)$ is defined as the derivative of $y(t)$, by the integral property we have

$$\begin{aligned}
\hat{y}(s) &= \frac{1}{j\omega}\hat{x}(s) + \hat{x}(0)\pi\delta(\omega) \\
&= \frac{1}{j\omega}\left(e^{j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right) - 2e^{-j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right) + e^{-3j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)\right) \\
&\quad + 0 \\
\hat{y}(s) &= \frac{1}{j\omega}\left(e^{j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right) - 2e^{-j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right) + e^{-3j\omega}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)\right)
\end{aligned}$$

Solving directly by superposition, we obtain $y(t)$:

- $u(t+1) \overset{\text{LTI}}{\to} \begin{cases} 0 & t \le -1 \\ 1+t & \text{otherwise} \end{cases}$
- $2u(t-1) \overset{\text{LTI}}{\to} \begin{cases} 0 & t \le 1 \\ -2+2t & \text{otherwise} \end{cases}$
- $u(t-3) \overset{\text{LTI}}{\to} \begin{cases} 0 & t \le 3 \\ -3+t & \text{otherwise} \end{cases}$

Hence the output $y(t)$ is similarly given by

$$y(t) = \begin{cases} 0 & t \le -1 \\ t+1 & -1 \le t \le 1 \\ 3t-1 & 1 \le t \le 3 \\ 4t-4 & 3 \le t \end{cases}$$

and we have found both $y(t)$ and its Fourier transform, $\hat{y}(s)$.

9. An LTI system has impulse response $h(t) = e^{-3t}u(t)$. What was the input $x(t)$, if the output is $e^{-3t}u(t) - e^{-4t}u(t)$?

   For a linear and time-invariant system, the Fourier transform of its impulse response is mathematically identical to its transfer function. We begin by taking the Fourier transform of the impulse response:

$$\begin{aligned} H(s) &= \int_{-\infty}^{\infty} u(t)e^{-3t}e^{-j\omega t}dt \\ &= \int_{0}^{\infty} e^{-t(3+j\omega)}dt \\ &= \frac{1}{3+j\omega} \end{aligned}$$

   Similarly, the Fourier transform of the response $y(t)$ is given by

$$\begin{aligned} \hat{y}(s) &= \int_{-\infty}^{\infty} u(t)e^{-3t} - u(t)e^{-4t}e^{-j\omega t}dt \\ &= \frac{1}{3+j\omega} - \frac{1}{4+j\omega} \end{aligned}$$

   In the frequency domain, we have the equality

$$\hat{x}(s)h(\omega) = \hat{y}(s)$$

   implying

$$\begin{aligned} \hat{x}(s) &= \frac{1}{h(\omega)}\hat{y}(s) \\ &= (3+j\omega)\left(\frac{1}{3+j\omega} - \frac{1}{4+j\omega}\right) \\ &= 1 - \frac{3+j\omega}{4+j\omega} \\ &= \frac{4+j\omega-3-j\omega}{4+j\omega} \\ &= \frac{1}{4+j\omega} \end{aligned}$$

   whose inverse transform yields

$$x(t) = u(t)e^{-4t}$$

   and hence we have found the input.

10. Let $x(t) = u(t)e^{-3t}$. Find $y(t)$ when

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} - 2y(t) = x(t)$$

   using partial fractions.

   Equating, we obtain

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} - 2y(t) = x(t) = u(t)e^{-3t}$$

Taking the Fourier transform of both sides and solving for the transform $\hat{y}(s)$ yields

$$\begin{aligned}
(j\omega)^2\hat{y}(s) + j\omega\hat{y}(s) - 2\hat{y}(s) &= \frac{1}{3+j\omega} \\
\hat{y}(s)\left(-\omega^2 + j\omega - 2\right) &= \frac{1}{3+j\omega} \\
\hat{y}(s) &= \frac{1}{(3+j\omega)(2+j\omega)(-1+j\omega)} \\
&= \frac{A}{3+j\omega} + \frac{B}{2+j\omega} + \frac{C}{-1+j\omega}
\end{aligned}$$

Equating coefficients and performing Gaussian elimination on the coefficient matrix, we obtain

$$\hat{y}(s) = \frac{1}{3}\cdot\frac{1}{3+j\omega} - \frac{1}{4}\cdot\frac{1}{2+j\omega} + \frac{1}{12}\cdot\frac{r}{-1+j\omega}$$

Since we know $\mathcal{F}\{u(t)e^{-\alpha t}\} = \frac{1}{\alpha+j\omega}$, we obtain that

$$y(t) = u(t)\left(\frac{e^{-3t}}{3} - \frac{e^{-2t}}{4} + \frac{e^{t}}{12}\right)$$

and hence we have found the output.

1 Matlab Problem 4

1. **Output 1.** Running `mod_play(noisy)` results in what sounds like many voices overlaid at once. The graph of the modulated signal is

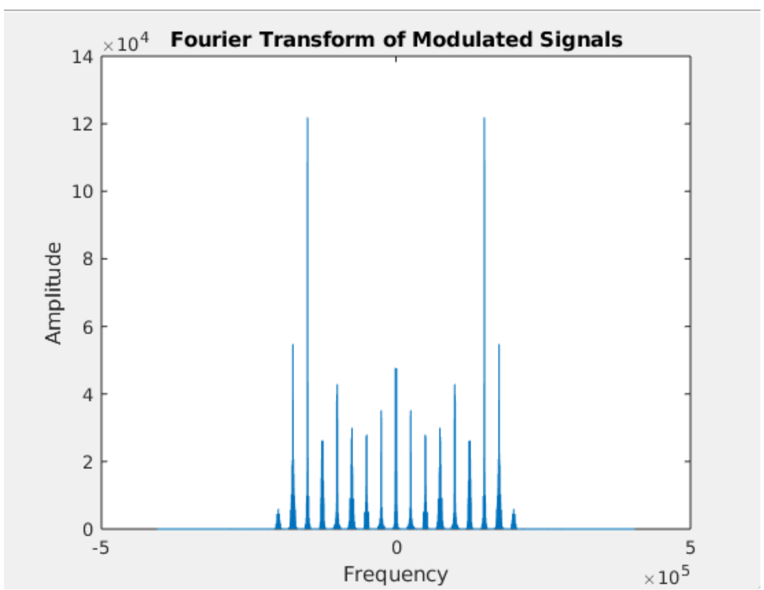

Figure 1: MATLAB frequency plot of modulated signals

and was generated with the commands

```
>> len = length(mod);
>> Fs = 811025;
```

```
>> f = Fs * (-len/2:len/2-1)/len;
>> modfreq = fft(mod);
>> plot(f, abs(fftshift(modfreq)))
>> title("Fourier Transform of Modulated Signals")
>> xlabel("Frequency")
>> ylabel("Amplitude")
```

2. **Output 2.** To generate frequency plots for the modulated signals, we use the commands

```
>> len = length(mod);
>> Fs = 811025;
>> f = Fs * (-len/2:len/2-1)/len;
>> modfreq = fft(mod);
>> plot(f, abs(fftshift(modfreq)))
>> title("Fourier Transform of Modulated Signals")
>> xlabel("Frequency")
>> ylabel("Amplitude")
```

and obtain

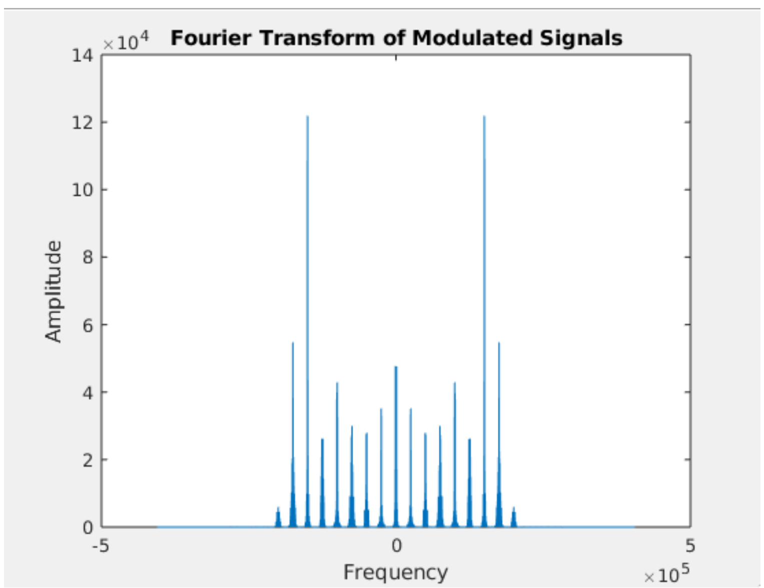

Figure 2: MATLAB frequency plot of modulated signals

For the unmodulated (noisy) signal, we use the commands

```
>> len = length(noisy);
>> Fs = 811025;
>> f = Fs * (-len/2:len/2)/len;
>> f = Fs * (-len/2:len/2-1)/len;
>> noisyfreq = fft(noisy);
>> plot(f, abs(fftshift(noisyfreq)))
>> title("Fourier Transform of Unmodulated Signals")
>> xlabel("Frequency")
>> ylabel("Amplitude")
```

and obtain

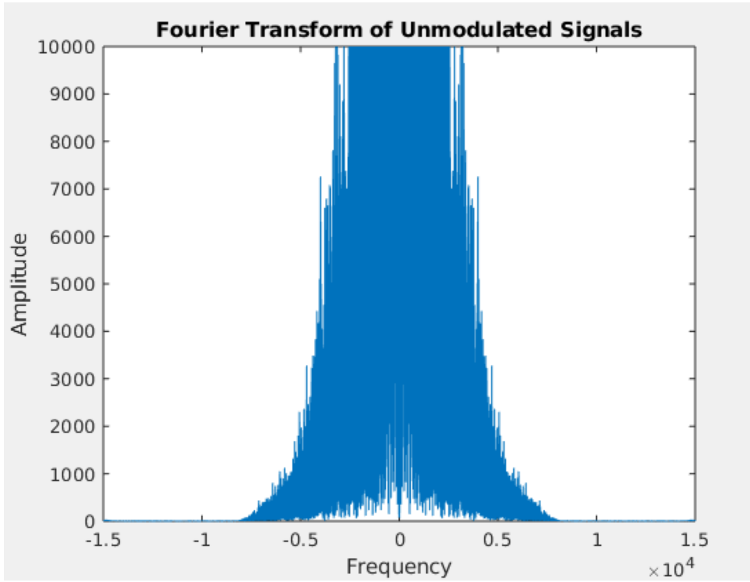

Figure 3: MATLAB frequency plot of unmodulated signals

3. **Outputs 3 and 4 for Selected Signals**

$k = 9$: The frequency plot of the message is

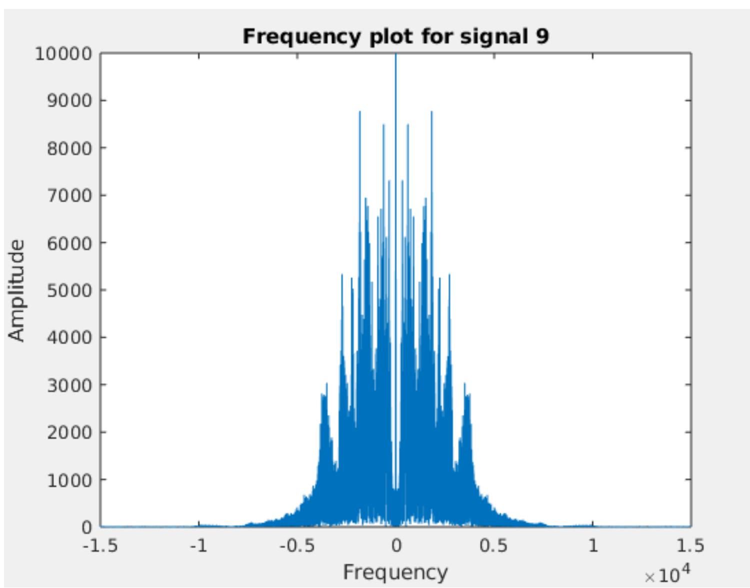

Figure 4: **Output 3:** MATLAB frequency plot of signal 9

and the audio message is

Output 4: Who died and made you my judge?

Demodulating the signal and producing a plot was done by

```
>> t = (0:len-1) / Fs;
>> fk = 200000;
```

```
>> Filtered = modfreq .* HW3_Filter(f, fk-15000, fk+15000);
>> filtered = real(ifft(Filtered));
>> demodded = filtered .* (2*cos(2*pi*fk*t));
>> Demodded = fft(demodded);
>> Message = Demodded .* HW3_Filter(f, -15000, 15000);
>> message = real(ifft(Message));
>> mod_play(message);
>> plot(f, abs(fftshift(Message)));
>> axis([-15000, 15000, 0, 10000]);
>> title(”Frequency plot for signal 9”)
>> xlabel(”Frequency”)
>> ylabel(”Amplitude”)
```

$k = 1$: The frequency plot of the message is

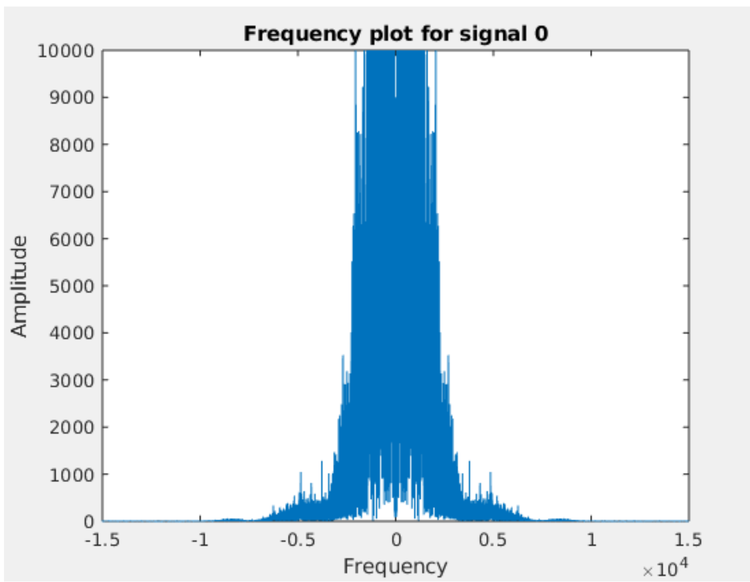

Figure 5: **Output 3:** MATLAB frequency plot of signal 1

and the audio message is

Output 4: I'll be back.

Demodulating the signal and producing a plot was done by

```
>> t = (0:len-1) / Fs;
>> fk = 0;
>> Filtered = modfreq .* HW3_Filter(f, fk-15000, fk+15000);
>> filtered = real(ifft(Filtered));
>> demodded = filtered .* (2*cos(2*pi*fk*t));
>> Demodded = fft(demodded);
>> Message = Demodded .* HW3_Filter(f, -15000, 15000);
>> message = real(ifft(Message));
>> mod_play(message);
>> plot(f, abs(fftshift(Message)));
>> axis([-15000, 15000, 0, 10000]);
>> title(”Frequency plot for signal 0”)
>> xlabel(”Frequency”)
>> ylabel(”Amplitude”)
```

$k = 4$: The frequency plot of the message is

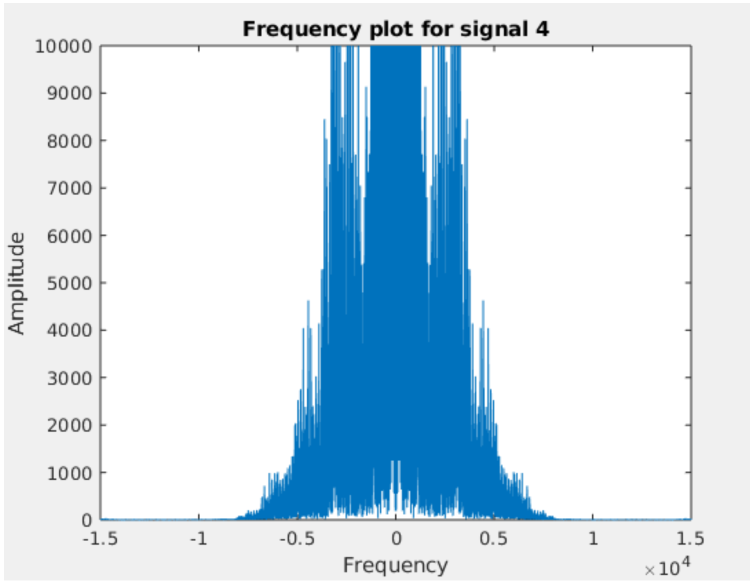

Figure 6: **Output 3:** MATLAB frequency plot of signal 4

and the audio message is

Output 4: You have failed me for the last time.

Demodulating the signal and producing a plot was done by

```
>> t = (0:len-1) / Fs;
>> fk = 75000;
>> Filtered = modfreq .* HW3_Filter(f, fk-15000, fk+15000);
>> filtered = real(ifft(Filtered));
>> demodded = filtered .* (2*cos(2*pi*fk*t));
>> Demodded = fft(demodded);
>> Message = Demodded .* HW3_Filter(f, -15000, 15000);
>> message = real(ifft(Message));
>> mod_play(message);
>> plot(f, abs(fftshift(Message)));
>> axis([-15000, 15000, 0, 10000]);
>> title("Frequency plot for signal 4")
>> xlabel("Frequency")
>> ylabel("Amplitude")
```