# Fluid Propertys

$c^\circ = \frac{5}{9}(F-32)$

$F^\circ = \frac{9}{5}(c-32)$

$K = C + 273.15$

$R^\circ = F + 459.67$ , $0R^\circ = 0K$

$P = \frac{F}{A}$ $\quad \frac{N}{m^2} = Pa$ , $\frac{lb}{ft^2} = Psi$

Conversions : 1 atm = 101.325 Pa = 14.7 psi

100,000 Pa = 1bar

$\rho = \frac{m}{V}$ $\quad \frac{kg}{m^3}$ , $\frac{slugs}{ft^3}$

$P = \rho R T$ (equation of state)

$R = 287 \frac{J}{kg K}$ , $1716 \frac{ft\,lb}{slug\,R}$

$dP = -\rho g\, dh$ (hydro static equation)

$P_2 - P_1 = -\rho g (h_2 - h_1)$

$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \longrightarrow$

$V_1 A_1 = V_2 A_2$

$Q = VA$ (volume flow Rate) $\frac{m^3}{s}$ , $\frac{ft^3}{s}$

$A_1 < A_2 \rightarrow V_1 > V_2$ decreasing

## Bernolli

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$ $\quad P_1 - P_2 = \frac{1}{2}\rho(V_1^2 - V_2^2)$

$\underset{\text{static}}{P_1} + \underset{\text{Dynamic}}{\frac{1}{2}\rho V_1^2} = \overset{\text{total}}{\text{constant}}$

- stagnation point : $V=0$ , measure tot p
- static p port : flow over not into , measure static p
- P Transducer Connected : measure p dif. $(P_s - P_T)$
- lift : flow top faster
- no viscouse, steady flow, incompressible, points are on same stream line

## Airfoils

C: chord $\quad \bar{C} = \frac{C_T + C_R}{2}$

$\alpha$ : angle of attack $\quad \bar{C} = \frac{S}{b}$

b: Wingspan

S: Wing Planform (area)

AR: Aspect Ratio $\longrightarrow AR = \frac{b^2}{S}$

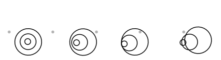
- ↑ AR, ↓ wingtip effects, ↑ efficiency

$\lambda$ : Taper Ratio $\longrightarrow \lambda = \frac{C_{tip}}{C_{root}} \longrightarrow 1$ for rectangles

NACA 2 4 12

- 2: max camber in chord 2%
- 4: Position of max cam $\frac{x}{c} = 0.4$
- 12: max thickness of chord 12%

## Compressibility




$M_\infty < 0.3$ Subsonic Incompressible

$0.3 < M_\infty < 0.7$ Subsonic compressible

$0.7 < M_\infty < 1.2$ Transonic

$1.2 < M_\infty < 5$ Supersonic

$5 < M_\infty$ Hypersonic

# Aerodynamics

- Compressibility : $\Delta$ in vol $(\rho)$
- Viscosity : resistance to shear itself
- Adiabatic : no heat transfer , can $\Delta$ temp
- Reversible : no $\Delta$ in global entropy, can $\Delta$ in local
- Isentropic : no $\Delta$ in entropy (= adia & rever)
- Lamiar flow : no mixing , smooth in 1 direction
- Turbulent flow : mixing, all directions , flow separates
- Boundary layer : near body , can have lamiar & turbulent

## Lift Curve

$\alpha_{L=0}$ : no lift $\alpha$

- 0° for symmetric, − for cambered
- $C_{l,\alpha=0} > 0$ for cam

$a_0$ : lift slope

$a_0 = \frac{C_l}{\alpha}$, linear , $\frac{1}{deg}$ or $\frac{1}{rad}$

$C_{l\,max}$ : at stall (peak)

$C_{D,0}$ : straight vertical part

## Coefficents

2D: Lower case , $q = \frac{1}{2}\rho v^2$

$C_l = \frac{L}{\frac{1}{2}\rho v^2 c} = a_0(\alpha - \alpha_{L=0})$

$C_d = \frac{D}{\frac{1}{2}\rho v^2 c}$

$C_m = \frac{M}{\frac{1}{2}\rho v^2 c^2}$

$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho v^2 c}$

3D: upper case

$C_L = \frac{L}{\frac{1}{2}\rho v^2 S} = a(\alpha - \alpha_{L=0})$

$C_D = \frac{D}{\frac{1}{2}\rho v^2 S}$

$C_M = \frac{M}{\frac{1}{2}\rho v^2 S c}$

$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty}$

$\mu_{sea\ level} = 1.789 \times 10^{-5}$ Pa s

$M_\infty = \frac{V_\infty}{a_\infty} = \frac{V_\infty}{\sqrt{\gamma R T_\infty}}$

$\gamma = 1.4$ , $R = 287 \frac{J}{kg K}$ , $1716 \frac{ft\,lb}{slug\,R}$

## Build up

- wing

turbulent

$C_f = \frac{0.455}{(\log Re)^{2.58} \; (1 + 0.144M^2)^{0.65}}$

$C_f = \frac{1.328}{\sqrt{R}}$ (laminar)

$FF = \left[1 + \frac{0.6}{x/c}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34 M^{0.18}(\cos\Lambda_m)^{0.28}\right]$

$t/c < 0.05$: $S_{wet} = \left[1.977 + 0.52\left(\frac{t}{c}\right)\right] S_{exposed}$

$t/c > 0.05$: $S_{wet} = 2.003\, S_{exposed}$

$C_{D,0} = C_f\, FF\, Q \frac{S_{wet}}{S_{wing}}$

- Fuselage

$C_f$ = same

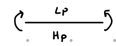
$FF = 0.9 + \frac{5}{f^{1.5}} + \frac{f}{400}$

$f = \frac{l}{d} = \frac{l}{\sqrt{4/\pi \underset{\text{Area}}{A_{max}}}}$

$C_{D,0}$ = same

# 3D wing

- wing tip effcts




- less lift at tips of wings , vortices
  - reduce vortices, ↑AR w/out ↑ b
- Downwash (w) : vortices create downward flow, tip

$\alpha_g = \alpha$ : geometric

$\alpha_i$ : induced from w

$\alpha_{eff}$ : effective , $\alpha_{eff} < \alpha$

$\alpha_{eff} = \alpha - \alpha_i$

- more drag

- AR > 4 , straight , imcompressible

$\alpha_{i\,rad} = \frac{C_L}{\pi e AR}$ $\quad \alpha_{eff} = \alpha - \alpha_i$

$\alpha_{i\,deg} = \frac{57.3\, C_L}{\pi e AR}$

e = 0.7 rect

e = 1 elliptical planform

e ≤ 1 flat wings

e can be > 1 winglets

$a_{rad} = \frac{a_0}{1 + \frac{a_0}{\pi e AR}}$ $\quad e_0$ : oswald (whole plane)

$a_{deg} = \frac{a_0}{1 + \frac{57.3\, a_0}{\pi e AR}}$ $\quad e$ : span eff (wing)

- lift induced drag

$D = q_\infty S C_{D,0} + q_\infty S \frac{C_L^2}{\pi e AR}$

$C_D = \underset{\text{2D drag}}{\overset{\text{Zero lift Parasite}}{C_{D,0}}} + \underset{\text{induced drag}}{\frac{C_L^2}{\pi e AR}}$ $\quad k = \frac{1}{\pi e AR}$

$= C_{D,0} + k C_L^2$

- AR > 4, straight , compressible

$a_{comp\,rad} = \frac{a_0}{\sqrt{1 - M_\infty^2} + \frac{a_0}{\pi e AR}}$

$C_L = a_{comp}(\alpha - \alpha_{L=0})$

- AR > 4, straight , supersonic

$a_{comp\,rad} = \frac{4}{\sqrt{M_\infty^2 - 1}}$

- AR < 4, straight , subsonic & incompressible

$a_{rad} = \frac{a_0}{\sqrt{1 + \left(\frac{a_0}{\pi AR}\right)^2} + \frac{a_0}{\pi AR}}$

- AR < 4, straight , subsonic & compessible

$a_{comp\,rad} = \frac{a_0}{\sqrt{1 - M_\infty^2 + \left(\frac{a_0}{\pi AR}\right)^2} + \frac{a_0}{\pi AR}}$

- AR < 4, straight , supersonic

$a_{comp\,rad} = \frac{4}{\sqrt{M_\infty^2 - 1}}\left(1 - \frac{1}{2AR\sqrt{M_\infty^2 - 1}}\right)$

- Swept wings , subsonic imp , com

$a_{rad} = \frac{a_0 \cos\Lambda}{\sqrt{1 + \left(\frac{a_0 \cos\Lambda}{\pi AR}\right)^2} + \frac{a_0 \cos\Lambda}{\pi AR}}$

$a_{comp\,rad} = \frac{a_0 \cos\Lambda}{\sqrt{1 - M_\infty^2 \cos^2\Lambda + \left(\frac{a_0 \cos\Lambda}{\pi AR}\right)^2} + \frac{a_0 \cos\Lambda}{\pi AR}}$

## Area of Circle

$A = \frac{\pi}{4} d^2$

## Conversions

5280 ft = 1mile $\quad$ ft/s $\quad$ m/s

deg · $\frac{\pi}{180} \rightarrow$ rad $\quad$ lb/f $\quad$ kg/m

# Steady flight

- Thrust = drag

$T = D = q_\infty S C_D$

$T = \frac{W}{C_L/C_D}$

$T = \underset{\substack{\text{Zero lift drag} \\ \infty = V_\infty \to \infty}}{q S C_{D,0}} + \underset{\substack{\text{Drag due to lift} \\ 0 = V_\infty \to \infty}}{\frac{K W^2}{q_\infty S}}$ @ dynamic pressure

$T_{min} = W\sqrt{4 K C_{D,0}}$

$\frac{C_L}{C_D}_{max} = \frac{1}{\sqrt{4 K C_{D,0}}}$

$V_\infty = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{k}{C_{D,0}}}}$ @ $T_R$ , min

- lift = weight

$W = L = q_\infty S C_L$

$C_L = \frac{W}{q_\infty S}$ @ dynamic pressure

$C_L = \frac{2W}{\rho_\infty V_\infty^2 S}$ @ airspeed and density (altitude)

- power

Power = $\frac{W}{t} = \frac{F \times D}{t}$ = Force × velocity

$= T V_\infty$

$P = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \frac{2kW^2}{\rho_\infty V_\infty S}$

$V_\infty = \sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\sqrt{\frac{K}{3C_{D,0}}}}$ for min power

$P_{min} = \sqrt{\frac{2}{\rho_\infty S}}\frac{W^{3/2}}{C_L^{3/2}/C_D}$

- Stall velocity

$V_{stall} = \sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{C_{L\,max}}}$

- weight build up

$1 = \frac{W_e}{W_0} \quad \frac{W_P}{W_0} \quad \frac{W_f}{W_0}$

$W_e$ = empty (no fuel / payload)

$W_0$ = Gross take off

$W_P$ = Payload (can $\Delta$)

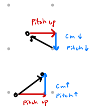
$W_f$ = fuel (can $\Delta$)

## Stability

neutral point : moment doesn't $\Delta$ w/ $\alpha$

stable : CG infront of neutral point

- Pitching moment
  - stable $\frac{\partial C_m}{\partial \alpha} < 0$
  - unstable $\frac{\partial C_m}{\partial \alpha} > 0$




$C_{HT} = \frac{\overset{\text{Top view Area}}{S_{HT}}\,\overset{\Delta \text{ b/w } C_g \text{ + } \frac{1}{4}\text{ chord HT}}{L_{HT}}}{\underset{\text{Avg chord}}{c}\,\underset{\text{wing Reference area}}{S_{Ref}}}$

$C_{VT} = \frac{\overset{\text{Side view area}}{S_{VT}}\,\overset{\Delta \text{ b/w } C_g \text{ + } \frac{1}{4}\text{ chord}}{L_{VT}}}{\underset{\text{wing span}}{b}\,\underset{\text{wing ref area}}{S_{Ref}}}$

## Drag polar

2D: $C_d = C_{d,0} + k_l C_l^2$

$C_d = C_{d,0}$ (simple)

- drag bucket → laminar flow
- slope = $\frac{C_D}{C_L}$ (aero dynamic efficency Ratio)
  - max for optimal aerodynamics

# High AR Straight wing (incomp)

$C_L = a(\alpha - \alpha_{L=0})$

$C_D = C_{D,0} + KC_L^2$

$k = \dfrac{1}{\pi e_0 AR}$

$e_0$: oswald (full aircraft, its $e$)

$e$: span eff

$e = 0.7$ rec, $e = 1$ ellip

$a(rad) = \dfrac{a_0}{1 + \dfrac{a_0}{\pi e AR}}$ match units

$a(deg) = \dfrac{a_0}{1 + \dfrac{57.3\, a_0}{\pi e AR}}$

# Range & Endurance

| | Battery | fuel Driven Prop | Turbojet / rocket |
|---|---|---|---|
| max end ration | $\frac{C_L^{3/2}}{C_D}$ max | $\frac{C_L^{3/2}}{C_D}$ max | $\frac{C_L}{C_D}$ max |
| max end form | $\frac{E\eta_m\eta_{pr}\sqrt{\rho_\infty S}}{\sqrt{2}\,W^{3/2}}$ | $\frac{\eta_{pr}\sqrt{2\rho_\infty S}}{c}\frac{C_L^{3/2}}{C_D}\left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}}\right)$ | $\frac{1}{C_T}\left(\frac{C_L}{C_D}\right)\ln\frac{W_0}{W_1}$ |
| max ran ratio | $\frac{C_L}{C_D}$ max | $\frac{C_L}{C_D}$ max | $\frac{C_L^{1/2}}{C_D}$ max |
| max end form | $\frac{E\eta_m\eta_{pr}}{W}\frac{C_L}{C_D}$ | $\frac{\eta_{pr}}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W_1}$ | $2\sqrt{\frac{2}{\rho_\infty S}}\frac{1}{C_T}\frac{C_L^{1/2}}{C_D}\left(\sqrt{W_0} - \sqrt{W_1}\right)$ |

E: Bat energy — C: (Brake) Spec fuel consup — W: const weight

$\eta_{pr}$: Prop eff — $C_T$: Thrust spec fuel consup — $W_0$: Gross Take off

$\eta_m$: Elc mot eff — $W_1$: weight w/ fuel

# Effciency Ratios

| use | $\frac{C_L^{3/2}}{C_D}$ max | $\frac{C_L}{C_D}$ max | $\frac{C_L^{1/2}}{C_D}$ max |
|---|---|---|---|
| situation | max end (prop) | max ran (prop) max end (jet) | max Ran (jet) |
| Position of req'd curve's | min on pow | min on thr, tangent on pow | Tangent on thr |
| $V_\infty$ to achive | $\sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\sqrt{\frac{K}{3C_{D,0}}}}$ | $\sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\sqrt{\frac{K}{C_{D,0}}}}$ | $\sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\sqrt{\frac{3K}{C_{D,0}}}}$ |
| relation $C_{D,0} : C_i$ | $3C_{D,0} = KC_L^2$ | $C_{D,0} = KC_L^2$ | $C_{D,0} = 3KC_L^2$ |

$\sqrt{\dfrac{1}{4KC_{D,0}}}$

- endurance: time in air w/ given fuel
- range: distance can fly given fuel

# Steady Flight

$L = W$ $\qquad C_L = \dfrac{W}{q_\infty S}$

$T = D$ $\qquad C_L = \dfrac{2W}{\rho_\infty V_\infty^2 S}$

$T = \dfrac{W}{C_L/C_D}$ $\qquad$ Power $P = TV_\infty$

$T_R = q_\infty S C_{D,0} + \dfrac{KW^2}{q_\infty S}$

$T_R = \frac{1}{2}\rho_\infty V_\infty^2 S C_{D,0} + \dfrac{2KW^2}{\rho_\infty V_\infty^2 S}$

$P_R = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \dfrac{2KW^2}{\rho_\infty V_\infty S}$

- Thrust
  - $T_{min}$, $C_L/C_D$ max
  - $T_A$ (avalible) depends on alt ($\rho$)
- Power
  - $P_R = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \dfrac{2KW^2}{\rho_\infty V_\infty S}$
  - $P_R$ min, $\dfrac{C_L^{3/2}}{C_D}$ max
  - Exess Power $= P_A - P_R$

$\left(\dfrac{C_L^{3/2}}{C_D}\right)_{max} : 3C_{D,0} = KC_L^2$

# Propulsion

$T = \dot{m}V_j = \dot{m}(V_j - V_\infty)$ ground ref

$\dot{m} = \rho VA$

$P_A = TV_\infty = \eta_p P = \eta_p \cdot P_{tot}$

$P_{tot} = TV_\infty + \frac{1}{2}\dot{m}(V_j - V_\infty)^2$

$\eta_p = \dfrac{P_A}{P_{tot}} = \dfrac{2}{1 + \frac{V_j}{V_\infty}}$

$\eta_{pr} = \dfrac{\text{aero pow}}{\text{mech pow}}$ prop

$J = \dfrac{V_\infty}{ND}$ Diameter, rev/sec

Thrust for turbojet

$T = (\dot{m}_{air} + \dot{m}_{fuel})V_j - \dot{m}_{air}V_\infty + (p_{exit} - p_\infty)A_{exit}$

Bypass ratio: $\dfrac{\dot{m}_{fan}}{\dot{m}_{core}}$

- turbojet
  - $P_A = (T_{prop} + T_j)V_\infty$
  - $P_A = \eta_{pr}P_s + T_jV_\infty$
  - $P_A = \eta_{ps}P_{es}$
  - $P_{es} = P_s + \dfrac{T_jV_\infty}{\eta_{pr}}$

# Fuel Consumption

- Pistion/Prop
  - SFC: $c$ $\left(\frac{1}{m}\text{ or }\frac{lb}{hp\,hr}\right)$
  - $c = \dfrac{\dot{W}_f}{P} = \dfrac{\text{fuel burned}}{\text{eng pow}}$
  - SFC = BSFC (g)
  - BSFC $\left(\frac{kg/s}{w}\right)$
- Jet
  - $C_T = \dfrac{\dot{W}_f}{T}$ $\left(\frac{1}{s}\text{ or }\frac{1}{hr}\right)$
- Turboprop
  - $C_T = \dfrac{\dot{W}_f}{T} = \dfrac{\dot{W}_f}{T_{prop} + T_j}$ TSFC
  - $C_A = \dfrac{\dot{W}_f}{P_A}$ SFC: Pow aval
  - $C_s = \dfrac{\dot{W}_f}{P_s}$ SFC: shaft pow
  - $C_{es} = \dfrac{\dot{W}_f}{P_{es}}$ SFC: equi shaft pow
- BSFC to TSFC
  - $C_T = \dfrac{cV_\infty}{\eta_{pr}}$

# Steady Gliding flight (no pow)

$\dfrac{1}{L/D} = \tan\theta$ rate of decent, sink

$\tan\theta_{min} = \dfrac{1}{L/D_{max}}$

- $\theta$ min, max glide, V L/D max

$d = \dfrac{h}{\tan\theta}$ : glide range

max range ~ h L/D max

# Takeoff

head wind: take off, landing, max endurcence

tail: max range

$V_1$: desision

$V_R$: rotation, pull up

$V_{LO}$: L=W pull up manuver ≈ 1.2 Vstall

- normal and friction (R)
- @ 0.7 $V_{LO}$

$S_g = \dfrac{1.44\frac{W}{S}}{g\rho_\infty C_{L,max}\left(\frac{T}{W} - \frac{D}{W} - \mu_r\left(1 - \frac{L}{W}\right)\right)}$

$L = \frac{1}{2}\rho_\infty(0.7V_{LO})^2 S C_L$

$D = \frac{1}{2}\rho_\infty(0.7V_{LO})^2 S(C_{D,0} + \phi KC_L^2)$

$\phi = \dfrac{(16\,h/b)^2}{(1 + 16h/b)^2}$ b = wing span

- pull up

$R = \dfrac{V_\infty^2}{g(n-1)} = \dfrac{1.44\,V_{stall}^2}{0.15g}$ changes with $V_{flare}$, $V_{LO}$

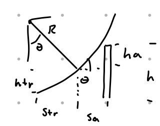
$h_{tr} = R - R\cos\theta$

$s_{tr} = R\sin\theta$

$h_a = h - h_{tr} = 35^{ft} - R + R\cos\theta$

$S_a = \dfrac{h_a}{\tan\theta}$

$S_t = S_g + S_{tr} + S_a$




tr: transition
g: ground
a: air
h: hight
s: distance

# Landing

$V_A$: approch ~ 1.3 Vstall

$V_F$: flare ~ 1.23 Vstall

$V_{TD}$: tochdown ~ 1.15 Vstall = $V_{FR}$ free roll

$h_f = R - R\cos\theta_f$ f: flare

$S_f = R\sin\theta_f$

$h_f + h_a = h = 50^{ft}$

$S_a = \dfrac{h_a}{\tan\theta}$

$S_g = \dfrac{1.69\frac{W}{S}}{g\rho_\infty C_{L,max}\left(\frac{D}{W} + \mu_r\left(1 - \frac{L}{W}\right)\right)}$

$S_L = S_a + S_f + S_g$

# Pull Up

$R = \dfrac{V_\infty^2}{g(n-1)}$ $\qquad$ load factor $n = \dfrac{L}{W}$

$\omega = \dfrac{g(n-1)}{V_\infty}$ $\qquad$ level turn $\omega = \dfrac{V_\infty}{R}$ rad/s

# Pull down

$R = \dfrac{V_\infty^2}{g(n+1)}$

$\omega = \dfrac{g(n+1)}{V_\infty}$

# Level turn

$R = \dfrac{V_\infty^2}{g\sqrt{n^2 - 1}}$

$\omega = \dfrac{g\sqrt{n^2 - 1}}{V_\infty}$

# Steady Climbing flight

rate of climb $R/C = V_V = V_\infty\sin\theta$

$R/C = \dfrac{P_A - P_R}{W}$ exess power

$R/C = \dfrac{TV_\infty - DV_\infty}{W} = V_\infty\sin\theta$

$T_R = D = q_\infty S C_{D,0} + \dfrac{KW^2\cos^2\theta}{q_\infty S}$

$\dfrac{T - T_R}{W} = \sin\theta = \theta$ for small $\theta$ in rad

Prop $P = TV_\infty$ $\qquad$ jets

$\dfrac{T}{W} - \dfrac{1}{(L/D)_{max}} = \sin\theta_{max}$

$\dfrac{T}{W} - 4\sqrt{C_{D,0}K} = \sin\theta_{max}$

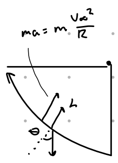
$V_{\theta max} = \sqrt{\dfrac{2}{\rho_\infty}\sqrt{\dfrac{K}{C_{D,0}}}\dfrac{W}{S}\cos\theta_{max}}$

$R/C_{\theta max} = V_{\theta max}\sin\theta_{max}$

# Weight Build up

$W_0 = W_e + W_p + W_f$ (empty, pay, fuel)

$1 = \dfrac{W_e}{W_0} + \dfrac{W_p}{W_0} + \dfrac{W_f}{W_0}$




Pull up


pull down