

# Example

Sinusoidal wave

$$A \cos(\omega t + \theta) \rightarrow \text{ang. freq.} = \omega$$

$$f = \frac{\omega}{2\pi} \quad T = \frac{2\pi}{\omega}$$

D4.1. Given the voltage  $v(t) = 120 \cos\left(314t + \frac{\pi}{4}\right)$  V, determine the frequency of the voltage in Hertz and the phase angle in degrees.

$$f = \frac{314}{2\pi} = 49.97 \text{ Hz} \quad \text{phase angle} = 45^\circ$$

D4.3. Convert the following voltage functions to phasors.

$$v_1(t) = 12 \cos(377t - 425^\circ) \text{ V}$$

$$v_2(t) = 18 \sin(2513t + 4.2^\circ) \text{ V}$$

$$\cos(\theta - 90^\circ) = \sin \theta$$

$$V_2 = 18 \cos(2513t - 85.8^\circ)$$

$$360 + 65$$

$$V_1(t) = 12 \cos(377t - 65^\circ)$$

$$V_1 = 12 \angle -65^\circ \text{ V} \quad V_2 = 18 \angle -85.8^\circ \text{ V}$$

D4.4. Convert the following phasors to the time domain if the frequency is 400 Hz.

$$V_1 = A \cos(2\pi f t + \phi)$$

$$A = 10$$

$$\phi = 20^\circ$$

$$V_1 = 10 \angle 20^\circ$$

$$V_2 = 12 \angle -60^\circ$$

$$V_1 = 10 \cos(800\pi t + 20^\circ)$$

$$V_2 = 12 \cos(800\pi t - 60^\circ)$$

# Example

**D4.5.** Compute the impedance  $Z_T$  in the network in Figure D4.5.

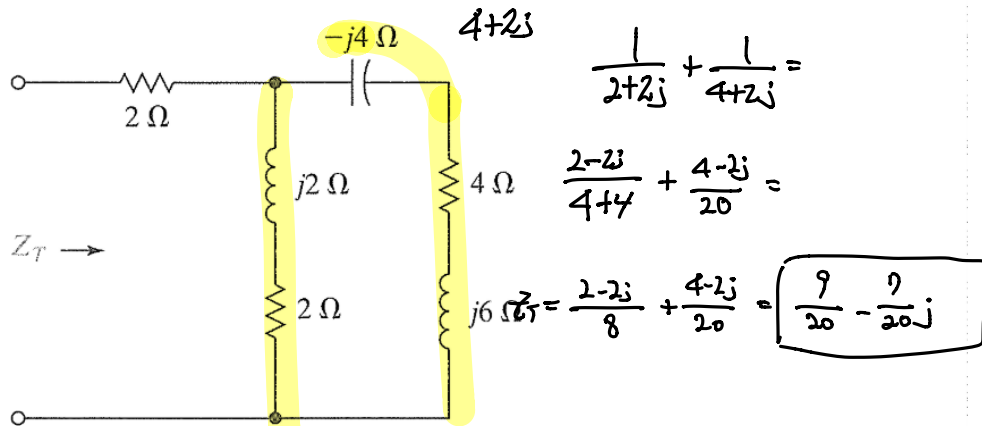


FIGURE D4.5

# Example

**D4.6.** Determine  $Z_{eq}$  at the terminals  $A - B$  of the network shown in Figure D4.6.

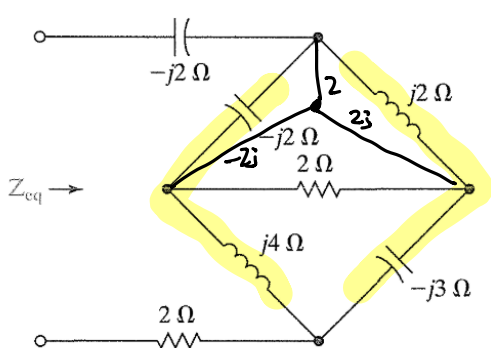


FIGURE D4.6

$$\frac{1}{\frac{1}{2j} + \frac{1}{-j}} = -2j$$

$$\underline{Z_{eq} = 4.43 \Omega}$$

# Example

**D4.8.** Use nodal analysis to find  $\mathbf{I}_0$  in the circuit in Figure D4.8.

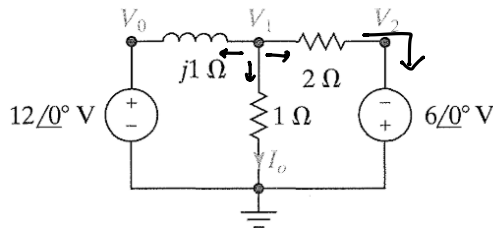


FIGURE D4.8

$$I_0 = V_1$$

$$V_1 = \frac{30}{13} - \frac{84}{13}j$$

$$I_0 = \left( \frac{30}{13} - \frac{84}{13}j \right) \text{ A}$$

at node  $V_1$  KCL  $\swarrow I_0$

$$\frac{V_1 - V_0}{j} + \frac{V_1 - V_2}{2} + \frac{V_1}{1} = 0$$

$$V_0 = 12 \angle 0^\circ \text{ V}$$

$$V_2 = -6 \angle 0^\circ \text{ V}$$

$$-j(V_1 - V_0) + \frac{3}{2}V_1 - \frac{V_2}{2} = 0$$

$$V_1 \left( +\frac{3}{2} - j \right) + V_0 \cdot j - \frac{V_2}{2} = 0$$

$$V_1 \left( +\frac{3}{2} - j \right) + 12 \angle 0^\circ \cdot j + 3 \angle 0^\circ = 0$$

$$V_1 = \frac{12 \angle 0^\circ j + 3 \angle 0^\circ}{\frac{3}{2} - j}$$

# Example

$$V = I \times R$$

**D4.9.** Use mesh equations to find  $V_0$  in the network in Figure D4.9.

$$I_0 = \sqrt{3} + j$$



$$2 \times (\cos 30^\circ + j \sin 30^\circ)$$

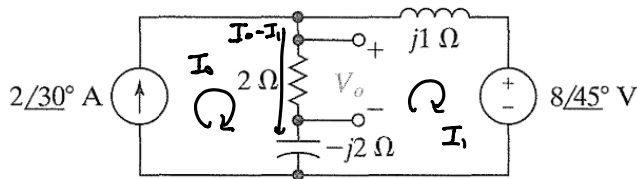


FIGURE D4.9

$$V_0 = 2 \cdot (I_0 - I_1)$$

KVL

$$(I_1 - 2\angle 30^\circ) \cdot (2 - j2) + I_1 \cdot j + 8\angle 45^\circ = 0$$

$$(2 - j)I_1 - 4\angle 30^\circ - 4j\angle 30^\circ + 8\angle 45^\circ = 0$$

$$(2 - j)I_1 = \frac{(4 + 4j)\angle 30^\circ}{2 - j} + \frac{8\angle 45^\circ}{2 - j}$$

$$\underline{V_0 = 0.99 + j1.99 \text{ V}}$$

# Example

**D4.10.** Using superposition, find  $V_0$  in the network in Figure D4.10.

KCL at  $V_1$

$$\frac{V_1 - 24\angle 0^\circ}{2} + \frac{V_1 - V_0}{-2j} + \frac{V_1 + 12\angle 0^\circ}{2j} = 0$$

$$\Rightarrow \underline{V_0 + V_1 j = (-12 + 24j)\angle 0^\circ}$$

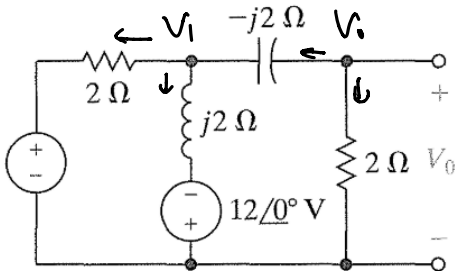


FIGURE D4.10

Matrix

$$\begin{bmatrix} 1 & j \\ j-1 & 1 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} = \begin{bmatrix} -12+24j \\ 0 \end{bmatrix}$$

KCL at  $V_0$

$$\frac{V_0}{2} + \frac{V_0 - V_1}{-2j} = 0$$

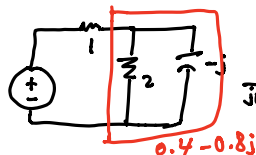
$$\underline{V_0(j-1) + V_1 = 0}$$

$$\boxed{V_0 = 12\angle 90^\circ \text{ V}}$$

# Example

$$\bar{i}_1(t) = \frac{10}{3}$$

$$8 \cdot e^{j0^\circ}$$



$$\frac{1}{j\omega C} = \frac{1}{j \cdot 2 \cdot \frac{1}{2}} = -j$$

total imp  
 $\bar{Z} = 1.4 - 0.8j$

$$\frac{1}{\bar{Z}} = \frac{1}{1.4 - 0.8j} \quad \bar{Z} = \frac{-2j(2+j)}{(2-j)(2+j)} = 0.4 - 0.8j$$

D4.14. Determine the expression for the steady-state current  $i(t)$  in Figure D4.14 if the input voltage  $v_s(t)$  is given by the expression

$$v_s(t) = 10 + 8 \cos(2t) + 6 \cos(4t - 60^\circ) + 4 \cos(6t - 45^\circ) \text{ V}$$

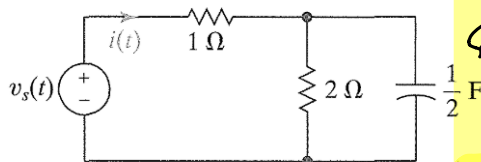
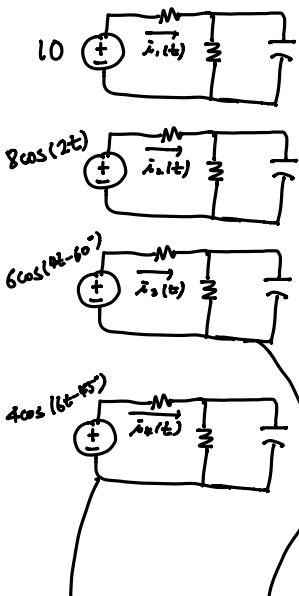


FIGURE D4.14

$$\bar{i}(t) = \bar{i}_1(t) + \bar{i}_2(t) + \bar{i}_3(t) + \bar{i}_4(t)$$

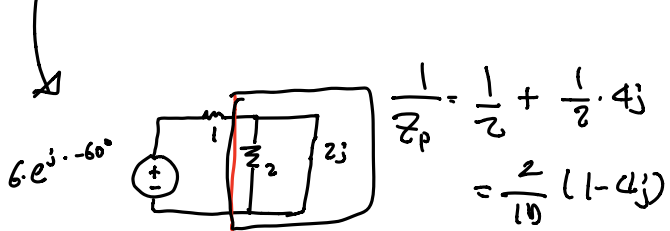
$$= \frac{10}{3} + 4.96 \cos(2t + 29.7^\circ) + 4.95 \cos(4t - 37.17^\circ) + 3.67 \cos(6t - 29.7^\circ)$$

$$\frac{8}{\bar{Z}} = 4.3077 + 2.4615j$$

$$\omega = 2 \quad \bar{i}_2(t) = 4.96 \cos(2t + 29.7^\circ)$$

$$\angle \bar{I} = \text{angle}(\bar{I}) \times \frac{180}{\pi} = 29.7^\circ$$

$$|\bar{I}| = \text{abs}(\bar{I}) = 4.96$$

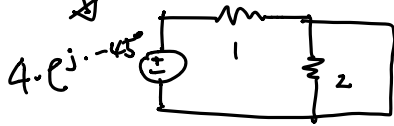


$$j\omega C = j \times 4 \times \frac{1}{2}$$

$$Z_p = \frac{10}{10} - \frac{8}{10}j$$

$$II = \frac{6e^{-j60^\circ}}{Z_p}$$

$$= \underline{\bar{x}_3(t) = 4.95 \cos(4t - 37.17^\circ)}$$



$$j\omega C = j \times 6 \times \frac{1}{2} = 3j$$

$$\frac{1}{Z_p} = \frac{1}{2} + 3j = \frac{2}{1+6j}$$

$$\Rightarrow II = \frac{4e^{j \cdot -45^\circ}}{Z_p} = \underline{\bar{x}_4(t) = 3.69 \cos(6t - 27.90^\circ)}$$