Example

D7.1. Determine the transfer function $\frac{\mathbf{V}_0}{\mathbf{V}_1}(j\omega)$ for the circuit in Figure D7.1 and locate the poles and zeros of the function.

$$\frac{V_o}{V_i}(jw) = \frac{4}{(jw)^2+5jw+4}$$

① no zeroes

② poles: $jw=-1, -4$

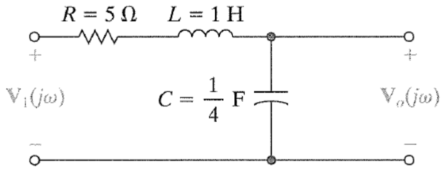


FIGURE D7.1

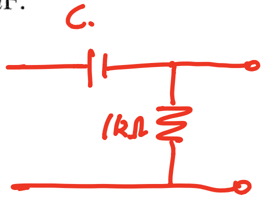
Ans: $\frac{\mathbf{V}_0}{\mathbf{V}_1}(j\omega) = \frac{4}{(j\omega)^2 + 5j\omega + 4}$, there are no zeros and the pole locations are $j\omega = -1$ and $j\omega = -4$.

Example

D7.2. Design a simple *RC* high-pass filter with a break frequency of 20 kHz. Use a resistor value of 1 kΩ.

Ans: $C = 8\,n\text{F}$.

$\frac{1}{RC}$



$$\omega_{3dB} = 2\pi \cdot 20kHz$$

$$C = \frac{1}{2\pi \cdot 20kHz \cdot 1k\Omega}$$

$$= \frac{1}{40\pi \cdot 10^6} = 8nF$$

Example

D7.3. Determine the half-power frequency for each of the filters in Figure D7.3 and identify the type of filter.

(a) Low Pass Filter.

$$\omega_{HI} = \frac{1}{\tau} = \frac{1}{RC} = \frac{1}{10 \times 20 \times 10^{-6}}$$

$$= 5 \times 10^{3} \text{ rad/s}$$

(b) High Pass Filter

$$\omega_{LO} = \frac{1}{\tau} = \frac{1}{RC} = \frac{1}{25 \times 10^{3} \times 20 \times 10^{-6}}$$

$$= 2 \text{ rad/s}$$

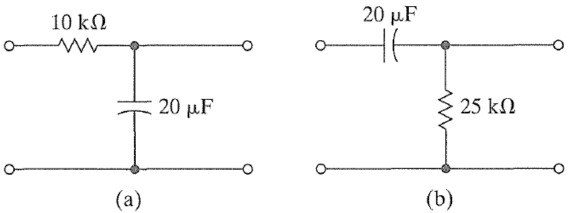


FIGURE D7.3

Ans: (a) Low-pass filter, $\omega_{HI} = 5$ rad/s; (b) high-pass filter $\omega_{LO} = 2$ rad/s.

Example

D7.4. Given the network in Figure D7.4, find the value of C that will place the circuit in resonance at 1800 rad/s.

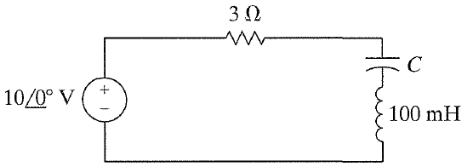


FIGURE D7.4

$W = \frac{1}{\sqrt{LC}} = 1800$ rad/s

$LC = \left(\frac{1}{1800}\right)^2$, $C = \frac{1}{1800^2 \cdot L}$

$= 3.09 \times 10^{-6}$ F

Ans: $C = 3.09\ \mu F$.

D7.5. Given the network in D7.4, determine the Q of the network and the magnitude of the voltage across the capacitor.

6x3x100

Ans: $Q = 60$, $|\mathbf{V}_c| = 600$ V.

$Q = 1800 \times \frac{L}{R} = 1800 \times \frac{100}{3} \times \frac{1}{1000}$

$= \boxed{60}$

$|V_c| = 10 \times \frac{j\omega L}{3 + j\omega L + \frac{1}{j\omega C}}$

$= 600j = 600\angle 90^\circ$ $= \boxed{600\text{V}}$

Example

$BW = \frac{R}{L} = \frac{2\Omega}{40mH} = \frac{1000}{20} = 50\ rad/s$

$W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{40\times10^{-3}\times100\times10^{-6}}} = 500\ rad/s$

D7.6. A series *RLC* circuit is composed of $R = 2\ \Omega$, $L = 40$ mH, and $C = 100\ \mu$F. Determine the bandwidth of this circuit about its resonant frequency.

Ans: $BW = 50$ rad/s, $\omega_0 = 500$ rad/s.

D7.7. A series *RLC* circuit has the following properties: $R = 4\ \Omega$, $\omega_0 = 4000$ rad/s, and $BW = 100$ rad/s. Determine the values of L and C.

Ans: $L = 40$ mH, $C = 1.56\ \mu$F.

$BW = \frac{R}{L} = 100$

$W_0 = \frac{1}{\sqrt{LC}} = 4000$

$L = \frac{4}{100} = 0.04H$

$= 40\ mH.$

$\frac{1}{LC} = 4^2\times10^6$

$C = \frac{1}{4^2\times10^6\times40} = 1.56\ uF$

Example

$$W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 150}} = \frac{1}{\sqrt{3uf}} = 577 \text{ rad/s}$$

$$BW = \frac{1}{RC} = \frac{1}{2\times10^3 \times 150 \times 10^{-6}} \approx 3.3 \text{ rad/s}$$

$$Q = \frac{W_0}{BW} = \frac{300m}{\sqrt{3uF}} \approx 173$$

D7.8. A parallel *RLC* circuit has the following parameters: $R = 2\ \text{k}\Omega$, $L = 20$ mH, and $C = 150\ \mu\text{F}$. Determine the resonant frequency, the Q, and the bandwidth of the circuit.

Ans: $\omega_0 = 577$ rad/s, $BW = 3.33$ rad/s, $Q = 173$.

D7.9. A parallel *RLC* circuit has the following parameters: $R = 6\ \text{k}\Omega$, $BW = 1000$ rad/s, and $Q = 120$. Determine the values of L, C, and ω_0.

Ans: $C = 0.167\ \mu\text{F}$, $L = 416.7\ \mu\text{H}$, $\omega_0 = 119760$ rad/s.

$$\frac{W_0}{BW} = 120 \qquad W_0 = 120 \times 1000 = 120000$$

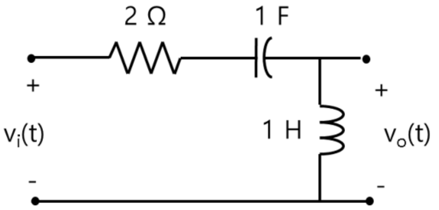
$$C = \frac{1}{600k} = 0.167 \times \frac{1}{10^6} = 0.167\,uF.$$

$$L = \frac{1}{C \cdot W_0^2} = \frac{6 \cdot 10^6}{12^2 \cdot 10^8} \approx 416.7\,uH$$

Example

- Ex1

(10 points) In the following circuit, the input signal is $v_i(t)$ and the output signal is $v_o(t)$.



$$FR = \frac{j\omega}{2 + j\omega + \frac{1}{j\omega}} = \frac{(j\omega)^2}{(j\omega)^2 + 2j\omega + 1}$$

(a) (4 points) Find the frequency response of $\mathbf{V}_o(j\omega)/\mathbf{V}_i(j\omega)$.

(b) (4 points) Find the magnitude and phase of the frequency response from (a).

(c) (2 points) Find the magnitude of the frequency response at ω = 0 rad/s, ω = 1 rad/s, and ω = ∞ rad/s.

(b) magnitude $\dfrac{w^2}{\sqrt{(w^2-1)^2 + 4w^2}}$, phase $= -\tan^{-1}\left(\dfrac{2w}{w^2-1}\right)$

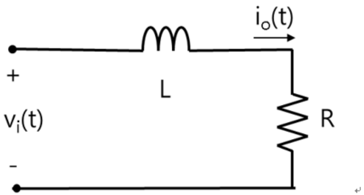
(c) $w = 0$, $|FR| = 0$, phase $= 0$ $w = 1$, $|FR| = \frac{1}{2}$ phase $= 90°$

$w = \infty$, $|FR| = 1$, phase $= 0$

Example

- Ex2

(10 points) Consider the following circuit. The input signal is $v_i(t)$ and the output signal is $i_o(t)$.



$FR = \frac{1}{R+j\omega L}$

(a) (4 points) Find the frequency response, $\mathbf{I}_o(j\omega)/\mathbf{V}_i(j\omega)$.

(b) (4 points) 3-dB bandwidth in the unit of Hz.

(c) (2 points) Determine the type of the filter. (low-pass filter, high-pass filter, bandpass filter, or band-rejection filter)

(b) $\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ = Magnitude $\frac{R}{2\pi L}$ Hz

$2R^2 = R^2 + (\omega L)^2$

$w = \frac{R}{c}$

(c) Low Pass filter