Midterm

Apr. 26, 2021

Time: 10:30 ~ 11:50

Name ____________________

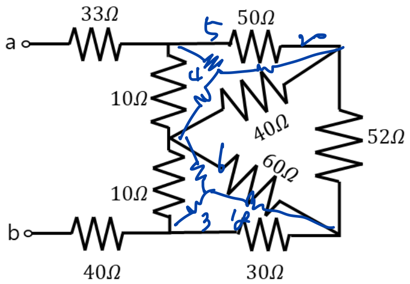
Student ID ____________________

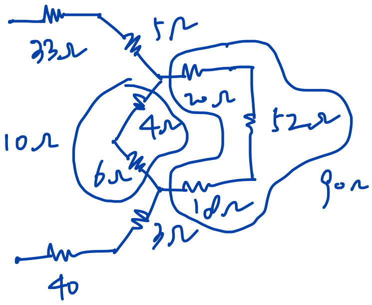
Signature ____________________

- **If there is no answer, you can get only partial credit for your work.**
- **Don't forget the units of your answers**

1. (15 points)

(a) (5 points) Find the resistance between terminals a-b in the following circuit.



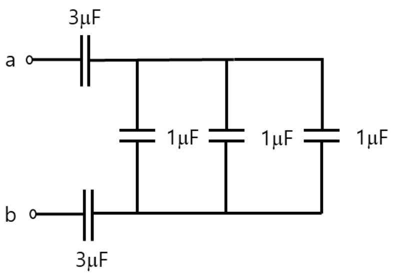


$$\frac{1}{R_p} = \frac{1}{10} + \frac{1}{90} = \frac{9+1}{90} = \frac{1}{9}$$

$$R_{eq} = 33+5+9+3+40.$$

Ans: 90Ω

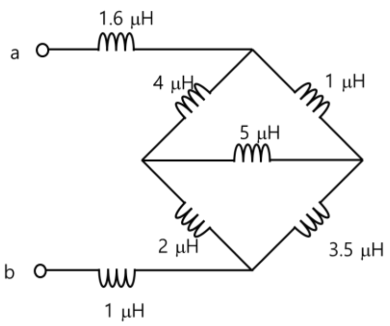
(b) (5 points) Find the capacitance between terminals a-b in the following circuit.

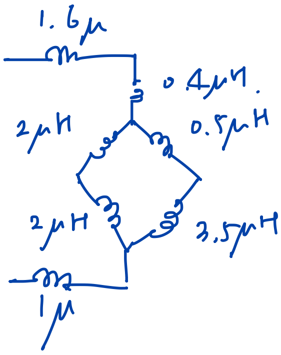


$$\frac{1}{C} = \frac{1}{3\mu} + \frac{1}{3\mu} + \frac{1}{3\mu} = \frac{1}{1\mu}$$

Ans: 1μF

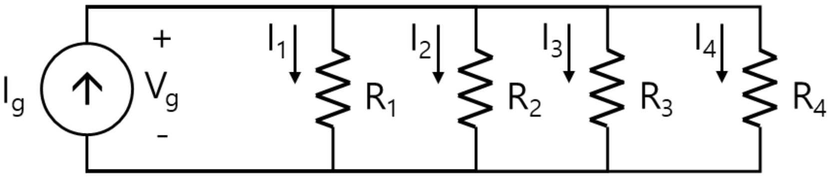
(c) (5 points) Find the inductance between terminals a-b in the following circuit.





Ans: 5μH.

2. (10 points) When I_g=8mA, V_g=1V, $I_1=2*I_2$, $I_2=10*I_3$, and $I_3=I_4$, find R_1, R_2, R_3, and R_4 in the following circuits.



$32 I_4 = 8mA$. ∴ $I_4 = 0.25mA$.

$\frac{1}{4k} = 0.25mA$

$\frac{1}{4k} = 0.25mA$

$\frac{1}{400} = 2.5mA$

$\frac{1}{200} = 5mA$.

∴ $R_4 = \frac{1}{0.25m} = \frac{1000}{0.25} = 4k\Omega$

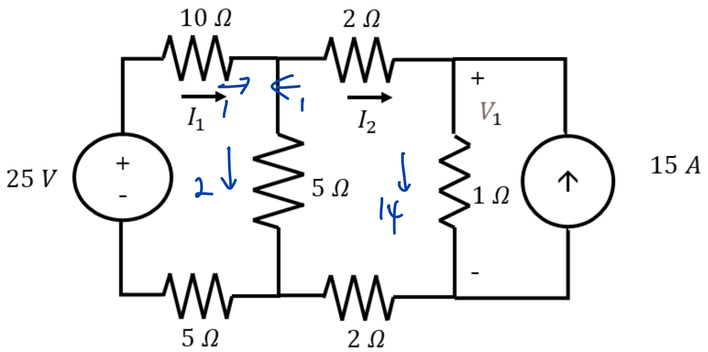
$R_3 = 4k\Omega$

$R_2 = \frac{1}{2.5m} = \frac{1000}{2.5} = 400\Omega$

$R_1 = \frac{1}{5m} = \frac{1000}{5} = 200\Omega$.

Ans: R_1= 200Ω, R_2= 400Ω, R_3= 4kΩ, R_4= 4kΩ

3. (10 points) Find V_1, I_1, and I_2.



$10I_1 + 5(I_1 - I_2) + 5I_1 = 25$.

$2I_2 + (I_2 + 15) + 2I_2 + 5(I_2 - I_1) = 0$.

$20I_1 - 5I_2 = 25$.

$-5I_1 + 10I_2 = -15$.

$+ \quad 40I_1 - 10I_1 = 50$.

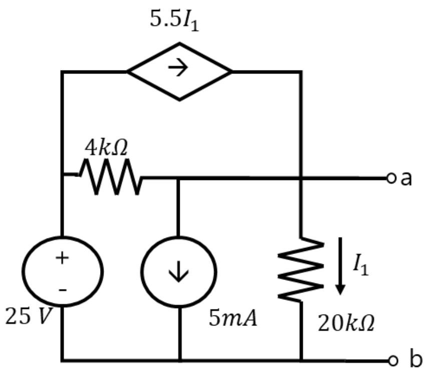
$35I_1 = 35$

∴ $I_1 = 1A$

$I_2 = -1A$

Ans: V_1= 14 V, I_1= 1A, I_2= -1A

4. (15 points) Find the Thevenin equivalent circuit between terminal a-b of the following circuit.



For Voc.

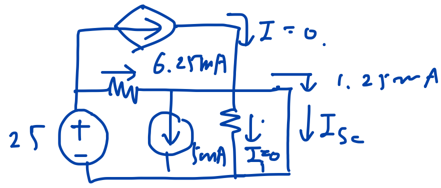
$$\frac{V_{oc}-25}{4k} + 5m + \frac{V_{oc}}{20k} - 5.5\frac{V_{oc}}{20k} = 0.$$

$$5V_{oc} - 125 + 100 + V_{oc} - 5.5V_{oc} = 0$$

$$0.5V_{oc} = 25$$

$$\therefore V_{oc} = 50V.$$

For Isc

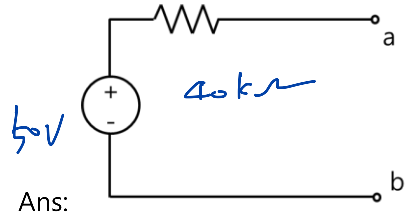


$$I_{sc} = 1.25mA.$$

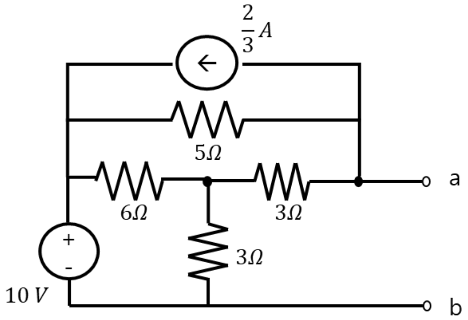
$$I_{sc} = \frac{25}{4k} - 5m = 6.25m - 5m = 1.25mA$$

$$R_{Th} = \frac{50}{1.25m} = 40k\Omega$$

Ans:



5. (10 points) Find the Norton equivalent circuit between terminal a-b of the following circuit.



For Voc

$$\frac{V_1-10}{6} + \frac{V_1}{3} + \frac{V_1-V_{oc}}{3} = 0.$$

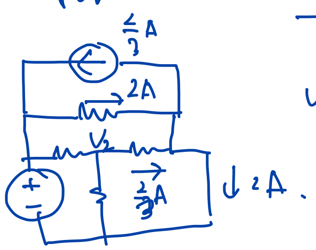
$$\frac{V_{oc}-V_1}{3} + \frac{V_{oc}-10}{5} + \frac{2}{3} = 0.$$

$$V_1 - 10 + 2V_1 + 2V_1 - 2V_{oc} = 0 \rightarrow 5V_1 - 2V_{oc} = 10$$

$$5V_{oc} - 5V_1 + 3V_{oc} - 30 + 10 = 0 \qquad -5V_1 + 8V_{oc} = 20$$

$$\therefore V_{oc} = 5V.$$

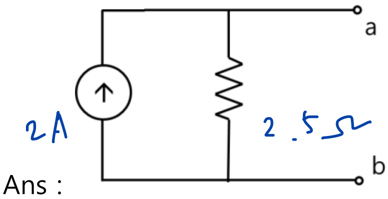
For Isc



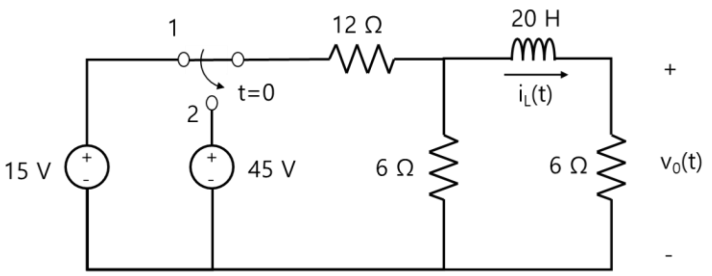
$$\frac{V_2-10}{6} + \frac{V_2}{3} + \frac{V_2}{3} = 0$$

$$V_2 - 10 + 2V_2 + 2V_2 = 0 \qquad \therefore V_2 = 2V.$$

Ans :



6. (10 points) In the following circuit, the switch is moved from terminal 1 to 2 at t=0.



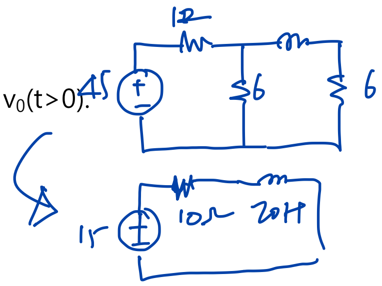
(a) (3 points) Find $i_L(t<0)$.

$R = 12 + 3 = 15\,\Omega$

$i = 1A$. $\quad i_L(t<0) = \frac{1}{2}\cdot i = 0.5A$

Ans: $i_L(t<0)=$ 0.5A

(b) (7 points) Find $v_0(t>0)$.

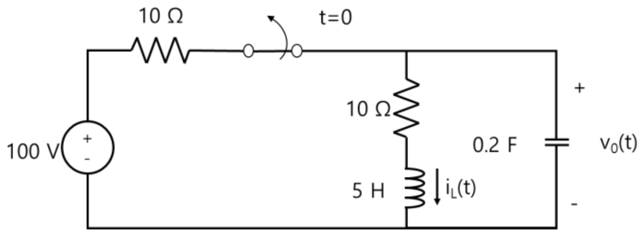


$i_L(t>0) = K_1 + K_2 e^{-t/2}$

$= 1.5 - e^{-t/2}$.

Ans: $v_0(t>0)=$ $(9 - 6e^{-t/2})$ V

7. (15 points) In the following circuit, the switch is open at t=0.



(a) (3 points) Find $i_L(t<0)$ and $v_0(t<0)$.

Ans: $i_L(t<0)=$ 5A., $v_0(t<0)=$ 50V

(b) (8 points) Find $i_L(t>0)$.

$5\frac{di_L}{dt} + 10\cdot i_L + \frac{1}{0.2}\int_{-\infty}^{t} i_L(\tau)d\tau = 0.$

$\frac{d^2 i_L}{dt} + 2 i_L + i_L = 0. \qquad s^2+2s+1 = 0 \quad \therefore s = -1.$

$i_L(t) = K_1 e^{-t} + K_2 t e^{-t}. \qquad i_L(t=0) = 5A. \quad \therefore K_1 = 5.$

$v_o(t) = 10\cdot i_L + 5\cdot\frac{di_L}{dt} = 10(K_1 e^{-t} + K_2 t e^{-t}) + 5(-K_1 e^{-t} + K_2 e^{-t} - K_2 t e^{-t})$

$v_o(t=0) = 10K_1 + 5(-K_1 + K_2) = 50$

$\therefore K_1 = K_2$

Ans: $i_L(t>0)=$ $(5e^{-t} + 5te^{-t})$ A.

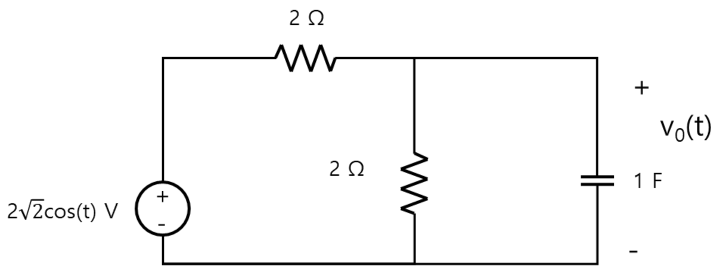
(c) (4 points) Find $v_o(t>0)$.

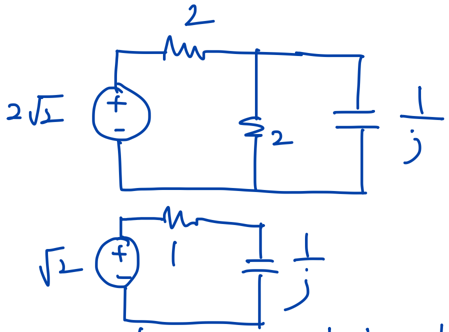
$$v_o(t) = 50e^{-t} + 50te^{-t} + 5\left(-\cancel{5}e^{-t} + 5\cancel{e}^{-t} - 5te^{-t}\right)$$
$$= 50e^{-t} + 25te^{-t}$$

Ans: $v_0(t>0)$= $\left(50e^{-t} + 25te^{-t}\right)$ V

8. (15 points)

(a) (10 points) In the following circuit, we want to find the voltage in steady state using phasor. Find the phasor of $v_o(t)$ and $v_o(t)$. ($\frac{1+j}{\sqrt{2}} = e^{j45°}, \frac{1-j}{\sqrt{2}} = e^{-j45°}$)

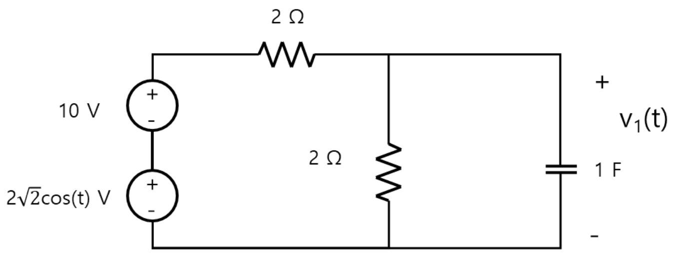




$$\text{phasor of } v_o(t) = \sqrt{2}\cdot\frac{\frac{1}{j}}{1+\frac{1}{j}} = \sqrt{2}\cdot\frac{1}{1+j} = \sqrt{2}\cdot\frac{1-j}{2} = \frac{1-j}{\sqrt{2}} = e^{-j45°}.$$

Ans: phasor of $v_0(t)$= $e^{-j45°}$ V, $v_0(t)$= $\cos(t-\pi/4)$ $\cos(t-45°)$ V

(b) (5 points) In the following circuit, we want to find the voltage, $v_1(t)$ in steady state using superposition principle.



Ans: $v_1(t)$= $5 + \cos(t-45°)$ V