Finance Quantitative

Modèlisation du Smile

Le groupe de travail

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In this problem set, you will use the following functions:

GBSPrice: Price of a vanilla option:

$$P = f(\text{PutCall}, S, K, T, r, b, \sigma)$$

where:

PutCall 'c' for a call, 'p' for a put

b cost of carry: ridk free rate r less dividend yield d

r risk-free rate

```
GBSPrice <- function(PutCall, S, K, T, r, b, sigma) {
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))
  d2 <- d1 - sigma*sqrt(T)

  if(PutCall == 'c')
    px <- S*exp((b-r)*T)*pnorm(d1) - K*exp(-r*T)*pnorm(d2)
  else
    px <- K*exp(-r*T)*pnorm(-d2) - S*exp((b-r)*T)*pnorm(-d1)

px
}
```

GBSVega: Vega ($\frac{\partial P}{\partial \sigma}$) of a Vanilla option:

```
GBSVega <- function(PutCall, S, K, T, r, b, sigma) {
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))
  S*exp((b-r)*T) * dnorm(d1)
}
```

Data

The spot is $S = 110$. We observe the following prices for calls and puts on an asset paying a continuous dividend.

Strike	Call	Put
70	38.496	0.017
75	33.656	0.055
80	28.863	0.143
85	24.193	0.336
90	19.722	0.736
95	15.493	1.395
100	11.704	2.499
105	8.529	4.213
110	5.851	6.390
115	3.831	9.279
120	2.372	12.702
125	1.374	16.542
130	0.768	20.823
135	0.414	25.315
140	0.208	29.994
145	0.093	34.765
150	0.049	39.594
155	0.020	44.445
160	0.008	49.309

Questions

Dividend yield and risk-free rate

Using the Call-Put parity, estimate by linear regression the implied risk-free rate (r) and dividend yield (d).

- Using the functions above, write a function that computes the implied volatility of a Vanilla option. The function should have the following signature:

where:

p price of the option

$sigma$ an optional initial value for the volatility

$maxiter$ an optional maximum number of iterations

tol an optional tolerance for the error $|g(\sigma)|$.

- Test the accuracy of your procedure on options that are deep in the money and deep out of the money, and report the results of your tests.
- Compute the implied volatility of the calls and puts in the data set.
- Fit a quadratic function to the call and put implied volatilities (one function for the calls, one for the puts), and plot actual vs. fitted data. Interpret the results.

Breeden-Litzenberger formula

Compute the implied density of S_T using the Breeden-Litzenberger formula. Estimate

$$\frac{\partial^2 f}{\partial K^2}$$

by finite difference. Remember that now σ is a function of strike. Plot the implied distribution and compare to the distribution implicit in the standard Black-Scholes model. Interpret your observations.

Shimko's Model

Compute the implied distribution of S_T using Shimko's model and the quadratic smile function estimated above. Plot this distribution and compare with the result of the Breeden-Litzenberger formula. Interpret your observations.

Pricing a digital call

Recall that a digital call with strike K pays one euro if $S_T \geq K$, and nothing otherwise.

Using the implied density computed above, compute the price of a digital call by numerical integration.

Perform this calculation for strikes ranging from 80 to 140. Compare with the price obtained using a log-normal distribution for S_T. Interpret your observations.